Solutions to RSPL/2

1. We have, $(12 - 9) = (15 - 12) = (18 - 15) = 3$. Therefore, the given sequence is an AP with common difference 3 and first term = 9

$\therefore$ 16th term $= a_{16} = 9 + (16 - 1)(3) = 54$ $\quad (\because\ a_n = a + (n-1)d)$

2. Given $\quad 2x + 3y - 7 = 0 \quad ...(i)$

$2\alpha x + (\alpha + \beta)y - 28 = 0 \quad ...(ii)$

Required condition for infinite number of solutions is

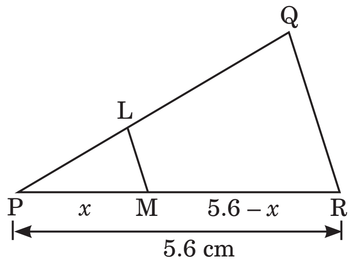
$$\frac{2\alpha}{2} = \frac{\alpha+\beta}{3} = \frac{-28}{-7} \Rightarrow \alpha = \frac{\alpha+\beta}{3} = 4$$

$\Rightarrow \quad \alpha = 4$ and $\alpha + \beta = 12$

$\Rightarrow \quad \alpha = 4$ and $\beta = 8$

3. Let PM = x cm

Then MR = $(5.6 - x)$ cm



Now, $\quad$ LM || QR $\quad$ (Given)

$\Rightarrow \quad \dfrac{PM}{MR} = \dfrac{PL}{LQ}$ $\quad$ (Using BPT)

$\Rightarrow \quad \dfrac{x}{5.6 - x} = \dfrac{3}{5} \Rightarrow 5x = 3(5.6 - x)$

$\Rightarrow \quad 8x = 3 \times 5.6 \Rightarrow 8x = 16.8$

$\Rightarrow \quad x = 2.1$ cm

Hence, PM = 2.1 cm.

4. LCM of 1, 2, 3, 4, 5, i.e. first five natural numbers = 60

Hence, the least positive integer divisible by first five natural numbers is 60.

5. We divide the polynomial $2x^3 - 3x^2 + 6x + 7$ by $x^2 - 4x + 8$.

$$\begin{array}{r|l} & (2x+5) = q(x) \\ \hline x^2 - 4x + 8 & 2x^3 - 3x^2 + 6x + 7 \\ & 2x^3 - 8x^2 + 16x \\ & - \quad + \quad - \\ \hline & \quad 5x^2 - 10x + 7 \\ & \quad 5x^2 - 20x + 40 \\ & \quad - \quad + \quad - \\ \hline & \quad\quad (10x - 33) = r(x) \end{array}$$

$\therefore$ Quotient $= 2x + 5$

and Remainder $= 10x - 33$.

6. We denote the event Sangeeta wins the tennis match by S and the event Reshma wins the tennis match by R. We are given that P(S) = 0.62

Now, $\quad$ P(S) + P(R) = 1

$\Rightarrow \quad 0.62 + P(R) = 1 \Rightarrow P(R) = 0.38$

7. The smallest number which, when divided by 35, 56 and 91, leaves the remainder zero is the LCM of 35, 56, 91. So, the required number is

LCM of (35, 56, 91) + 7

Now, $35 = 5 \times 7$

$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$

$91 = 7 \times 13$

So, $\text{LCM}(35, 56, 91) = 2^3 \times 5 \times 7 \times 13 = 3640$

Thus, the required number is 3640 + 7 = 3647.

8. Let the four parts be $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$.

Then sum = 32

$\Rightarrow (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow a = 8$

Also, $$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\Rightarrow \quad \frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15} \Rightarrow \frac{64 - 9d^2}{64 - d^2} = \frac{7}{15} \qquad (\because a = 8)$$

$$\Rightarrow \quad 960 - 135d^2 = 448 - 7d^2$$

$$\Rightarrow \quad 960 - 448 = 135d^2 - 7d^2$$

$$\Rightarrow \quad 128d^2 = 512 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

So, when $a = 8$ and $d = 2$, the numbers are 2, 6, 10, 14.

When $a = 8$ and $d = -2$, the numbers are 14, 10, 6, 2.

Thus, the four parts are 2, 6, 10, 14.

9. $$\text{LHS} = \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta + 1} = \text{cosec}^2\theta\left\{\frac{1}{\text{cosec}\,\theta - 1} - \frac{1}{\text{cosec}\,\theta + 1}\right\}$$

$$= \text{cosec}^2\theta\left\{\frac{\text{cosec}\,\theta + 1 - (\text{cosec}\,\theta - 1)}{(\text{cosec}\,\theta - 1)(\text{cosec}\,\theta + 1)}\right\}$$

$$= \text{cosec}^2\theta\left\{\frac{2}{\text{cosec}^2\theta - 1}\right\} = \text{cosec}^2\theta\left\{\frac{2}{\cot^2\theta}\right\} = \text{cosec}^2\theta\{2\tan^2\theta\}$$

$$= \frac{1}{\sin^2\theta}\left\{\frac{2\sin^2\theta}{\cos^2\theta}\right\}$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta = \text{RHS}$$

Hence proved.

10. Given

$$ax + by = 2a - 3b \qquad \ldots(i)$$

and $$bx - ay = 3a + 2b \qquad \ldots(ii)$$

Multiplying (i) by a and (ii) by b, then adding both, we get

$$a(ax + by) + b(bx - ay) = a(2a - 3b) + b(3a + 2b)$$

$$\Rightarrow \quad a^2x + aby + b^2x - aby = 2a^2 - 3ab + 3ab + 2b^2$$

$$\Rightarrow \quad (a^2 + b^2)x = 2(a^2 + b^2)$$

$$\Rightarrow \quad x = 2$$

Substituting $x = 2$ in (i), we get

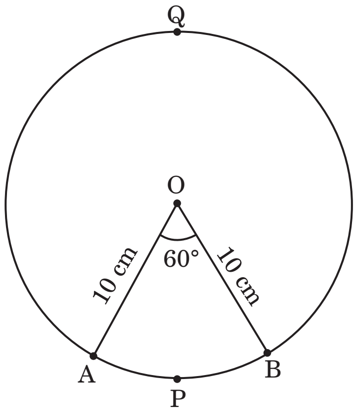
$$2a + by = 2a - 3b$$

$$\Rightarrow \quad by = -3b \quad \Rightarrow y = -3$$

Hence, the required solution is $x = 2, y = -3$

11. Here, $r = 10$ cm is the radius of the circle having centre at O.

$$\angle AOB = 60°$$



The area of the sector OAPB $= \frac{60}{360} \times \pi r^2$

$$= \frac{1}{6} \times 3.14 \times (10)^2 \text{ cm}^2$$

$$= \frac{314}{6} \text{ cm}^2 = \frac{157}{3} \text{ cm}^2$$

The area of the major sector OAQB $= \left(\frac{360 - 60}{360}\right)\pi r^2$

$$= \frac{300}{360} \times 3.14 \times (10)^2$$

$$= \frac{5}{6} \times 3.14 \times 100$$

$$= \frac{5 \times 157}{3} \text{ cm}^2 = \frac{785}{3} \text{ cm}^2$$

12. LHS $= \sin 35° \sin 55° - \cos 35° \cos 55°$

$= \sin(90° - 55°)\sin(90° - 35°) - \cos 35° \cos 55°$

$= \cos 55° \cos 35° - \cos 35° \cos 55°$ $\quad [\because \sin(90° - \theta) = \cos \theta]$

$= 0 =$ RHS

Hence proved.

13. We have

$2x - y = 2 \quad ...(i)$

$4x - y = 8 \quad ...(ii)$

Now, $2x - y = 2$

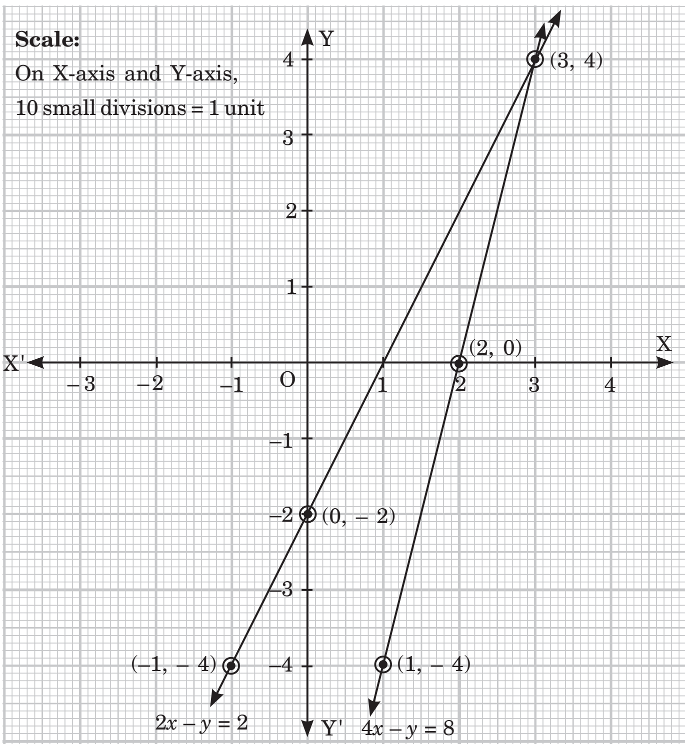
$\Rightarrow \quad y = 2(x - 1)$

x	-1	0	3
y	-4	-2	4

Also, $4x - y = 8$

$\Rightarrow \quad y = 4(x - 2)$

x	1	2	3
y	-4	0	4



The two lines intersect at the point (3, 4). So, $x = 3$ and $y = 4$ is the required solution of the pair of linear equations.

14. Since $x = 2$ is a root of the equation $2x^2 + kx - 6 = 0$,

$$\therefore \quad 2(2)^2 + 2k - 6 = 0$$

$$\Rightarrow \quad 8 + 2k - 6 = 0$$

$$\Rightarrow \quad 2k + 2 = 0$$

$$\Rightarrow \quad k = -1$$

Putting $k = -1$ in the equation $2x^2 + kx - 6 = 0$, we get

$$2x^2 - x - 6 = 0$$

$$\Rightarrow \quad 2x^2 - 4x + 3x - 6 = 0$$

$$\Rightarrow \quad 2x(x - 2) + 3(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)(2x + 3) = 0$$

$$\Rightarrow \quad x - 2 = 0 \text{ or } 2x + 3 = 0$$

$$\Rightarrow \quad x = 2 \text{ or } x = \frac{-3}{2}$$

Hence, the other root is $\frac{-3}{2}$.

15. Let the diagonals AC and BD of parallelogram ABCD intersect at O. Then

Mid-point of AC = Mid-point of BD

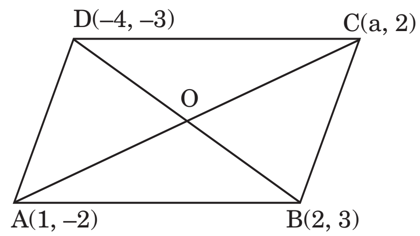
$$\Rightarrow \quad \left(\frac{1+a}{2}, \frac{2-2}{2}\right) = \left(\frac{2-4}{2}, \frac{3-3}{2}\right)$$

$$\Rightarrow \quad \left(\frac{1+a}{2}, 0\right) = (-1, 0)$$

$$\Rightarrow \quad \frac{1+a}{2} = -1 \Rightarrow 1 + a = -2 \Rightarrow a = -3$$

Thus, the value of a is -3.

$\therefore$ Coordinates of C are $(-3, 2)$



If h be the height of the parallelogram and taking AB as base, then

Area of parallelogram ABCD = AB × h

$$\Rightarrow \quad 2 \times \text{ar}(\Delta ABD) = AB \times h$$

$$\Rightarrow \quad h = \frac{2 \times \text{ar}(\Delta ABD)}{AB}$$

Now, we calculate the area of ΔABD.

$$\text{ar}(\Delta ABD) = \frac{1}{2}\,|\,1(3 + 3) + 2(-3 + 2) + (-4)(-2 - 3)\,|$$

$$= \frac{1}{2} \times 24 = 12 \text{ sq units}$$

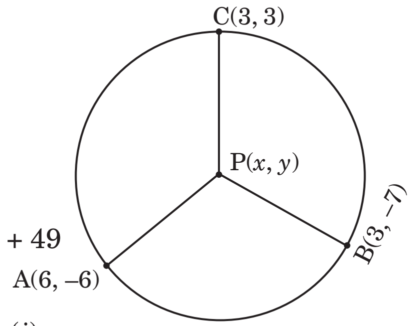
Also, $$AB = \sqrt{(1-2)^2 + (-2-3)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26} \text{ units}$$

Thus, $$h = \frac{2 \times 12}{\sqrt{26}} = \frac{12}{13}\sqrt{26} \text{ units}$$

$\therefore$ Height of parallelogram ABCD = $\frac{12}{13}.\sqrt{26}$ units

OR

Let P (x, y) be the centre of the circle passing through the points A(6, – 6), B(3, –7) and C(3, 3).



Then $AP = BP = CP$

$\Rightarrow$ $(AP)^2 = (BP)^2 = (CP)^2$

So, $(AP)^2 = (BP)^2$ gives

$$(x - 6)^2 + (y + 6)^2 = (x - 3)^2 + (y + 7)^2$$

$\Rightarrow$ $x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$

$\Rightarrow$ $-12x + 6x + 12y - 14y + 72 - 58 = 0$

$\Rightarrow$ $-6x - 2y + 14 = 0$ or $3x + y - 7 = 0$ ...(*i*)

Similarly, $BP^2 = CP^2$ gives

$$20y + 40 = 0 \quad \Rightarrow y = -2$$

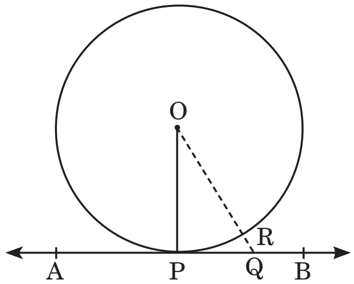
Putting this value of y in (*i*), we have $x = 3$

Hence, the centre of the circle is (3, – 2).

16. **Given:** A circle C(O, r) and a tangent AB at a point P.

To prove: $OP \perp AB$.

Construction: Take any point Q, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.



Proof: We know that among all line segments joining the point O to a point on AB, the shortest one is perpendicular to AB. So, to prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly, $OP = OR$ (Radii of the same circle)

Now, $OQ = OR + RQ$

$\Rightarrow$ $OQ > OR$

$\Rightarrow$ $OQ > OP$ ($\because$ OP = OR)

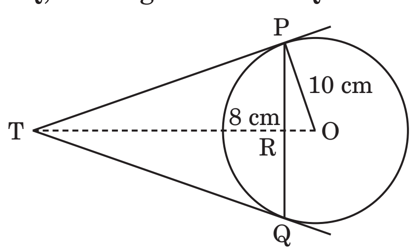
$\Rightarrow$ $OP < OQ$

Thus, OP is shorter than any other segment joining O to any point on AB.

So, $OP \perp AB$. Hence proved.

OR

Join OT. Let it intersect chord PQ at the point R. Then, ΔPTQ is isosceles and TO is the bisector of $\angle PTQ$. So, $OT \perp PQ$ and therefore OT bisects PQ, which gives PR = RQ = 8 cm



Also, $OR = \sqrt{OP^2 - PR^2}$

$= \sqrt{10^2 - 8^2} = 6$ cm

Now, $\angle TPR + \angle RPO = 90° = \angle TPR + \angle PTR$

So, $\angle RPO = \angle PTR$ ...(*i*)

Now, in right-angled triangles TRP and PRO, we have

$$\angle PTR = \angle RPO \quad \text{(By } (i)\text{)}$$

and $$\angle TRP = \angle PRO \quad \text{(Each } 90°\text{)}$$

$\therefore$ By AA similarity criterion, we have

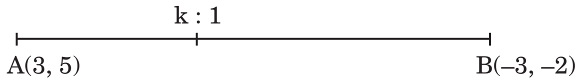
$$\Delta TRP \sim \Delta PRO$$

$$\Rightarrow \quad \frac{TP}{PO} = \frac{RP}{RO}$$

$$\Rightarrow \quad \frac{TP}{10} = \frac{8}{6}$$

Thus, the length of TP is $\frac{40}{3}$ cm.

17. Let the point C divides AB in the ratio $k : 1$, then, the coordinates of C are



$$\left(\frac{-3k+3}{k+1}, \frac{-2k+5}{k+1}\right)$$

But, the coordinates of C are given as $\left(\frac{3}{5}, \frac{11}{5}\right)$.

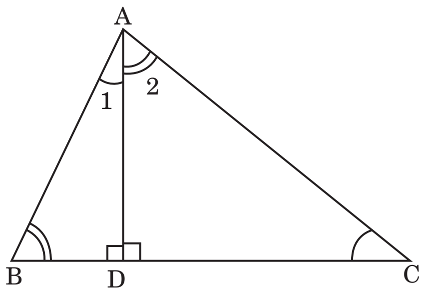
$$\therefore \quad \frac{-3k+3}{k+1} = \frac{3}{5} \text{ and } \frac{-2k+5}{k+1} = \frac{11}{5}$$

$$\Rightarrow \quad -15k + 15 = 3k + 3 \text{ and } -10k + 25 = 11k + 11$$

$$\Rightarrow \quad 18k = 12 \text{ and } 21k = 14 \Rightarrow k = \frac{2}{3}$$

Hence, the point C divides AB in the ratio 2 : 3

18. We have



$$AD^2 = BD \times DC$$

$$\Rightarrow \quad AD \times AD = BD \times DC$$

$$\Rightarrow \quad \frac{AD}{DC} = \frac{BD}{AD}$$

Thus, in ΔDBA and ΔDAC, we have

$$\frac{AD}{DC} = \frac{BD}{AD}$$

and $$\angle BDA = \angle CDA \quad \text{(Each equal to } 90°\text{)}$$

So, by SAS criterion of similarity, we get

$$\Delta DBA \sim \Delta DAC$$

$\Rightarrow$ ΔDBA and ΔDAC are equiangular

$$\Rightarrow \quad \angle 1 = \angle C \text{ and } \angle 2 = \angle B$$

$$\Rightarrow \quad \angle 1 + \angle 2 = \angle B + \angle C$$

$$\Rightarrow \quad \angle A = \angle B + \angle C$$

But $$\angle A + \angle B + \angle C = 180° \quad \text{(Angle sum property of triangle)}$$

$$\Rightarrow \quad \angle A + \angle A = 180°$$

$$\Rightarrow \quad 2\angle A = 180° \Rightarrow \angle A = 90°$$

Thus, $$\angle BAC = 90°$$ Hence proved.

OR

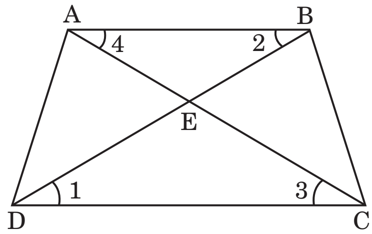
In ΔEDC and ΔEBA, we have

$$\angle 1 = \angle 2 \quad \text{(Alternate interior angles)}$$

$$\angle 3 = \angle 4 \quad \text{(Alternate interior angles)}$$

and $$\angle CED = \angle AEB \quad \text{(Vertically opposite angles)}$$

$\therefore$ $$\Delta EDC \sim \Delta EBA \quad \text{(AAA similarity)}$$



$\Rightarrow$ $$\frac{ED}{EB} = \frac{EC}{EA} \Rightarrow \frac{ED}{EC} = \frac{EB}{EA} \quad ...(i)$$

It is given that $\Delta AED \sim \Delta BEC$

$\Rightarrow$ $$\frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\frac{EB}{EA} = \frac{EA}{EB}$$

$\Rightarrow$ $$(EB)^2 = (EA)^2 \Rightarrow EB = EA$$

Substituting EB = EA in (*ii*), we get

$$\frac{EA}{EA} = \frac{AD}{BC}$$

$\Rightarrow$ $$\frac{AD}{BC} = 1$$

$\Rightarrow$ $$AD = BC$$ Hence proved.

19. In a leap year, there are 366 days.

We have, 366 days = 52 weeks and 2 days. Thus, a leap year has always 52 Sundays.

The remaining 2 days can be:

(*i*) Sunday and Monday

(*ii*) Monday and Tuesday

(*iii*) Tuesday and Wednesday

(*iv*) Wednesday and Thursday

(*v*) Thursday and Friday

(*vi*) Friday and Saturday

(*vii*) Saturday and Sunday

Clearly, there are seven elementary events associated with this random experiment.

Let A be the event that a leap year has 53 Sundays.

Clearly, the event A will happen if the last two days of the leap year are either Sunday and Monday or Saturday and Sunday.

$\therefore$ Favourable number of elementary events = 2

Hence, required probability = $\frac{2}{7}$

OR

Let there be x blue balls in the bag.

$\therefore$ Total number of balls in the bag = $(5 + x)$

Now,

P_1 = Probability of drawing a blue ball = $\dfrac{x}{5+x}$

P_2 = Probability of drawing a red ball $\dfrac{5}{5+x}$

It is given that

$$P_1 = 2P_2$$

$$\Rightarrow \quad \frac{x}{5+x} = 2\times\frac{5}{5+x} \Rightarrow \frac{x}{5+x} = \frac{10}{5+x} \Rightarrow x = 10$$

Hence, there are 10 blue balls in the bag.

20. Let a be the first term and d be the common difference of the given AP. Then,

$$a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n} \qquad ...(i)$$

and $$a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m} \qquad ...(ii)$$

Subtracting (ii) from (i), we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow \quad (m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$$

Putting $d = \dfrac{1}{mn}$ in (i), we get

$$a + (m-1)\frac{1}{mn} = \frac{1}{n}$$

$$\Rightarrow \quad a + \frac{1}{n} - \frac{1}{mn} - \frac{1}{n} = 0 \Rightarrow a - \frac{1}{mn} = 0 \Rightarrow a = \frac{1}{mn}$$

Now, $$S_{mn} = \frac{mn}{2}[2a + (mn-1)d]$$

$$\Rightarrow \quad S_{mn} = \frac{mn}{2}\left[\frac{2}{mn} + (mn-1)\times\frac{1}{mn}\right]$$

$$\Rightarrow \quad S_{mn} = \frac{1}{2}(mn+1)$$

Hence proved.

21. Let n be the maximum number of columns in which the two groups can march.

Then, n is the HCF of 616 and 32, i.e.

$$\text{HCF}(616, 32) = n$$

Let us determine the HCF of 616 and 32

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

$$\therefore \quad \text{HCF}(616, 32) = 8$$

Hence, the maximum number of columns in which they can march is 8.

22. Let the assumed mean a be 150.

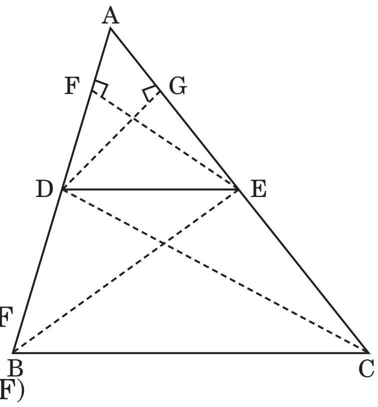
The frequency distribution table for the given data is as shown below:

Daily wages (₹)	Number of workers (f_i)	Class mark (x_i)	$d_i = x_i - a$	$f_i d_i$
100 – 120	12	110	– 40	– 480
120 – 140	14	130	– 20	– 280
140 – 160	8	150	0	0
160 – 180	6	170	20	120
180 – 200	10	190	40	400
	$\Sigma f_i = 50$			$\Sigma f_i d_i = -240$

$\therefore$ $$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 150 - \left(\frac{240}{50}\right) = 150 - 4.8 = 145.2$$

Thus, the mean daily wages is ₹ 145.20

23. **Given:** A triangle ABC in which DE | | BC and intersects AB in D and AC in E.



To prove: $$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction: Join BE, CD. Draw EF ⊥ BA and DG ⊥ CA.

Proof: Since EF is perpendicular to AB, therefore, EF is the height of triangles ADE and DBE.

Now, $\text{Area}(\Delta ADE) = \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2} \times AD \times EF$

and $\text{Area}(\Delta DBE) = \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2} \times (DB \times EF)$

$\therefore$ $$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DBE)} = \frac{\frac{1}{2} \times (AD \times EF)}{\frac{1}{2} \times (DB \times EF)} = \frac{AD}{DB} \quad ...(i)$$

Similarly, we have

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DEC)} = \frac{\frac{1}{2} \times (AE \times DG)}{\frac{1}{2} \times (EC \times DG)} = \frac{AE}{EC} \quad ...(ii)$$

But, ΔDBE and ΔDEC are on the same base DE and between the same parallels DE and BC.

$\therefore$ $$\text{Area}(\Delta DBE) = \text{Area}(\Delta DEC)$$

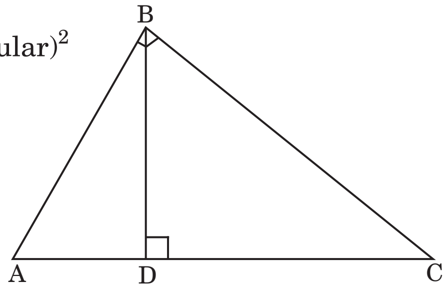
$\Rightarrow$ $$\frac{1}{\text{Area}(\Delta DBE)} = \frac{1}{\text{Area}(\Delta DEC)} \quad \text{[Taking reciprocals of both sides]}$$

$\Rightarrow$ $$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DBE)} = \frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta DEC)}$$

$\Rightarrow$ $$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{(Using } (i) \text{ and } (ii))$$

Hence proved.

OR



Given: A right-angled triangle ABC in which $\angle B = 90°$

To prove: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e. $(AC)^2 = (AB)^2 + (BC)^2$

Construction: From B, draw $BD \perp AC$.

Proof: In triangles ADB and ABC, we have

$\angle ADB = \angle ABC$ (Each equal to 90°)

and $\angle A = \angle A$ (Common)

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ ($\because$ In similar triangles, corresponding sides are proportional)

$\Rightarrow AB^2 = AD \times AC$ ...(*i*)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ (Each equal to 90°)

and $\angle C = \angle C$ (Common)

So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$ ($\because$ In similar triangles, corresponding sides are proportional)

$\Rightarrow BC^2 = AC \times DC$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AB^2 + BC^2 = (AC)^2$

So, $(AC)^2 = (AB)^2 + (BC)^2$ Hence proved.

24. Given, $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$ and $0° < \theta < 90°$,

$\Rightarrow (\cos\theta - \sin\theta)(1 + \sqrt{3}) = (1 - \sqrt{3})(\cos\theta + \sin\theta)$

$\Rightarrow \cos\theta\,(1 + \sqrt{3}) - \sin\theta(1 + \sqrt{3}) = (1 - \sqrt{3})\cos\theta + (1 - \sqrt{3})\sin\theta$

$\Rightarrow \cos\theta\,(1 + \sqrt{3} - 1 + \sqrt{3}) = \sin\theta\,[1 - \sqrt{3} + 1 + \sqrt{3}]$

$\Rightarrow 2\sqrt{3}\cos\theta = 2\sin\theta$

$\Rightarrow \sqrt{3} = \frac{\sin\theta}{\cos\theta} \Rightarrow \sqrt{3} = \tan\theta \Rightarrow \tan 60° = \tan\theta \Rightarrow \theta = 60°$

OR

$$\text{LHS} = \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$$

$$= \frac{\operatorname{cosec}\theta + \cot\theta}{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta)} - \operatorname{cosec}\theta \qquad \left(\because \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right)$$

$$= \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} - \operatorname{cosec}\theta$$

$$= \frac{\operatorname{cosec}\theta + \cot\theta - \operatorname{cosec}\theta}{1} \qquad (\because \operatorname{cosec}^2\theta - \cot^2\theta = 1)$$

$$= \cot\theta$$

$$\text{RHS} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$= \operatorname{cosec}\theta - \left(\frac{\operatorname{cosec}\theta - \cot\theta}{(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}\right)$$

$$= \operatorname{cosec}\theta - \left(\frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta}\right)$$

$$= \operatorname{cosec}\theta - \left(\frac{\operatorname{cosec}\theta - \cot\theta}{1}\right)$$

$$= \operatorname{cosec}\theta - \operatorname{cosec}\theta + \cot\theta = \cot\theta$$

$\therefore$ LHS = RHS Hence proved.

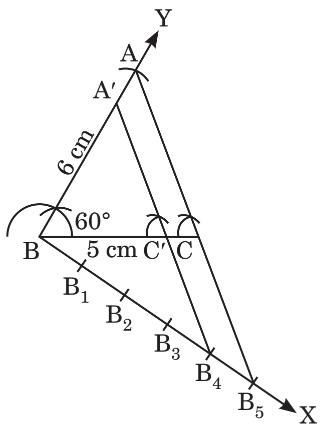
25. Steps of construction:

(*i*) Draw a line segment BC of length 5 cm.

(*ii*) Draw $\angle CBY = 60°$.

(*iii*) Cut off BA = 6 cm from the vertex B. Join AC.

(*iv*) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.



(*v*) Locate 5 (the greater of 4 and 5 in $\frac{4}{5}$) points

B_1, B_2, B_3, B_4, B_5 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$

(*vi*) Join B_5C and draw a line through B_4 (the 4th point, 4 being smaller of 4 and 5 in $\frac{4}{5}$) parallel to B_5C to intersect BC at C′.

(*vii*) Draw a line through C′ parallel to the line CA to intersect BA at A′. Then A′BC′ is required triangle.

26. Let the digits at unit's and ten's place in the given number be x and y respectively. Then,

Number = $10y + x$ ...(*i*)

Number formed by interchanging the digits = $10x + y$

According to the given conditions, we have

$$(10y + x) + (10x + y) = 110$$

and $$(10y + x) - 10 = 5(x + y) + 4$$

$\Rightarrow$ $$11x + 11y = 110$$

and $$4x - 5y + 14 = 0$$

$\Rightarrow$ $$x + y - 10 = 0$$

and $$4x - 5y + 14 = 0$$

By cross-multiplication method, we have

$$\frac{x}{14-50} = \frac{y}{-40-14} = \frac{1}{-5-4}$$

$\Rightarrow$ $$\frac{x}{-36} = \frac{y}{-54} = \frac{1}{-9}$$

$\Rightarrow$ $$x = \frac{36}{9} = 4 \text{ and } y = \frac{54}{9} = 6$$

Putting the values of x and y in (*i*), we get

$$\text{Number} = 10 \times 6 + 4 = 64$$

OR

Let the digit in the unit's place be x and the digit at the ten's place by y.

Then,

$$\text{Number} = 10y + x$$

The number obtained by reversing the order of the digits is $10x + y$

According to the given conditions, we have

$$(10y + x) + (10x + y) = 121$$

$\Rightarrow$ $$11(x + y) = 121$$

$\Rightarrow$ $$x + y = 11$$

and $$x - y = \pm 3$$ ($\because$ Difference of digits is 3)

Thus, we have the following sets of simultaneous equations:

$x + y = 11$ ...(*i*)
and $x - y = 3$ ...(*ii*)

or

$x + y = 11$ ...(*iii*)
$x - y = -3$ ...(*iv*)

On solving (*i*) and (*ii*), we get $x = 7, y = 4$

On solving (*iii*) and (*iv*), we get $x = 4, y = 7$

When $x = 7, y = 4$, we have

Number = $10y + x = 10 \times 4 + 7 = 47$

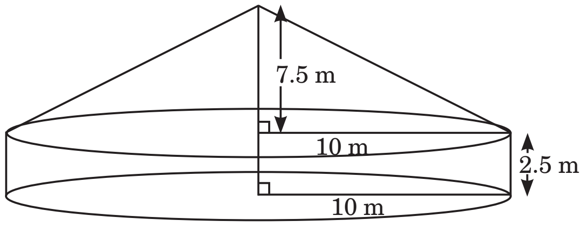
When $x = 4, y = 7$, we have

Number $= 10y + x = 10 \times 7 + 4 = 74$

Hence, the required number is either 47 or 74.

27. $$\text{Capacity of the tent} = [\pi(10)^2 (2.5) + \frac{1}{3}\pi(10)^2 (7.5)] \text{ m}^3$$

$$= (250\pi + 250\pi) \text{ m}^3 = 500\pi \text{ m}^3$$



$$\text{Curved surface area of the tent} = [\pi (10) \sqrt{10^2 + (7.5)^2} + 2\pi(10)(2.5)] \text{ m}^2$$

$$= 175\pi \text{ m}^2$$

$\therefore$ $$\text{Cost of canvas required} = ₹\ (175\pi \times 100)$$

$$= ₹ \left(175 \times \frac{22}{7} \times 100\right) = ₹\ 55{,}000$$

Values reflected: Philanthropy, concern for homeless.

28. We have the following table:

Marks	Frequency	Cumulative frequency
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$

It is given that $n = 60$

$\Rightarrow$ $$\frac{n}{2} = \frac{60}{2} = 30$$

$\therefore$ $$45 + x + y = 60 \Rightarrow x + y = 15 \qquad ...(i)$$

Since the median is given to be 28.5, the median class is 20 – 30.

Thus, $l = 20$, $h = 10$, $f = 20$ and cf $= 5 + x$

$\therefore$ $$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$\Rightarrow$ $$28.5 = 20 + \left(\frac{30 - (5 + x)}{20}\right) \times 10$$

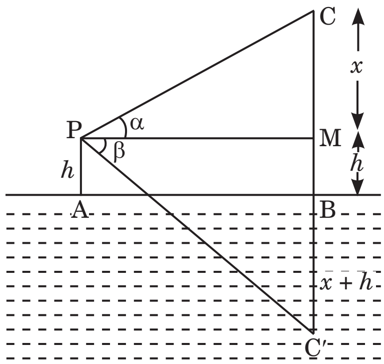
$$\Rightarrow \quad 28.5 = 20 + \frac{25 - x}{2}$$

$$\Rightarrow \quad x = 8$$

From (i), we have $y = 7$

Thus, $x = 8$ and $y = 7$

29. Let AB be the surface of the lake and let P be a point of observation such that AP = h metres. Let C be the position of the cloud and C′ be its reflection in the lake, then CB = C′B. Let PM be perpendicular from P on CB. Then, $\angle CPM = \alpha$ and $\angle MPC' = \beta$.



Let $\quad CM = x$

Then CB = CM + MB = CM + PA = $(x + h)$,

In right-angled ΔCPM, we have $\tan \alpha = \frac{CM}{PM}$

$$\Rightarrow \quad \tan \alpha = \frac{x}{AB} \qquad (\because PM = AB)$$

$$\Rightarrow \quad AB = x \cot \alpha \qquad ...(i)$$

In right-angled ΔPMC′, we have

$$\tan \beta = \frac{C'M}{PM}$$

$$\Rightarrow \quad \tan \beta = \frac{x + 2h}{AB} \qquad (\because C'M = C'B + BM = x + h + h)$$

$$\Rightarrow \quad AB = (x + 2h) \cot \beta \qquad ...(ii)$$

From (i) and (ii), we have

$$x \cot \alpha = (x + 2h) \cot \beta$$

$$\Rightarrow \quad x \cot \alpha = x \cot \beta + 2h \cot \beta$$

$$\Rightarrow \quad (x \cot \alpha - x \cot \beta) = 2h \cot \beta$$

$$\Rightarrow \quad x\left(\frac{1}{\tan \alpha} - \frac{1}{\tan \beta}\right) = \frac{2h}{\tan \beta}$$

$$\Rightarrow \quad x\frac{(\tan \beta - \tan \alpha)}{\tan \alpha \tan \beta} = \frac{2h}{\tan \beta} \quad \Rightarrow x = \frac{2h \tan \alpha}{\tan \beta - \tan \alpha}$$

Thus, $\quad$ height of the cloud $= x + h$

$$= \frac{2h \tan \alpha}{\tan \beta - \tan \alpha} + h$$

$$= \frac{2h \tan \alpha + h \tan \beta - h \tan \alpha}{\tan \beta - \tan \alpha}$$

$$= \frac{h \tan \alpha + h \tan \beta}{\tan \beta - \tan \alpha}$$

$$= \frac{h(\tan \alpha + \tan \beta)}{\tan \beta - \tan \alpha}$$

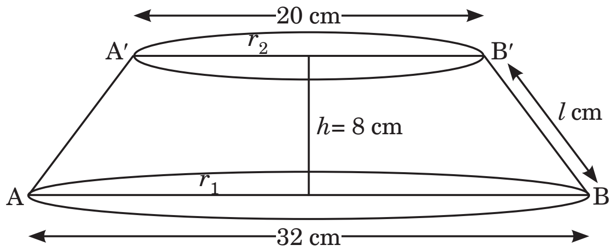
Hence proved.

30. Let ABB′A′ be the friction clutch of slant height l cm.

We have, $r_1 = 16$ cm, $r_2 = 10$ cm and $h = 8$ cm

$\therefore \quad l^2 = h^2 + (r_1 - r_2)^2$

$\Rightarrow \quad l^2 = 64 + 36 \Rightarrow l^2 = 100 \Rightarrow l = 10$ cm



Now, Lateral surface area of the frustum $= \pi(r_1 + r_2)l$

$= \frac{22}{7} \times (16 + 10) \times 10 \text{ cm}^2$

$= 817.14 \text{ cm}^2$

Volume of the clutch $= \frac{1}{3}\pi h\left(r_1^2 + r_1 r_2 + r_2^2\right)$

$= \frac{1}{3} \times \frac{22}{7} \times 8 \times ((16)^2 + 16 \times 10 + 10^2) \text{ cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 8 \times (256 + 160 + 100) \text{ cm}^3$

$= \frac{176}{21} \times 516 \text{ cm}^3$

$= 4324.57 \text{ cm}^3$