# Solutions to RSPL/1

**1.** It is given that 3 is a zero of $f(x) = 2x^2 - 3x + p$.

$\therefore$ $(x - 3)$ is a factor of $f(x)$.

So, $2(3)^2 - 3(3) + p = 0$

$\Rightarrow$ $18 - 9 + p = 0$

$\Rightarrow$ $p = -9$

Thus, the polynomial is $2x^2 - 3x - 9$.

Now, $2x^2 - 3x - 9 = 2x^2 - 6x + 3x - 9 = 2x(x - 3) + 3(x - 3) = (x - 3)(2x + 3)$

So, zeroes are 3 and $\frac{-3}{2}$

Thus, the other zero is $\frac{-3}{2}$ and the value of $p$ is $-9$.

**2.** We have $27300 = 2 \times 13650$

$= 2 \times 2 \times 6825$

$= 2 \times 2 \times 3 \times 2275$

$= 2 \times 2 \times 3 \times 5 \times 455$

$= 2 \times 2 \times 3 \times 5 \times 5 \times 91$

$= 2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 13$

$= 2^2 \times 3 \times 5^2 \times 7 \times 13$

**3.** AP of two-digit numbers are 14, 21, 28, ..., 98.

Here, $a = 14$

Common difference, $d = 21 - 14 = 7$

Let there are $n$ terms in AP.

$\therefore$ $n$th term $(a_n) = a + (n - 1)d$

$\Rightarrow$ $98 = 14 + (n - 1)7$

$\Rightarrow$ $84 = (n - 1)7$

$\Rightarrow$ $(n - 1) = 12$

$\Rightarrow$ $n = 13$

Hence, two-digit numbers that are divisible by 7 are 13.

**4.** $$\text{Class mark} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

$$= \frac{10 + 25}{2} = \frac{35}{2} = 17.5$$

**5.** Since $x = a$ and $y = b$ is the solution of the pair of equations $x - y = 2$ and $x + y = 4$,

Then $a - b = 2$ ...(*i*)

$a + b = 4$ ...(*ii*)

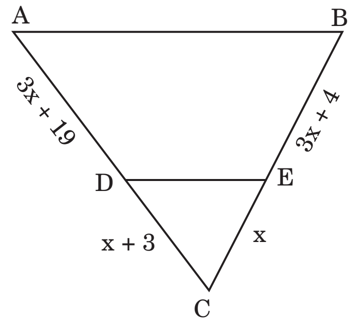
Adding (*i*) and (*ii*), we get

$$2a = 2 + 4 \Rightarrow a = 3$$

Subtracting (*i*) from (*ii*), we get

$$2b = 4 - 2 \Rightarrow 2b = 2 \Rightarrow b = 1$$

**6.** In triangle CAB, if DE divides CA and CB in the same ratio, then DE || AB.




$\therefore \quad \dfrac{CD}{DA} = \dfrac{CE}{EB}$

$\Rightarrow \quad \dfrac{x+3}{3x+19} = \dfrac{x}{3x+4}$

$\Rightarrow \quad (x + 3)(3x + 4) = x(3x + 19)$

$\Rightarrow \quad 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

$\Rightarrow \quad 19x - 4x - 9x = 12$

$\Rightarrow \quad 6x = 12$

$\Rightarrow \quad x = 2$

**7.** We have $\quad 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5(7 \times 6 \times 4 \times 3 \times 2 + 1)$

$= 5(42 \times 24 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

Hence, 5 is a factor of given number and it has more than 2 factors, so it is a composite number.

**8.** Given $\quad \cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \quad \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \quad \sin\theta = \cos\theta(\sqrt{2} - 1)$

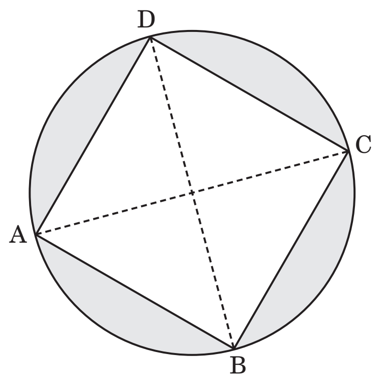
$\Rightarrow \quad \sin\theta = \left(\dfrac{\sqrt{2}-1}{\sqrt{2}+1} \times \sqrt{2}+1\right)\cos\theta$

$\Rightarrow \quad (\sqrt{2}+1)\sin\theta = \left[(\sqrt{2})^2 - (1)^2\right]\cos\theta$

$\Rightarrow \quad \sqrt{2}\sin\theta + \sin\theta = (2 - 1)\cos\theta$

$\Rightarrow \quad \sqrt{2}\sin\theta = \cos\theta - \sin\theta$ Hence proved.

**9.** It is given that ABCD is a square of side 6 cm.




Diagonal AC or BD of a square $= \sqrt{6^2 + 6^2}$ cm

$= \sqrt{36+36}$

$= \sqrt{72}$ cm

$= 6\sqrt{2}$ cm

$\therefore \quad$ Diameter of a circle $= 6\sqrt{2}$ cm

$\Rightarrow \quad$ Radius $= 3\sqrt{2}$ cm

$\therefore$ Area of shaded region $=$ Area of a circle of radius $3\sqrt{2}$ cm – Area of a square ABCD

$$= [\pi(3\sqrt{2})^2 - (6)^2] \text{ cm}^2$$

$$= \left(\frac{22}{7} \times 18 - 36\right) \text{cm}^2 = \left(\frac{396 - 252}{7}\right) \text{cm}^2$$

$$= \frac{144}{7} \text{ cm}^2 = 20.57 \text{ cm}^2$$

**10.**

| **Monthly consumption (in units)** | **Number of consumers (frequency) (*f*)** | **Monthly consumption less than (in units)** | **Cumulative frequency (*cf*)** |
|---|---|---|---|
| 65 – 85 | 4 | 85 | 4 |
| 85 – 105 | 5 | 105 | 9 |
| 105 – 125 | 13 | 125 | 22 |
| 125 – 145 | 20 | 145 | 42 |
| 145 – 165 | 14 | 165 | 56 |
| 165 – 185 | 8 | 185 | 64 |
| 185 – 205 | 4 | 205 | 68 |

**11.** If $\alpha$ and $\beta$ are zeroes of $p(x) = 2x^2 - x - 6$, so

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{2} = \frac{1}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{-6}{2} = -3$$

Now, $$\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{1}{2}}{-3} = \frac{-1}{6}$$

**12.** We have $$3x^2 - 2\sqrt{6}x + 2 = 0$$

$$\Rightarrow \quad 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$\Rightarrow \quad (3x^2 - \sqrt{6}x) + (-\sqrt{6}x + 2) = 0$$

$$\Rightarrow \quad \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow \quad (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow \quad x = \sqrt{\frac{2}{3}} \text{ or } x = \sqrt{\frac{2}{3}}$$

Therefore, the roots of $3x^2 - 2\sqrt{6}x + 2 = 0$ are $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

**13.** Let $p(x)$ be the given polynomial, then

$$p(x) = 9x^4 - 6x^3 - 35x^2 + 24x - 4$$

Since zeroes of a polynomial p(x) are 2 and – 2, therefore

$$(x - 2)(x + 2) = x^2 - 4$$

$\Rightarrow x^2 - 4$ is a factor of $p(x) = 9x^4 - 6x^3 - 35x^2 + 24x - 4$

Now, we divide the given polynomial by $x^2 - 4$.

$$
\begin{array}{r|l}
 & 9x^2 - 6x + 1 \\
\hline
x^2 - 4 & 9x^4 - 6x^3 - 35x^2 + 24x - 4 \\
 & 9x^4 \qquad - 36x^2 \\
 & - \qquad\quad + \\
\hline
 & - 6x^3 + x^2 + 24x - 4 \\
 & - 6x^3 \qquad + 24x \\
 & + \qquad\quad - \\
\hline
 & x^2 \qquad - 4 \\
 & x^2 \qquad - 4 \\
 & - \qquad\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

[First term of quotient is $\frac{9x^4}{x^2} = 9x^2$]

[Second term of quotient is $\frac{-6x^3}{x^2} = -6x$]

[Third term of quotient is $\frac{x^2}{x^2} = 1$]

So, $\quad 9x^4 - 6x^3 - 35x^2 + 24x - 4 = (x^2 - 4)(9x^2 - 6x + 1)$

$$= (x - 2)(x + 2)[9x^2 - 3x - 3x + 1]$$

$$= (x - 2)(x + 2)[3x(3x - 1) - 1(3x - 1)]$$

$$= (x - 2)(x + 2)(3x - 1)(3x - 1)$$

Hence, all the zeroes of the given polynomial $9x^4 - 6x^3 - 35x^2 + 24x - 4$ are $2, -2, \frac{1}{3}$ and $\frac{1}{3}$.

**14.** Let N be any positive integer.

$\therefore$ N = $4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$

(*i*) When N = $4q = 2(2q)$ = even

(*ii*) When N = $4q + 1 = 2(2q) + 1$ = even + 1 = odd

(*iii*) When N = $4q + 2 = 2(2q + 1)$ = even

(*iv*) When N = $4q + 3 = 4q + 2 + 1 = 2(2q + 1) + 1$ = even + 1 = odd

$\therefore$ When N = $4q + 1$ or $4q + 3$, then it is odd.

$\Rightarrow$ Any positive odd integer is of the form $4q + 1$ or $4q + 3$, where $q$ is some integer.

**15.**

$$\text{LHS} = \frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\operatorname{cosec} A + \cot A - 1}$$

$$= \frac{\sin A \operatorname{cosec} A + \sin A \cot A - \sin A + \cos A \sec A + \cos A \tan A - \cos A}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{\frac{\sin A}{\sin A} + \sin A \frac{\cos A}{\sin A} - \sin A + \frac{\cos A}{\cos A} + \cos A \frac{\sin A}{\cos A} - \cos A}{(\sec A + \tan A - 1)(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{1 + \cos A - \sin A + 1 + \sin A - \cos A}{\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1\right)\left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1\right)}$$

$$= \frac{2}{\left(\frac{1 + \sin A - \cos A}{\cos A}\right)\frac{(1 + \cos A - \sin A)}{\sin A}}$$

$$= \frac{2 \sin A \cos A}{[1 + (\sin A - \cos A)]\,[1 - (\sin A - \cos A)]}$$

$$= \frac{2 \sin A \cos A}{1 - (\sin A - \cos A)^2} = \frac{2 \sin A \cos A}{1 - (\sin^2 A + \cos^2 A - 2 \sin A \cos A)}$$

$$= \frac{2\sin A\cos A}{1-(1-2\sin A\cos A)} = \frac{2\sin A\cos A}{1-1+2\sin A\cos A}$$

$$= \frac{2\sin A\cos A}{2\sin A\cos A} = 1 = \text{RHS}$$ Hence proved.

**16.** $\sin(50° + \theta) - \cos(40° - \theta) + \tan 1° \tan 10° \tan 20° \tan 70° \tan 80° \tan 89°$

$= \sin(90° - 40° + \theta) - \cos(40° - \theta) + \tan 1° \tan 10° \tan 20° \tan(90° - 20°) \tan(90° - 10°) \tan(90° - 1°)$

$= \sin[(90° - (40° - \theta))] - \cos(40° - \theta) + \tan 1° \tan 10° \tan 20° \cot 20° \cot 10° \cot 1°$

[$\because \tan(90° - \theta) = \cot\theta$]

$$= \cos(40° - \theta) - \cos(40° - \theta) + \tan 1° \tan 10° \tan 20° \frac{1}{\tan 20°} \cdot \frac{1}{\tan 10°} \cdot \frac{1}{\tan 1°}$$

$= 0 + 1.1.1 = 1$ [$\because \tan\theta.\cot\theta = 1, \sin(90° - \theta) = \cos\theta$]

**OR**

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{(\sin\theta/\cos\theta)}{(\sin\theta-\cos\theta)/\sin\theta} + \frac{\cos\theta/\sin\theta}{(\cos\theta-\sin\theta)/\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \frac{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$ (Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$)

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + 1 = 1 + \sec\theta \operatorname{cosec}\theta = \text{RHS}$$ Hence proved.

**17.** Given AP is $\frac{-4}{3}, -1, \frac{-2}{3}, ..., 4\frac{1}{3}$

Here, $a = \frac{-4}{3}$, $d = -1 - \left(\frac{-4}{3}\right) = -1 + \frac{4}{3} = \frac{1}{3}$,

$a_n = n\text{th term of AP} = 4\frac{1}{3} = \frac{13}{3}$

$\therefore \quad \frac{13}{3} = a_n = a + (n-1)d$

$\Rightarrow \quad \frac{13}{3} = \frac{-4}{3} + (n-1)\left(\frac{1}{3}\right)$

$$\Rightarrow \quad \frac{13}{3} = \frac{-4}{3} + \frac{(n-1)}{3}$$

$$\Rightarrow \quad \frac{13}{3} = \frac{n-5}{3}$$

$$\Rightarrow \quad 13 = n - 5$$

$$\Rightarrow \quad n = 18$$

Since $n = 18$, therefore, there are two middle terms of the AP, i.e.

$\left(\frac{18}{2}\right)$th term and $\left(\frac{18}{2}+1\right)$th term

i.e. 9th term and 10th term, i.e. $a_9$ and $a_{10}$

$\therefore$ Sum of the two middle terms of AP $= a_9 + a_{10}$

$$= a + 8d + a + 9d$$

$$= 2a + 17d$$

$$= 2 \times \left(\frac{-4}{3}\right) + 17 \times \frac{1}{3}$$

$$= \frac{-8}{3} + \frac{17}{3} = \frac{9}{3} = 3$$

**OR**

Let $a$ be the first term and $d$ be common difference of an AP $a, a + d, a + 2d$ ...

Then $a_{12}$ = 12th term of an AP $= a + (12 - 1)d$

$$\Rightarrow \quad -13 = a + 11d \qquad ...(i) \; [\because a_{12} = -13 \text{ (given)}]$$

and $S_4$ = Sum of first four terms = 24 (Given)

$$\Rightarrow \quad 24 = \frac{4}{2}[2a + (4-1)d]$$

$$\Rightarrow \quad 24 = 2[2a + 3d]$$

$$\Rightarrow \quad 12 = 2a + 3d \qquad ...(ii)$$

$$\Rightarrow \quad 12 = 2 \times (-13 - 11d) + 3d \qquad \text{(Using } (i))$$

$$\Rightarrow \quad 12 = -26 - 22d + 3d$$

$$\Rightarrow \quad -19d = 38$$

$$\Rightarrow \quad d = -2$$

Substituting $d = -2$ in $(i)$, we get

$$a = -13 - 11 \times (-2)$$

$$a = -13 + 22 = 9$$

$$\therefore \quad S_{10} = \text{Sum of first 10 terms}$$

$$= \frac{10}{2}[2a + (10-1)d] = 5 \times [2 \times 9 + 9 \times (-2)]$$

$$= 5 \times [18 - 18]$$

$$= 5 \times 0 = 0$$

**18.** Since the given points A$(-6, 10)$, B$(-4, k)$ and C$(3, -8)$ are collinear, therefore, the area of the triangle formed by them must be zero.

$$\therefore \quad \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0,$$

where $x_1 = -6, y_1 = 10, x_2 = -4, y_2 = k, x_3 = 3, y_3 = -8$

$$\Rightarrow \qquad \frac{1}{2}|(-6)(k+8)+(-4)(-8-10)+3(10-k)| = 0$$

$$\Rightarrow \qquad \frac{1}{2}|-6k-48+72+30-3k| = 0$$

$$\Rightarrow \qquad \frac{1}{2}|-9k+54| = 0$$

$$\Rightarrow \qquad -9k+54 = 0$$

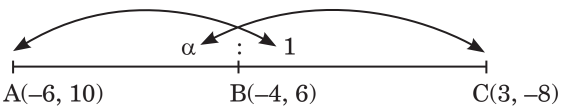
$$\Rightarrow \qquad k = \frac{54}{9} = 6$$

Hence, the value of $k = 6$ and the coordinates of B are $(-4, 6)$.

Let the point B$(-4, 6)$ divides the line segment joining the points A $(-6, 10)$ and C$(3, -8)$ in the ratio $\alpha : 1$

Then the coordinates of B are

$$\left(\frac{3\alpha-6}{\alpha+1}, \frac{-8\alpha+10}{\alpha+1}\right)$$




But the coordinates of B are $(-4, 6)$

$$\Rightarrow \frac{3\alpha-6}{\alpha+1} = -4 \text{ and } \frac{-8\alpha+10}{\alpha+1} = 6$$

$\Rightarrow$ $3\alpha - 6 = -4\alpha - 4$ and $-8\alpha + 10 = 6\alpha + 6$

$\Rightarrow$ $7\alpha = 2$ and $-14\alpha = -4$

$$\Rightarrow \alpha = \frac{2}{7} \text{ and } \alpha = \frac{4}{14} \Rightarrow \alpha = \frac{2}{7}$$

Thus, B$(-4, 6)$ divide the line segment AC in the ratio $2 : 7$

Now, $\quad$ length AC $= \sqrt{(3+6)^2+(-8-10)^2}$

$$= \sqrt{81+324} = \sqrt{405} = \sqrt{81\times 5} = 9\sqrt{5} \text{ units.}$$

**19. Steps of construction:**

(*i*) Draw a line segment AC = 7 cm.

(*ii*) With A as centre and radius = 4 cm, draw an arc.

(*iii*) With C as centre and radius = 5 cm, draw another arc cutting the previous arc at point B.

(*iv*) Join AB and CB to obtain the triangle ABC.

(*v*) Below AC, make an acute angle CAX.

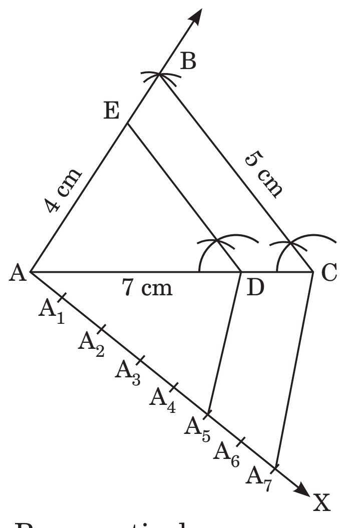
(*vi*) Along AX, mark off 7 points $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$

(*vii*) Join $A_7C$.

(*viii*) From $A_5$, draw $A_5D \,||\, A_7C$ meeting AC at D.

(*ix*) From D, draw DE || CB, meeting AB at E.

Then we have $\Delta$ADE, which is required triangle.




**20.** Let '$a$' and '$d$' be the first term and common difference of an A.P. respectively.

It is given that the ratio of 5th and 3rd term is 5:2

$$\therefore \qquad \frac{a_5}{a_3} = \frac{5}{2}$$

$$\Rightarrow \qquad \frac{a+4d}{a+2d} = 2.5$$

$\Rightarrow \quad a + 4d = 2.5a + 5d$

$\Rightarrow \quad 1.5a = -d$

$\Rightarrow \quad -1.5a = d$

Now, $\quad \frac{a_{15}}{a_7} = \frac{a+14d}{a+6d}$

$\Rightarrow \quad \frac{a_{15}}{a_7} = \frac{a+14(-1.5a)}{a+6(-1.5a)}$

$\Rightarrow \quad \frac{a_{15}}{a_7} = \frac{a-21a}{a-9a}$

$\Rightarrow \quad \frac{a_{15}}{a_7} = \frac{-20a}{-8a} = \frac{5}{2}$

Hence, the ratio of the 15th and 7th term is 5 : 2

**OR**

Let $a$ and $d$ be the first term and common difference of an AP $a, a+d, a+2d, \ldots$ respectively.

It is given that:

14th term of an AP is twice its 8th term

$\therefore \quad t_{14} = 2t_8$

$\Rightarrow \quad a + (14-1)d = 2[a + (8-1)d]$

$\Rightarrow \quad a + 13d = 2a + 14d$

$\Rightarrow \quad 2a - a = -14d + 13d$

$\Rightarrow \quad a = -d \quad \ldots(i)$

Also, it is given that

$\Rightarrow \quad t_6 = -8$

$\Rightarrow \quad a + (6-1)d = -8$

$\Rightarrow \quad -d + 5d = -8 \quad$ (Using $(i)$)

$\Rightarrow \quad 4d = -8$

$\Rightarrow \quad d = -2$

Substituting $d = -2$ in $(i)$, we have

$\Rightarrow \quad a = -d = -(-2) = 2$

$\therefore$ AP is 2, 0, – 2, – 4, ...

$\therefore \quad S_{20} = \frac{20}{2}[2 \times 2 + (20-1)(-2)] = 10[4-38] = 10(-34) = -340$

**21.** A(2, 3), B(– 2, 2), C(– 1, – 2) and D(3, – 1) are the given four points.

$\therefore \quad AB = \sqrt{(-2-2)^2+(2-3)^2}$

$= \sqrt{(-4)^2+(-1)^2} = \sqrt{16+1} = \sqrt{17}$ units

$BC = \sqrt{(-1+2)^2+(-2-2)^2} = \sqrt{(1)^2+(-4)^2} = \sqrt{17}$ units

$CD = \sqrt{(3+1)^2+(-1+2)^2} = \sqrt{(4)^2+(1)^2} = \sqrt{17}$ units

$DA = \sqrt{(2-3)^2+(3+1)^2} = \sqrt{(-1)^2+(4)^2} = \sqrt{17}$ units

Thus, AB = BC = CD = DA = $\sqrt{17}$ units

All four sides are equal in length.

Now, $AC = \sqrt{(-1-2)^2+(-2-3)^2} = \sqrt{(-3)^2+(-5)^2} = \sqrt{9+25} = \sqrt{34}$ units

$BD = \sqrt{(3+2)^2+(-1-2)^2} = \sqrt{(5)^2+(-3)^2} = \sqrt{25+9} = \sqrt{34}$ units

Thus, length of diagonal AC = $\sqrt{34}$ units = length of diagonal BD.

Since all four sides and diagonals of a quadrilateral ABCD are equal,

Hence, the points A(2, 3), B(– 2, 2), C(– 1, – 2) and D(3, – 1) are the vertices of a square ABCD.

**OR**

Since points are collinear, therefore, the area of triangle formed by them must be zero.

$\Rightarrow \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)| = 0$

Here, $x_1 = 1, y_1 = 5, x_2 = p, y_2 = 1, x_3 = 4, y_3 = 11$

$\Rightarrow \frac{1}{2}|1(1-11)+p(11-5)+4(5-1)| = 0$

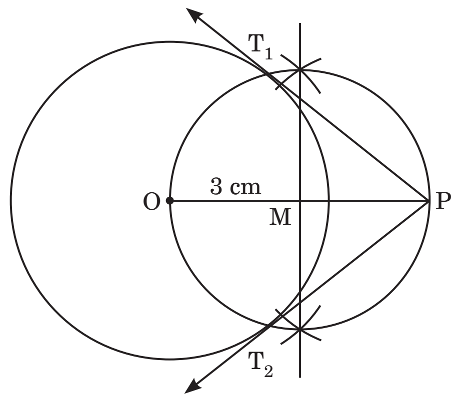
$\Rightarrow \frac{1}{2}|-10+6p+16| = 0$

$\Rightarrow \frac{1}{2}|6+6p| = 0$

$\Rightarrow 6+6p = 0$

$\Rightarrow p = -1$

**22. Steps of construction:**




(*i*) Taking a point O as centre, draw a circle of radius 3 cm.

(*ii*) Take a point P, 7 cm away from its centre O such that OP = 7 cm.

(*iii*) Bisect OP and let M be the mid-point of OP.

(*iv*) Draw a circle with M as centre and OM as radius to intersect the circle at $T_1$ and $T_2$.

(*v*) Join $PT_1$ and $PT_2$. Then, $PT_1$ and $PT_2$ are the required tangents.

∴ Length of each tangent. $PT_1 = PT_2 = 6.32$ cm

**23.**

| **Variable** | **Frequency** | **Cumulative frequency** |
|---|---|---|
| 10 – 20 | 12 | 12 |
| 20 – 30 | 30 | 42 |
| 30 – 40 | $f_1$ | $42 + f_1$ |
| 40 – 50 | $65 = f$ | $107+f_1$ |
| 50 – 60 | $f_2$ | $107 + f_1 + f_2$ |
| 60 – 70 | 25 | $132 + f_1 + f_2$ |
| 70 – 80 | 18 | $150 + f_1 + f_2 = 229$ |

$\therefore \quad 150 + f_1 + f_2 = 229$

$\Rightarrow \quad f_1 + f_2 = 79 \quad \quad ...(i)$

Since the median is given to be 46, the class (40 – 50) is median class.

Therefore, $l = 40$, $cf = 42 + f_1$, $N = 229$, $h = 10$

$$\Rightarrow \quad \text{Median} = l + \frac{(N/2 - cf)}{f} \times h$$

$$\Rightarrow \quad 46 = 40 + \frac{\left(\frac{229}{2} - 42 - f_1\right)}{65} \times 10$$

$$\Rightarrow \quad 6 = \frac{(229 - 84 - 2f_1)}{2 \times 65} \times 10$$

$$\Rightarrow \quad 78 = 229 - 84 - 2f_1$$

$$\Rightarrow \quad 2f_1 = 67 \Rightarrow f_1 = \frac{67}{2} = 33.5 \simeq 34$$

Putting the value of $f_1$ in $(i)$, we have

$$34 + f_2 = 79 \Rightarrow f_2 = 45$$

Hence, $f_1 = 34$ and $f_2 = 45$

**24.** Let the speed of the boat in still water be $x$ km/h

Let the speed of the stream be $y$ km/h

Speed upstream = $(x - y)$ km/h

Speed downstream = $(x + y)$ km/h

Now, time taken to cover 32 km upstream = $\frac{32}{x-y}$ hours

Time taken to cover 36 km downstream = $\frac{36}{x+y}$ hours

The total time journey is 7 hours.

$$\therefore \quad \frac{32}{x-y} + \frac{36}{x+y} = 7 \quad \quad ...(i)$$

Time taken to cover 40 km upstream = $\frac{40}{x-y}$ hours

Time taken to cover 48 km downstream = $\frac{48}{x+y}$ hours

In this case, total time of journey is 9 hours

$$\therefore \quad \frac{40}{x-y} + \frac{48}{x+y} = 9 \quad \quad ...(ii)$$

Put $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in $(i)$ and $(ii)$, we get

$$32u + 36v = 7 \Rightarrow 32u + 36v - 7 = 0$$

$$40u + 48v = 9 \Rightarrow 40u + 48v - 9 = 0$$

By cross-multiplication method, we have

$$\frac{u}{36 \times (-9) - 48 \times (-7)} = \frac{v}{-7 \times 40 - (-9) \times 32} = \frac{1}{32 \times 48 - 40 \times 36}$$

$$\Rightarrow \quad \frac{u}{-324+336} = \frac{v}{-280+288} = \frac{1}{1536-1440}$$

$$\Rightarrow \quad \frac{u}{12} = \frac{v}{8} = \frac{1}{96}$$

$$\Rightarrow \quad \frac{u}{12} = \frac{1}{96}; \frac{v}{8} = \frac{1}{96}$$

$$\Rightarrow \quad u = \frac{1}{8}; v = \frac{1}{12}$$

$$\Rightarrow \quad \frac{1}{x-y} = \frac{1}{8}; \frac{1}{x+y} = \frac{1}{12}$$

$$x - y = 8; x + y = 12$$

Solving these two equations, we have

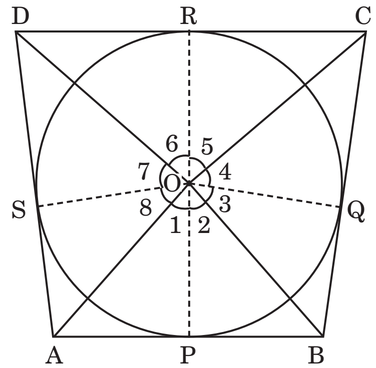
$$x - y = 8$$
$$x + y = 12$$
$$- \; - \quad -$$
$$-2y = -4$$

$$\Rightarrow \quad y = 2$$

and $\quad x = 10$

Hence, the speed of boat in still water is 10 km/h and speed of the stream is 2 km/h.

**25. Given:** Let ABCD be a quadrilateral such that opposite sides AB and CD as well as, sides BC and AD of a quadrilateral, circumscribing a circle at points P, Q, R and S respectively with centre O.




**To prove:** $\angle AOB + \angle COD = 180°$

and $\angle AOD + \angle BOC = 180°$

**Construction:** Join OP, OQ, OR and OS

**Proof:** Since two tangents drawn from an external point to circle subtend equal angles at the centre,

$\angle 8 = \angle 1, \angle 2 = \angle 3, \angle 4 = \angle 5$ and $\angle 6 = \angle 7$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

($\because$ Sum of all angles subtended at a point is 360°)

$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) + (\angle 5 + \angle 6) + (\angle 7 + \angle 8) = 360°$

$\Rightarrow \quad (\angle 1 + \angle 2) + (\angle 2 + \angle 5) + (\angle 5 + \angle 6) + (\angle 6 + \angle 1) = 360°$

$\Rightarrow \quad 2(\angle 1 + \angle 2 + \angle 5 + \angle 6) = 360°$ and

$\Rightarrow \quad 2(\angle 3 + \angle 4 + \angle 7 + \angle 8) = 360°$

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 5 + \angle 6 = 180°$

$\Rightarrow \quad \angle 3 + \angle 4 + \angle 7 + \angle 8 = 180°$

$\Rightarrow \quad (\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$ and $(\angle 3 + \angle 4) + (\angle 7 + \angle 8) = 180°$

$\Rightarrow \quad \angle AOB + \angle COD = 180°$ and $\angle BOC + \angle AOD = 180°$

[$\because \quad \angle 1 + \angle 2 = \angle AOB, \angle 5 + \angle 6 = \angle COD, \angle 3 + \angle 4 = \angle BOC, \angle 7 + \angle 8 = \angle AOD$]

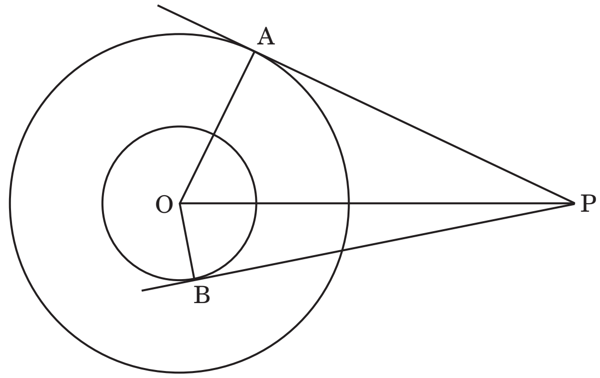
Hence proved.

**OR**

Radius of smaller circle OB = 3 cm

Radius of bigger circle OA = 5 cm

Length of the tangent PA = 12 cm




Since radius OA of a bigger circle is perpendicular to tangent line PA, then OA ⊥ PA.

In right-angled ΔOAP, we have

$\Rightarrow \quad OP^2 = OA^2 + PA^2$

(Using Pythagoras theorem)

$\Rightarrow \quad OP^2 = (5)^2 + (12)^2$

$\Rightarrow \quad OP^2 = 25 + 144$

$\Rightarrow \quad OP^2 = 169 \quad \Rightarrow OP = 13$ cm

Also, OB ⊥ PB, Then ∠OBP = 90°

In right-angled ΔOBP, we have

$$OP^2 = OB^2 + PB^2 \qquad \text{(Using Pythagoras theorem)}$$

$$\Rightarrow \quad (13)^2 = (3)^2 + (PB)^2$$

$$\Rightarrow \quad (PB)^2 = 169 - 9 = 160$$

$$\Rightarrow \quad PB = \sqrt{160}$$

$$\Rightarrow \quad PB = 4\sqrt{10} \text{ cm}$$

∴ Perimeter of quadrilateral PAOB = PA + OA + OB + PB

$$= (12 + 5 + 3 + 4\sqrt{10}) \text{ cm}$$

$$= (20 + 4\sqrt{10}) \text{ cm}$$

**26.** Let A and B be the positions of the two ships.

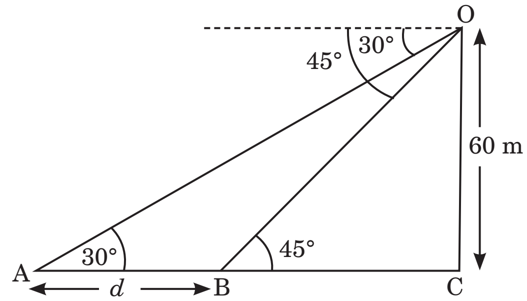
Let d be the distance between the two ships, i.e. AB = *d* metres

Let the observer be at O, the top of the lighthouse CO

It is given that CO = 60 m and the angles of depression from O to A and B are 30° and 45° respectively.

∴ $\angle OAC = 30°$ and $\angle OBC = 45°$

In right-angled ΔOBC, we have




$$\tan 45° = \frac{OC}{BC}$$

$$\Rightarrow \quad 1 = \frac{60}{BC}$$

$$\Rightarrow \quad BC = 60 \text{ m}$$

In right-angled $\Delta$OCA, we have

$$\tan 30° = \frac{OC}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60}{d + BC}$$

$$\Rightarrow \quad d + BC = 60\sqrt{3}$$

$$\Rightarrow \quad d + 60 = 60\sqrt{3}$$

$$\Rightarrow \quad d = 60\sqrt{3} - 60$$

$$\Rightarrow \quad d = 60(\sqrt{3} - 1)$$

$$\Rightarrow \quad d = 60(1.732 - 1)$$

$$\Rightarrow \quad d = 60 \times 0.732 \Rightarrow d = 43.92 \text{ m}$$

Hence, the distance between the two ships = 43.92 m

**OR**

$$\text{LHS} = \left(\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}\right) \cdot \left(\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}\right)$$

$$= \left(\frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)\sin A}\right) \cdot \left(\frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}\right)$$

$$= \frac{(\sin^2 A + 1 + \cos^2 A + 2\cos A) \cdot (\cos^2 A + 1 + \sin^2 A + 2\sin A)}{\sin A\cos A(1+\sin A)(1+\cos A)}$$

$$= \frac{(2+2\cos A)(2+2\sin A)}{\sin A\cos A(1+\sin A)(1+\cos A)}$$

$$= \frac{4(1+\cos A)(1+\sin A)}{\sin A\cos A(1+\cos A)(1+\sin A)}$$

$$= \frac{4}{\sin A\cos A}$$

$$= 4\sec A \operatorname{cosec} A$$

$$= \text{RHS}$$

Hence proved.

**27.** Depth of the bucket ($h$) = 24 cm

Diameter of upper circular end = 30 cm

Radius of upper circular end = R = 15 cm

Diameter of lower circular end = 10 cm

Radius of lower circular end = $r$ = 5 cm

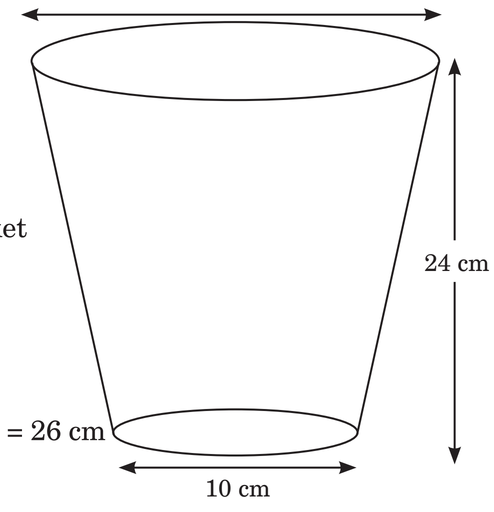
Hence, $l$ = slant height of bucket

$$\Rightarrow \quad l = \sqrt{(R - r)^2 + h^2}$$

$$\Rightarrow \quad l = \sqrt{(15 - 5)^2 + (24)^2}$$

$$\Rightarrow \quad l = \sqrt{(10)^2 + (24)^2}$$

$$\Rightarrow \quad l = \sqrt{100 + 576} = \sqrt{676} = 26 \text{ cm}$$



Total surface area of the frustum $= \pi(r + R)l + \pi r^2$

$= [3.14\,(5 + 15) \times 26 + 3.14 \times (5)^2]\ \text{cm}^2$

$= [3.14 \times 20 \times 26 + 3.14 \times 25]\ \text{cm}^2$

$= 3.14[520 + 25]\ \text{cm}^2$

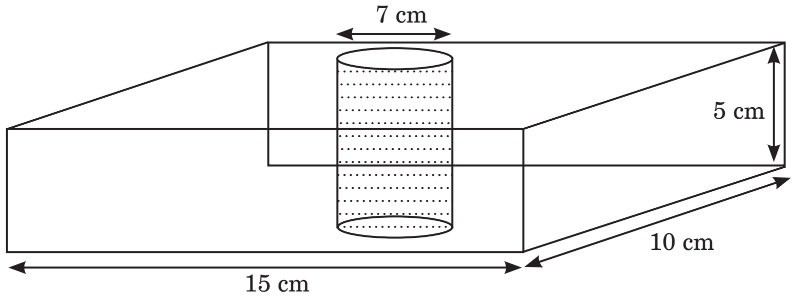
$= 3.14 \times 545 = 1711.3\ \text{cm}^2$

$\therefore$ Cost of the metal sheet used $= ₹\left(\dfrac{1711.3}{100} \times 10\right)$

$= ₹\ \dfrac{17113}{100} = ₹\ 171.13$

**OR**

Given a cuboidal solid metallic block of dimensions




$= (15 \times 10 \times 5)\ \text{cm} = l \times b \times h$

Total surface area of solid metallic cuboidal block

$= 2(lb + bh + hl) - 2\pi r^2$

$= 2[15 \times 10 + 10 \times 5 + 5 \times 15] - 2 \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}$

$= 2[150 + 50 + 75] - 77$

$= 2 \times 275 - 77 = 550 - 77 = 473\ \text{cm}^2$

Diameter of cylindrical hole = 7 cm

Radius of cylindrical hole $= r = \dfrac{7}{2}$ cm

Total surface area of cylindrical hole of radius $\dfrac{7}{2}$ cm = Curved surface area of a cylinder

$= 2\pi rh\ \text{cm}^2$

$= 2\pi \times \dfrac{7}{2} \times 5\ \text{cm}^2$ [$h$ = 5 cm = height of cylinder]

$= \pi \times 35\ \text{cm}^2$

$= \dfrac{22}{7} \times 35\ \text{cm}^2$

$= 22 \times 5\ \text{cm}^2$

$= 110\ \text{cm}^2$

$\therefore$ Required surface area of a remaining block

= Total surface area of a cuboidal block + Total surface area of a cylindrical hole

= (473 + 110) $cm^2$

= 583 $cm^2$

**28.** Angle described by minute hand in 35 minutes = 6° × 35 = 210°

Area of a sector of angle 210° in a circle of radius 5 cm

$$= \frac{\theta\pi r^2}{360} cm^2 = \left[\frac{210}{360} \times \frac{22}{7} \times (5)^2\right] cm^2$$

$$= \frac{275}{6} cm^2 = 45.83 \ cm^2$$

Area of a circle of radius 5 cm $= \pi r^2$

$$= \frac{22}{7} \times (5)^2 = \frac{550}{7} cm^2$$

$\therefore$ Area does not covered by the minute hand

$$= \frac{550}{7} - \frac{275}{6}$$

$$= \frac{3300 - 1925}{42} \ cm^2$$

$$= \frac{1375}{42} \ cm^2 = 32.74 \ cm^2 \text{ (approx).}$$

**29.** Total number of fruits in a bag = 20 oranges + 10 apples + 40 mangoes = 70

Total number of outcomes = 70

(*i*) There are 10 apples in a bag

$\therefore$ Number of outcomes favourable to an apple = 10

Hence, $$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{10}{70} = \frac{1}{7}$$

(*ii*) Number of oranges in a bag = Number of outcomes favourable to oranges in a bag = 20

Hence, P(an orange) = $\frac{20}{70} = \frac{2}{7}$

(*iii*) Total number of mangoes in a bag = 40

Number of outcomes favourable to mangoes = 40

Hence, P(that the drawn fruit is a mango) = $\frac{40}{70} = \frac{4}{7}$

(*iv*) There are 10 + 40 = 50 fruits which are not oranges in a bag.

$\therefore$ Number of outcomes favourable to not an orange = 50

Hence, P(that the drawn fruit is not an orange) = $\frac{50}{70} = \frac{5}{7}$

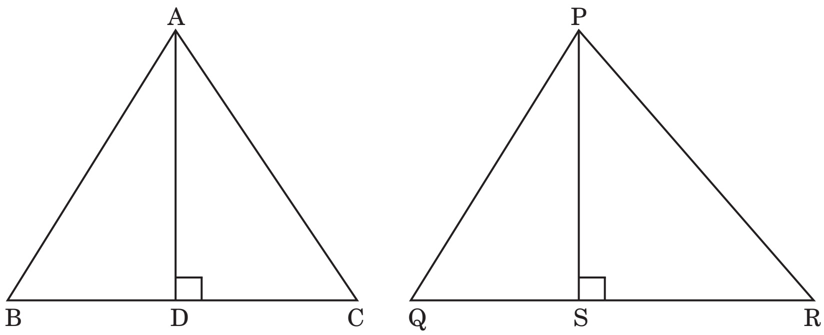
Key points for a student to remain healthy for long:

- Avoiding junk food
- Eating healthy balanced diet
- Doing physical exercise

**30. Given:** $\Delta ABC$ and $\Delta PQR$ such that $\Delta ABC \sim \Delta PQR$

**To prove:** $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$

**Construction:** Draw $AD \perp BC$ and $PS \perp QR$




**Proof:** $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of triangle = $\frac{1}{2}$ × base × height]

$\Rightarrow \quad \dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC \times AD}{QR \times PS}$ ...(*i*)

Now, in $\Delta ADB$ and $\Delta PSQ$, we have

$\angle B = \angle Q$ (As $\Delta ABC \sim \Delta PQR$)

$\angle ADB = \angle PSQ$ (Each 90°)

Thus, $\Delta ADB$ and $\Delta PSQ$ are equiangular and hence, they are similar.

Consequently, $\dfrac{AD}{PS} = \dfrac{AB}{PQ}$ ...(*ii*)

[If triangles are similar, the ratio of their corresponding sides is same]

But $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$

$\Rightarrow \quad \dfrac{AD}{PS} = \dfrac{BC}{QR}$ ...(*iii*) (Using (*ii*))

From (*i*) and (*iii*), we get

$\Rightarrow \quad \dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC}{QR} \times \dfrac{AD}{PS}$

$\Rightarrow \quad \dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC}{QR} \times \dfrac{BC}{QR}$ (Using (*iii*))

$\Rightarrow \quad \dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC^2}{QR^2}$ ...(*iv*)

Since $\Delta ABC \sim \Delta PQR$, therefore

$\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$ ...(*v*)

Thus, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$ (From (*iv*) and (*v*))

Hence proved.