Solutions to RSPL/3

1. $$5x + 4y = 9 \quad ...(i)$$
$$x + 2y = 3 \quad ...(ii)$$

Multiplying (*ii*) by 2, we get

$$2x + 4y = 6 \quad ...(iii)$$

Subtracting (*iii*) from (*i*), we obtain

$$(5x + 4y) - (2x + 4y) = 9 - 6$$
$$\Rightarrow \quad 5x - 2x = 3$$
$$\Rightarrow \quad 3x = 3$$
$$\Rightarrow \quad x = 1$$

Substituting $x = 1$ in (*ii*), we get

$$1 + 2y = 3$$
$$\Rightarrow \quad 2y = 2$$
$$\Rightarrow \quad y = 1$$

Hence, $x = 1$ and $y = 1$

2. We have

$$n = 2^7 \times 5^6 \times 13$$
$$\Rightarrow \quad n = 128 \times 15625 \times 13$$
$$\Rightarrow \quad n = 26000000$$

Hence, the number of zeroes it contains are 6.

3. Here, points are P(6, 12) and origin O(0, 0)

The coordinates of the mid-point of the line segment joining the point P(6, 12) to the origin O(0, 0) = $\left(\frac{6+0}{2}, \frac{12+0}{2}\right)$ = (3, 6)

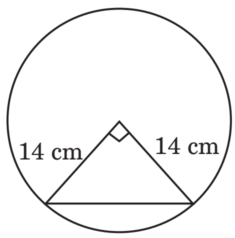
4. We know that, the empirical relationship is

$$\text{Mode} = 3\text{ Median} - 2\text{ Mean}$$
$$= 3(9.6) - 2(10.5)$$
$$= 28.8 - 21 = 7.8$$

Hence, mode = 7.8

5. Area of the minor sector of an angle 90° in a circle of radius 14 cm.

$$= \frac{\theta}{360} \times \pi r^2$$
$$= \frac{90}{360} \times \frac{22}{7} \times (14)^2$$
$$= \frac{1}{4} \times 22 \times 14 \times 2 \text{ cm}^2 = 154 \text{ cm}^2$$



6. Since the square circumscribing a circle of radius a cm, so the diameter of a circle is equal to the side of a square.

But the diameter of a circle = 2 × Radius of a circle

$= (2 \times a)$ cm $= 2a$ cm

$\Rightarrow$ Side of a square $= 2a$ cm

$\therefore$ Perimeter of a square = 4 × Side of a square

$= 4 \times 2a$ cm $= 8a$ cm

7. Given integers are 144 and 180

Since $180 > 144$

Applying Euclid's division lemma to 180 and 144, we get

$$180 = 144 \times 1 + 36 \quad ...(i)$$

$$144 = 36 \times 4 + 0 \quad ...(ii)$$

In (*ii*), the remainder is zero. So, the last divisor or the non-zero remainder at the earliest stage, i.e. in (*i*) is 36. Therefore, HCF of 180 and 144 is 36.

Comparing 36 with $13m - 3$, we get

$$36 = 13m - 3$$

$\Rightarrow$ $13m = 39$

$\Rightarrow$ $m = 3$

8. It is given that the point P(6, – 6) lies on the line $3x + k(y + 1) = 0$

Then $3(6) + k(-6 + 1) = 0$

$\Rightarrow$ $18 - 5k = 0$

$\Rightarrow$ $k = \frac{18}{5}$

Hence, the value of k is $\frac{18}{5}$.

9. Here, $S_n = \frac{3n^2}{2} + \frac{13n}{2}$

For $n = 1$, $S_1 = \frac{3(1)^2}{2} + \frac{13(1)}{2}$

$= \frac{3}{2} + \frac{13}{2} = \frac{16}{2} = 8 = t_1$ (first term).

For $n = 2$, $S_2 = \frac{3(2)^2}{2} + \frac{13(2)}{2} = 6 + 13 = 19$

For $n = 3$, $S_3 = \frac{3(3)^2}{2} + \frac{13(3)}{2} = \frac{27}{2} + \frac{39}{2} = \frac{66}{2} = 33$

Second term, $t_2 = S_2 - S_1 = 19 - 8 = 11$

Third term, $t_3 = S_3 - S_2 = 33 - 19 = 14$

So, $t_1 = 8, t_2 = 11, t_3 = 14$

Now, $t_2 - t_1 = 11 - 8 = 3$

$t_3 - t_2 = 14 - 11 = 3$

Clearly, common difference $(d) = t_2 - t_1 = 3 = t_3 - t_2$

Thus, AP is 8, 11, 14, ...

$$\therefore \quad 25\text{th term} = a + (25-1)d$$
$$= 8 + 24(3) = 8 + 72 = 80$$

10. $$l \sin\theta + m\cos\theta + n = 0$$
$$l'\sin\theta + m'\cos\theta + n' = 0$$

By cross-multiplication method, we have

$$\frac{\sin\theta}{\begin{vmatrix} m & n \\ m' & n' \end{vmatrix}} = \frac{\cos\theta}{\begin{vmatrix} n & l \\ n' & l' \end{vmatrix}} = \frac{1}{\begin{vmatrix} l & m \\ l' & m' \end{vmatrix}}$$

$$\Rightarrow \quad \frac{\sin\theta}{mn' - m'n} = \frac{\cos\theta}{nl' - n'l} = \frac{1}{lm' - l'm}$$

$$\Rightarrow \quad \sin\theta = \frac{mn' - m'n}{lm' - l'm}$$

and $$\cos\theta = \frac{nl' - n'l}{lm' - l'm}$$

We know that $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \quad \left(\frac{mn' - m'n}{lm' - l'm}\right)^2 + \left(\frac{nl' - n'l}{lm' - l'm}\right)^2 = 1$$

$$\Rightarrow \quad (mn' - m'n)^2 + (nl' - ln')^2 = (lm' - l'm)^2$$ Hence proved.

11. We have

$\sec^2 37° - \cot^2 53°. \tan 21°. \tan 69° - \sin 41°. \cos 49° - \cos 41°. \sin 49°$

$= \sec^2 37° - \cot^2(90° - 37°).\tan 21°. \tan(90° - 21°) - \sin(90° - 49°). \cos 49°$
$- \cos(90° - 49°). \sin 49°$

$= \sec^2 37° - \tan^2 37°. \tan 21.\cot 21° - \cos 49°. \cos 49° - \sin 49° \sin 49°$

$= \sec^2 37° - \tan^2 37°(\tan 21°. \cot 21°) - \cos^2 49° - \sin^2 49°$

$= \sec^2 37° - \tan^2 37° - (\cos^2 49° + \sin^2 49°)$ $(\because \tan\theta\cot\theta = 1)$

$= (1 + \tan^2 37°) - \tan^2 37° - 1$ $(\because \sec^2\theta = 1 + \tan^2\theta)$

$= 1 + 0 - 1 = 0$

12. Radius of cylindrical beaker (R) = 7 cm

Diameter of marble = 1.4 cm

Radius of marble = r = 0.7 cm

Suppose n marbles are required to raise the level of water by H = 28 cm

$$\text{Number of marbles } (n) = \frac{\text{Volume of water raised in a beaker}}{\text{Volume of 1 marble dropped}}$$

$$= \frac{\pi R^2 H}{\frac{4}{3}\pi r^3} = \frac{\frac{22}{7} \times 7 \times 7 \times 28}{\frac{4}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 0.7}$$

$$= \frac{7 \times 7 \times 28 \times 3 \times 10 \times 10 \times 10}{4 \times 7 \times 7 \times 7} = 3000$$

Hence, 3000 marbles will be dropped in the beaker.

13. Let the present ages of B and A be x years and y years respectively.

Then

B's age 5 years ago = $(x - 5)$ years

and A's age 5 years ago = $(y - 5)$ years

$\therefore \qquad (y - 5) = 3(x - 5) \Rightarrow 3x - y = 10 \qquad ...(i)$

B's age 10 years hence = $(x + 10)$ years

A's age 10 years hence = $(y + 10)$ years

$\therefore \qquad y + 10 = 2(x + 10) \Rightarrow 2x - y = -10 \qquad ...(ii)$

Subtracting (ii) from (i), we get $x = 20$

Putting $x = 20$ in (i), we get

$$3 \times 20 - y = 10 \Rightarrow y = 60 - 10 = 50$$

Hence, B's present age = 20 years

and A's present age = 50 years

OR

Let the required numbers be x and y

Then $\qquad x + y = 8 \qquad ...(i)$

and $\qquad \frac{1}{x} + \frac{1}{y} = \frac{8}{15} \Rightarrow \frac{x+y}{xy} = \frac{8}{15}$

$\Rightarrow \qquad \frac{8}{xy} = \frac{8}{15} \Rightarrow xy = 15$

$\therefore \qquad x - y = \sqrt{(x+y)^2 - 4xy}$

$$= \sqrt{8^2 - 4 \times 15} = \sqrt{64 - 60}$$

$$= \sqrt{4} = \pm 2$$

Thus, we have

$$\left.\begin{aligned} x + y &= 8 \quad ...(i) \\ x - y &= 2 \quad ...(ii) \end{aligned}\right\} \text{ or } \left\{\begin{aligned} x + y &= 8 \quad ...(iii) \\ x - y &= -2 \quad ...(iv) \end{aligned}\right.$$

On solving (i) and (ii), we get $x = 5$ and $y = 3$

On solving (iii) and (iv), we get $x = 3$ and $y = 5$

Hence, the required numbers are 5 and 3.

14. Let $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Let $\alpha = 2 + \sqrt{3}$ and $\beta = 2 - \sqrt{3}$. Then

$\therefore \qquad \alpha + \beta = 4$ and $\alpha\beta = 1$

So, the quadratic polynomial whose roots are α and β is given by

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 4x + 1$$

$\therefore \quad (x^2 - 4x + 1)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 - 4x + 1)$, we get

$$\begin{array}{r|l} & x^2 - 2x - 35 \\ x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\ & x^4 - 4x^3 + x^2 \\ & - \quad + \quad - \\ \hline & -2x^3 - 27x^2 + 138x - 35 \\ & -2x^3 + 8x^2 - 2x \\ & + \quad - \quad + \\ \hline & -35x^2 + 140x - 35 \\ & -35x^2 + 140x - 35 \\ & + \quad - \quad + \\ \hline & 0 \\ \hline \end{array}$$

$\therefore \quad f(x) = (x^2 - 4x + 1)(x^2 - 2x - 35)$

$\therefore$ The other two zeroes of $f(x)$ are given by $x^2 - 2x - 35 = 0$

Now, $\quad x^2 - 2x - 35 = 0$

$\Rightarrow \quad x^2 - 7x + 5x - 35 = 0$

$\Rightarrow \quad x(x - 7) + 5(x - 7) = 0$

$\Rightarrow \quad (x - 7)(x + 5) = 0$

$\Rightarrow \quad x - 7 = 0$ or $x + 5 = 0 \quad \Rightarrow x = 7$ or $x = -5$

Hence, the other two zeroes of $f(x)$ are 7 and -5.

OR

Let $f(x) = (x^3 - 3x^2 + x + 2)$, $q(x) = x - 2$ and $r(x) = (-2x + 4)$

Then, $\quad f(x) = g(x).q(x) + r(x)$

Now, $\quad \{f(x) - r(x)\} = (x^3 - 3x^2 + x + 2) - (-2x + 4)$

$\quad = x^3 - 3x^2 + 3x - 2$

$\therefore \quad g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2}$

On dividing $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$, we get $g(x)$

$$\begin{array}{r|l} & x^2 - x + 1 \\ x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

$\therefore \quad g(x) = x^2 - x + 1$

15. Let n be an arbitrary positive integer.

On dividing n by 3, let q be the quotient and r be the remainder.

Then, by Euclid's division lemma, we have

$$n = 3q + r, \text{ where } 0 \le r < 3$$

$$\therefore \quad n^2 = 9q^2 + r^2 + 6qr, \quad \text{...}(i)$$

where $0 \le r < 3$

Case I: When $r = 0$

Putting $r = 0$ in (i), we get $n^2 = 9q^2 = 3(3q^2) = 3m$, where $m = 3q^2$ is an integer.

Case II: When $r = 1$

Putting $r = 1$ in (i), we get $n^2 = (9q^2 + 1 + 6q) = 3(3q^2 + 2q) + 1 = 3m + 1$

where $m = (3q^2 + 2q)$ is an integer.

Case III: When $r = 2$

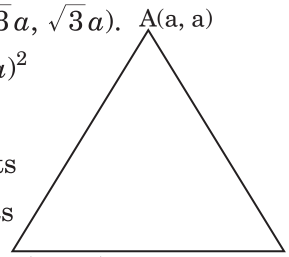
Putting $r = 2$ in (i), we get $n^2 = (9q^2 + 4 + 12q) = 9q^2 + 12q + 3 + 1$

$$= 3(3q^2 + 4q + 1) + 1 = 3m + 1,$$

where $m = 3q^2 + 4q + 1$ is an integer.

Hence, the square of any positive integer is either of the form $3m$ or $(3m + 1)$ for some integer m.

16. Let the given points be A(a, a), B$(-a, -a)$ and C$(-\sqrt{3}a, \sqrt{3}a)$.



Then $(AB)^2 = (-a - a)^2 + (-a - a)^2 = (-2a)^2 + (-2a)^2$

$= 8a^2$ units

$(BC)^2 = (-\sqrt{3}a + a)^2 + (\sqrt{3}a + a)^2 = 8a^2$ units

and $(AC)^2 = (-\sqrt{3}a - a)^2 + (\sqrt{3}a - a)^2 = 8a^2$ units

$\therefore$ $AB = BC = AC = 2\sqrt{2}a$ units

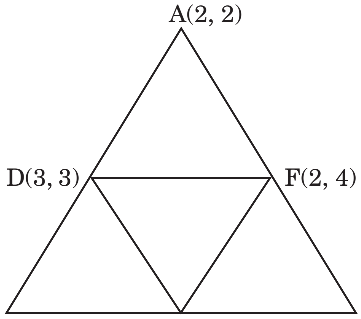
Hence, the given points are the vertices of an equilateral triangle.

Now, area of an equilateral $\Delta ABC = \frac{\sqrt{3}}{4}(\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times (AB)^2 = \frac{\sqrt{3}}{4} \times 8a^2 = 2\sqrt{3}a^2 \text{ sq. units}$$

OR

Let A (2, 2), B(4, 4) and C(2, 6) be the vertices of the given ΔABC. Let D, E and F be the mid-points of AB, BC and CA respectively.



Then, the coordinates of D, E and F are

$$D\left(\frac{2+4}{2}, \frac{2+4}{2}\right), E\left(\frac{4+2}{2}, \frac{4+6}{2}\right) \text{ and } F\left(\frac{2+2}{2}, \frac{2+6}{2}\right)$$

i.e. D(3, 3), E(3, 5) and F(2, 4).

For ΔDEF, we have

$(x_1 = 3, y_1 = 3)$, $(x_2 = 3, y_2 = 5)$ and $(x_3 = 2, y_3 = 4)$

$$\therefore \quad ar(\Delta DEF) = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}|3(5-4) + 3(4-3) + 2(3-5)|$$

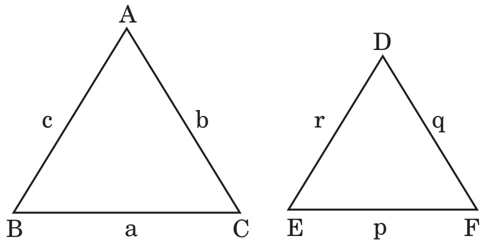
$$= \frac{1}{2}|(3 \times 1) + (3 \times 1) + 2 \times (-2)|$$

$$= \frac{1}{2}|3 + 3 - 4| = \frac{1}{2} \times 2 = 1 \text{ sq unit}$$

17. **Given:** ΔABC and ΔDEF in which BC = a, CA = b, AB = c and EF = p, DF = q, DE = r

Also, ΔABC ~ ΔDEF

To prove: $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r}$



Proof: Since ΔABC and ΔDEF are similar, therefore, their corresponding sides are proportional.

$\therefore \quad \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k$ (say) ...(i)

$\Rightarrow \quad a = kp, b = kq$ and $c = kr$

$\therefore \quad \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{a+b+c}{p+q+r} = \frac{kp+kq+kr}{p+q+r} = \frac{k(p+q+r)}{p+q+r} = k$ ...(ii)

From (i) and (ii), we get

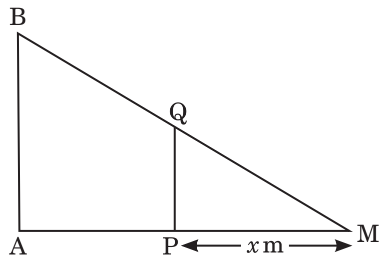
$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{a+b+c}{p+q+r}$$

$$= \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$$ (Each equal to k)

Hence proved.

OR

Let AB be the lamp post and PQ be the boy, where P is the position of the boy after 4 seconds.



AP = Distance in 4 s at 0.8 m/s

= (4 × 0.8) = 3.2 m

PM is the length of shadow of the boy.

Let PM = x m

In ΔAMB and ΔPMQ, we have

$\angle$MAB = $\angle$MPQ = 90° ($\because$ Both the lamp and the boy stand vertically erect)

$\angle$AMB = $\angle$PMQ (Common)

$\therefore$ ΔAMB ~ ΔPMQ (AA similarity)

and so, $\frac{AM}{PM} = \frac{AB}{PQ}$ (Corresponding sides of similar triangles are proportional)

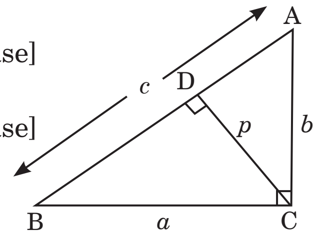
$\Rightarrow \quad \frac{AP+PM}{PM} = \frac{AB}{PQ} \Rightarrow \frac{3.2+x}{x} = \frac{3.3}{1.1}$ ($\because$ AB = 3.3 m, PQ = 110 cm = 1.1m)

$\Rightarrow \quad 3.2 + x = 3x \Rightarrow 2x = 3.2 \Rightarrow x = 1.6$ m

$\therefore$ The length of the shadow of the boy after 4 seconds is 1.6 m.

18. (*i*) We have

$$\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times CD = \frac{1}{2}cp \quad \text{[Taking AB as base]}$$



and $$\text{ar}(\Delta ABC) = \frac{1}{2} \times BC \times AC = \frac{1}{2}ab \quad \text{[Taking BC as base]}$$

$$\therefore \quad \frac{1}{2}cp = \frac{1}{2}ab \Rightarrow cp = ab$$

(*ii*) $$cp = ab \Rightarrow p = \frac{ab}{c} \Rightarrow \frac{1}{p} = \frac{c}{ab}$$

Squaring both sides, we get

$$\Rightarrow \quad \frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} \quad (\because AB^2 = AC^2 + BC^2)$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence, $$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ Hence proved.

19. Putting $\frac{1}{2x+3y} = u$ and $\frac{1}{3x-2y} = v$, the given equations become

$$\frac{u}{2} + \frac{12v}{7} = \frac{1}{2} \Rightarrow 7u + 24v = 7 \quad ...(i)$$

and $$7u + 4v = 2 \quad ...(ii)$$

Subtracting (*ii*) from (*i*), we get

$$20v = 5 \Rightarrow v = \frac{5}{20} = \frac{1}{4}$$

Putting $v = \frac{1}{4}$ in (*i*), we get

$$7u + \left(24 \times \frac{1}{4}\right) = 7 \Rightarrow 7u + 6 = 7 \Rightarrow 7u = 1 \Rightarrow u = \frac{1}{7}$$

Now, $$u = \frac{1}{7} \Rightarrow \frac{1}{2x+3y} = \frac{1}{7}$$

$$\Rightarrow \quad 2x + 3y = 7 \quad ...(iii)$$

and $$v = \frac{1}{4} \Rightarrow \frac{1}{3x-2y} = \frac{1}{4}$$

$$\Rightarrow \quad 3x - 2y = 4 \quad ...(iv)$$

Multiplying (*iii*) by 2 and (*iv*) by 3 and then adding the results, we have

$$4x + 9x = 14 + 12$$

$$\Rightarrow \quad 13x = 26 \Rightarrow x = 2$$

Putting $x = 2$ in (*iii*), we get

$$(2 \times 2) + 3y = 7 \Rightarrow 3y = (7 - 4) = 3 \Rightarrow y = 1$$

Hence, $x = 2$ and $y = 1$

20. Radius of the sphere, R = 3 cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi R^3 = \left(\frac{4}{3}\pi \times 3 \times 3 \times 3\right) \text{cm}^3$

$= 36\pi \text{ cm}^3$

Length of the wire, h = 36 m = 3600 cm

Let the radius of the wire be r cm.

Volume of the wire $= \pi r^2 h = (\pi r^2 \times 3600) \text{ cm}^3$

But, volume of the wire = volume of the sphere

$\Rightarrow \; 3600\pi r^2 = 36\pi \Rightarrow r^2 = \frac{1}{100} \Rightarrow r = \sqrt{\frac{1}{100}} \text{cm} = \frac{1}{10} \text{cm} = 1\text{mm}$

Hence, thickness of the wire = its diameter = 2 mm.

21. We have $5 + 8 + f_1 + 20 + f_2 + 2 = 50 \Rightarrow f_2 = 15 - f_1$

Now, we may prepare the table given below:

Class interval	**Frequency (f_i)**	**Class mark x_i**	**($f_i x_i$)**
10 – 30	5	20	100
30 – 50	8	40	320
50 – 70	f_1	60	$60f_1$
70 – 90	20	80	1600
90 – 110	$15 - f_1$	100	$1500 - 100f_1$
110 – 130	2	120	240
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 3760 - 40f_1$

$\therefore$ Mean $= \frac{\Sigma(f_i x_i)}{\Sigma f_i}$

$\Rightarrow$ $65.6 = \frac{3760 - 40f_1}{50} \Rightarrow 3760 - 40f_1 = 3280$

$\Rightarrow$ $40f_1 = 480 \Rightarrow f_1 = 12$

Thus, $f_1 = 12$ and $f_2 = 15 - 12 = 3$

22. Number of all possible outcomes = 36

(*i*) Let E_1 be the event of getting two numbers whose sum is 5. Then, the favourable outcomes are (1, 4), (2, 3), (3, 2), (4, 1)

Number of favourable outcomes = 4

$\therefore$ P(getting two numbers whose sum is 5) $= P(E_1) = \frac{4}{36} = \frac{1}{9}$

(*ii*) Let E_2 be the event of getting a even number on both dice. Then, the favourable outcomes are

(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6).

Number of favourable outcomes = 9

$\therefore$ P(getting even number on both dice) $= P(E_2) = \frac{9}{36} = \frac{1}{4}$

(*iii*) Let E_3 be the event of getting a doublet.

Then, the favourable outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Number of favourable outcomes = 6

$\therefore$ P(getting a doublet) = $P(E_3) = \frac{6}{36} = \frac{1}{6}$

23. LHS $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2(1)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\sin^4\theta - 2\sin^2\theta\cos^2\theta + 2\cos^4\theta - 3\sin^4\theta - 3\cos^4\theta + 1$

$= -\sin^4\theta - 2\sin^2\theta\cos^2\theta - \cos^4\theta + 1$

$= 1 - (\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta)$

$= 1 - (\sin^2\theta + \cos^2\theta)^2$ $\quad (\because (a^2 + b^2)^2 = a^4 + 2a^2b^2 + b^4)$

$= 1 - (1)^2 = 0 =$ RHS $\quad$ Hence proved.

OR

We have

$$\frac{\sec^2 54° - \cot^2 36°}{\text{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \sec^2 52° - \sin^2 45°$$

$$= \frac{\sec^2(90° - 36°) - \cot^2 36°}{\text{cosec}^2(90° - 33°) - \tan^2 33°} + 2\sin^2 38°.\sec^2(90° - 38°) - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{\text{cosec}^2 36° - \cot^2 36°}{\sec^2 33° - \tan^2 33°} + 2\sin^2 38° \text{cosec}^2 38° - \frac{1}{2}$$

$$= \frac{1 + \cot^2 36° - \cot^2 36°}{1 + \tan^2 33° - \tan^2 33°} + 2\sin^2 38° \text{cosec}^2 38° - \frac{1}{2}$$

$$= 1 + 2\sin^2 38° \cdot \frac{1}{\sin^2 38°} - \frac{1}{2}$$

$$= 1 + 2(1) - \frac{1}{2}$$

$$= \left(1 - \frac{1}{2}\right) + 2$$

$$= \frac{1}{2} + 2 = \frac{5}{2}$$

24. Let the usual speed be x km/h

Actual speed = $(x + 250)$ km/h

Time taken at usual speed = $\left(\frac{1500}{x}\right)$hours

Time taken at actual speed = $\left(\frac{1500}{x + 250}\right)$hours

Difference between the two times taken = $\frac{1}{2}$ hour

$$\therefore \quad \frac{1500}{x} - \frac{1500}{x + 250} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x+250} = \frac{1}{3000} \Rightarrow \frac{x+250-x}{x(x+250)} = \frac{1}{3000}$$

$$\Rightarrow \quad \frac{250}{x^2+250x} = \frac{1}{3000} \Rightarrow x^2 + 250x - 750000 = 0$$

$$\Rightarrow \quad x^2 + 1000x - 750x - 750000 = 0$$

$$\Rightarrow \quad x(x + 1000) - 750(x + 1000) = 0$$

$$\Rightarrow \quad (x + 1000)(x - 750) = 0$$

$$\Rightarrow \quad (x + 1000) = 0 \text{ or } (x - 750) = 0$$

$$\Rightarrow \quad x = -1000 \text{ or } x = 750$$

$$\Rightarrow \quad x = 750 \qquad (\because \text{ speed cannot be negative})$$

Hence, the usual speed of the aeroplane was 750 km/h.

OR

Suppose B alone takes x days to finish the work and A alone can finish it in $(x - 6)$ days.

$$\text{B's 1 day's work} = \frac{1}{x}$$

$$\text{A's 1 day's work} = \frac{1}{x-6}$$

$$\text{(A + B)'s 1 day's work} = \frac{1}{4}$$

$$\therefore \quad \frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{x-6+x}{x(x-6)} = \frac{1}{4} \Rightarrow \frac{2x-6}{x^2-6x} = \frac{1}{4}$$

$$\Rightarrow \quad x^2 - 6x = 8x - 24 \Rightarrow x^2 - 14x + 24 = 0$$

$$\Rightarrow \quad x^2 - 12x - 2x + 24 = 0$$

$$\Rightarrow \quad x(x - 12) - 2(x - 12) = 0$$

$$\Rightarrow \quad (x - 12)(x - 2) = 0$$

$$\Rightarrow \quad x = 12 \text{ or } x = 2$$

$$\Rightarrow \quad x = 12 \qquad (\because x = 2 \Rightarrow x - 6 < 0)$$

Hence, B alone can finish the work in 12 days.

25. Steps of construction:

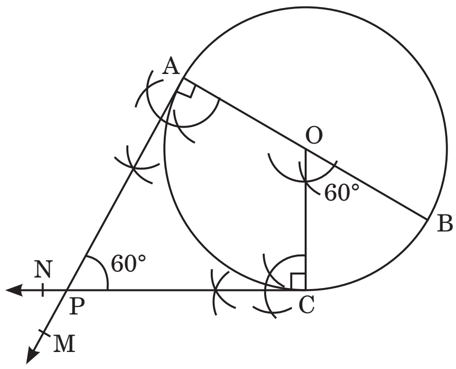
Step 1: Draw a circle with O as centre and radius = 3 cm

Step 2: Draw any diameter AOB of this circle.

Step 3: Construct ∠BOC = 60° such that radius OC meets the circle at C.

Step 4: Draw AM⊥ AB and CN ⊥ OC. Let AM and CN intersect each other at P.

Then, PA and PC are the desired tangents to the given circle, inclined at an angle of 60°.



26. Let a be the first term and d be the common difference of the given AP. Then,

S_1 = Sum of first n terms of the given AP

S_2 = Sum of first $2n$ terms of the given AP

S_3 = Sum of first $3n$ terms of the given AP

$$S_1 = \frac{n}{2}[2a + (n-1)d]$$

$$S_2 = \frac{2n}{2}[2a + (2n-1)d]$$

and $$S_3 = \frac{3n}{2}[2a + (3n-1)d]$$

$\Rightarrow$ $$3(S_2 - S_1) = 3.[\{2an + n(2n-1)d\} - \{na + \frac{1}{2}(n)(n-1)d\}]$$

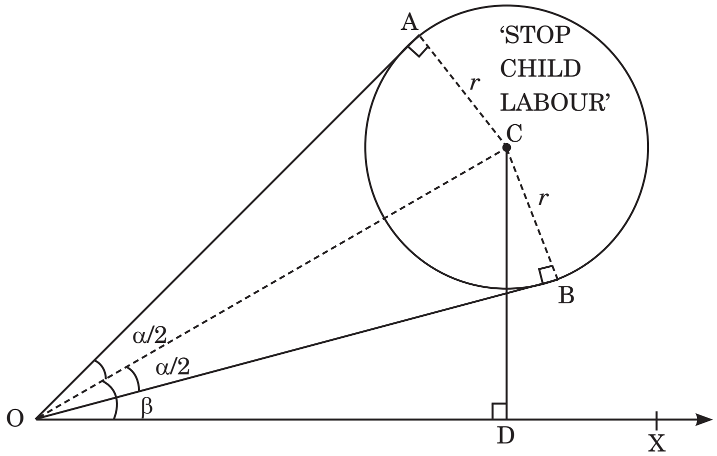
$$= 3 \cdot \left[na + \frac{3}{2}n^2 d - \frac{1}{2}nd\right]$$

$$= \frac{3n}{2}[2a + 3nd - d]$$

$$= \frac{3n}{2}\{2a + (3n-1)d\} = S_3$$

$\therefore$ $$S_3 = 3(S_2 - S_1)$$ Hence proved.

27. Let us represent the balloon by a circle with centre C and radius r. Let OX be the horizontal ground and let O be the point of observation. From O, draw tangents OA and OB to the circle. Join CA, CB and CO. Draw CD $\perp$ OX.



$\therefore$ $$\angle AOB = \alpha,\ \angle DOC = \beta \text{ and}$$

$$\angle AOC = \angle BOC = \frac{\alpha}{2}$$

In right-angled ΔOAC, we have

$$\frac{OC}{AC} = \text{cosec}\frac{\alpha}{2}$$

$\Rightarrow$ $$\frac{OC}{r} = \text{cosec}\frac{\alpha}{2} \Rightarrow OC = r\,\text{cosec}\,\frac{\alpha}{2} \qquad ...(i)$$

In right-angled ΔODC, we have

$\Rightarrow$ $$\frac{CD}{OC} = \sin\beta$$

$\Rightarrow \qquad CD = OC \times \sin \beta$

$\Rightarrow \qquad CD = r \sin \beta . \text{cosec } \frac{\alpha}{2}$ (Using (*i*))

Hence, the height of the centre of the balloon from the ground is $r \sin \beta \text{ cosec } \frac{\alpha}{2}$.

Child labour is illegal, harmful, offense and curse for our society. Each individual should be sent to school for education so that they can contribute for nation's growth and development.

28. Total number of all possible outcomes = 52

(*i*) Let E_1 be the event of getting either a red card or a king.

There are 26 red cards (including 2 kings) and there are 2 more kings.

So, the number of favourable outcomes = 26 + 2 = 28

$\therefore$ P(getting either a red card or a king) = $P(E_1) = \frac{28}{52} = \frac{7}{13}$

(*ii*) Let E_2 be the event of getting a card which is neither a red card nor a queen

There are 26 red cards (including 2 queens) and there are 2 more queens.

So, the number of non-favourable outcomes = 26 + 2 = 28

$\therefore$ The number of favourable outcomes = 52 – 28 = 24

$\therefore \; P(E_2) = \frac{24}{52} = \frac{6}{13}$

OR

Total number of pens = 144

Number of defective pens = 20

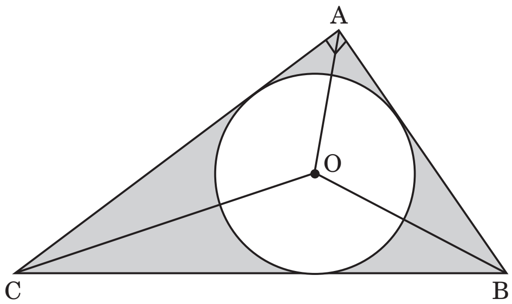
$\therefore$ Number of good pens = 144 – 20 = 124

Let E be the event that the pen is good and Nuri will buy it.

Then $\overline{E}$ (not E) is the event that the pen is defective and Nuri will not buy it. Hence,

(*i*) $P(E) = \frac{124}{144} = \frac{31}{36}$ (*ii*) $P(\overline{E}) = \frac{20}{144} = \frac{5}{36}$

29. Since ΔABC is a right-angled triangle,



$$AC^2 = BC^2 - AB^2$$

$$\Rightarrow \quad AC^2 = 10^2 - 6^2 = 64$$

$$\Rightarrow \quad AC = 8 \text{ m}$$

$$\therefore \quad ar(\Delta ABC) = \frac{1}{2} \times AC \times AB$$

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ m}^2 \qquad ...(i)$$

Let *r* m be the radius of the inscribed circle from OA, OB and OC

Then $\quad ar(\Delta OBC) = \frac{1}{2} \times 10 \times r \text{ m}^2 = 5r \text{ m}^2$

$$ar(\Delta OAC) = \frac{1}{2} \times 8 \times r \text{ m}^2 = 4r \text{ m}^2$$

$$\text{ar}(\Delta OAB) = \frac{1}{2} \times 6 \times r \text{ m}^2 = 3r \text{ m}^2$$

$$\therefore \quad \text{Area}(\Delta ABC) = \text{ar}(\Delta OAB) + \text{ar}(\Delta OBC) + \text{ar}(\Delta OAC)$$

$$= (3r + 5r + 4r) \text{ m}^2$$

$$= 12r \text{ m}^2 \quad \text{...}(ii)$$

From (*i*) and (*ii*), we have

$$r = 2 \text{ m}$$

Hence, area of the incircle $= \pi(2)^2 \text{ m}^2 = 4\pi \text{ m}^2$

$\Rightarrow$ Area of shaded region = Area of ΔABC – Area of the incircle

$$= (24 - 4\pi) \text{ m}^2 = (24 - 4 \times 3.14) \text{ m}^2$$

$$= 11.44 \text{ m}^2$$

30. We know that the lengths of tangents from an external point to a circle are equal.

$\therefore$ AP = AR ...(*i*) (Tangents from A)

BP = BQ ...(*ii*) (Tangents from B)

CQ = CR ...(*iii*) (Tangents from C)

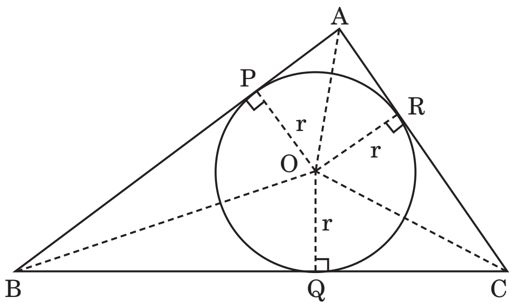
(*i*) AB + CQ = AP + BP + CQ

= AR + BQ + CR (Using (*i*), (*ii*) and (*iii*))

= (AR + CR) + BQ

= AC + BQ

(*ii*) Join OA, OB and OC.



Area (ΔABC) = area (ΔOAB) + area (ΔOBC) + area(ΔOCA)

$$= \left(\frac{1}{2} \times AB \times OP\right) + \left(\frac{1}{2} \times BC \times OQ\right) + \left(\frac{1}{2} \times CA \times OR\right)$$

$$= \left(\frac{1}{2} \times AB \times r\right) + \left(\frac{1}{2} \times BC \times r\right) + \left(\frac{1}{2} \times CA \times r\right)$$

$$= \frac{1}{2}(AB + BC + CA) \times r$$

$$= \frac{1}{2}(\text{Perimeter of } \Delta ABC) \times r$$

Hence proved.