Solutions to RMT/Set-1

1. (*d*) 2 and (–3) are zeroes of the polynomial it means when $x = 2$ or -3 then $y = 0$

Therefore point on the x-axis is (2, 0).

2. (*a*) System of equations $2x - ky = 14$ and $3x + y = 10$ has unique solution.

Therefore $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (for $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$)

$$\Rightarrow \quad \frac{2}{3} \neq \frac{-k}{1}$$

$$\Rightarrow \quad k \neq \frac{-2}{3}$$

3. (*d*) Secant intersects the circle at more than two points.

4. (*a*) $S_n = n^2 - 3n$

$S_1 = (1)^2 - 3 \times 1 = -2$

$S_2 = (2)^2 - 3 \times 2 = -2$

Therefore $a_2 = S_2 - S_1$

$\Rightarrow a_2 = -2 -(-2) = 0$ and $a_1 = -2$

Common difference $= 0 - (-2) = 2$

5. (*d*)

$$\text{Volume of cone} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$= \frac{1}{3} \times 156 \times 8$$

$$= 416 \text{ cm}^3$$

6. (*b*) We have $\sec\theta = \frac{5}{3} \Rightarrow \cos\theta = \frac{3}{5}$

Now, $$\frac{4 + \cos\theta}{4 - \cos\theta} = \frac{4 + \frac{3}{5}}{4 - \frac{3}{5}} = \frac{23}{17}$$

7. (*a*) $\angle OPT = 900$ (Radius is perpendicular to tangent at the point of contact)

and $\angle OPQ = \angle TPQ - \angle OPT$

$= 100° - 90° = 10°$

8. (*c*) We have,

$$p(x) = x^2 - 3x - 4$$

$$= x^2 - 4x + x - 4$$

$$= x(x - 4) + 1(x - 4)$$

$$= (x - 4)(x + 1)$$

For zeroes $p(x) = 0$

$\Rightarrow \quad x - 4 = 0$ or $x + 1 = 0 \Rightarrow x = 4$ or $x = -1$

Hence zeroes are 4, –1.

9. (*a*) The highest frequency is of class 40-60.

Therefore modal class is 40-60.

Hence, upper limit of modal class is 60.

10. (*d*) $\angle PTO = 90°$ (Radius is perpendicular to tangent at the point of contact)

In ΔPTO, $\angle PTO + \angle TPO = x$ (Exterior angle property of triangle)

$\Rightarrow$ $90° + 25° = x$

$\Rightarrow$ $x = 115°$

11. (*c*) We have, $x^2 - 4 = 0$

$$\text{Discriminant, D} = b^2 - 4ac$$
$$= (0)^2 - 4\,(1)(-4)$$
$$= 16 > 0$$

Therefore, roots are real and distinct.

12. (*d*) By putting $\theta = 60°$ in $\tan^2\theta + \cot^2\theta$, we get

$$\tan^2 60° + \cot^2 60° = \left(\sqrt{3}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$
$$= 3 + \frac{1}{3} = \frac{10}{3}$$

13. (*b*) Volume of a solid sphere is 616 cm^3.

$$V = \frac{4}{3}\pi r^3$$
$$\Rightarrow \quad 616 = \frac{4}{3}\pi r^3$$
$$\Rightarrow \quad 616 = \frac{4}{3} \times \frac{22}{7} \times r^3$$
$$\Rightarrow \quad r^3 = 147 \text{ cm}^3$$

14. (*d*) $0 \le P(E) \le 1$, where $P(E)$ is the probability of an event E.

Here $\frac{17}{16} > 1$.

Therefore $\frac{17}{16}$ cannot be the probability of an event.

15. (*b*) Let the points be P(–4, 0), Q(4, 0) and R(0, 3)

$PQ^2 = (-4 - 4)^2 + (0 - 0)^2$ (Using distance formula)

$PQ^2 = 64$ or $PQ = 8$ units

$RQ^2 = (4 - 0)^2 + (0 - 3)^2$ (Using distance formula)

$RQ^2 = 25$ or $RQ = 5$ units

and $RP^2 = (-4 - 0)^2 + (0 - 3)^2$ (Using distance formula)

$RP^2 = 25$ or $RP = 5$ units.

Since, $RQ = RP$

Hence, the given points are vertices of an isosceles triangle.

16. (*c*) Using empirical formula: mode = 3 median – 2 mean

$\Rightarrow$ $12 = 3 \text{ Median} - 2 \times 24$

$\Rightarrow$ $3 \text{ Median} = 60$

$\Rightarrow$ $\text{Median} = 20$

17. (*c*) Centre is the mid-point of diameter.

Therefore coordinates of centre are $\left(\frac{-6+6}{2}, \frac{3+4}{2}\right)$ i.e. $\left(0, \frac{7}{2}\right)$

18. (*d*) Total cards = 52

Face cards = 12

Not face cards = 52 – 12 = 40

P(getting not a face card) = $\frac{40}{52} = \frac{10}{13}$

19. (*c*) Smallest composite numbers are 4 and 6.

$4 = 2 \times 2$

$6 = 2 \times 3$

Therefore HCF = 2,

Assertion (A) is true but reason (R) is false.

20. (*a*) Area of sector = $\frac{1}{2}$ × radius × length of arc

Area of sector = $\frac{1}{2} \times 7 \times 4 = 14$ cm^2

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

21. (*a*)

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$
$$\text{HCF} = 2 \times 2 \times 2 \times 3 = 24$$
$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$$

OR

(*b*) M = $p^5q^3r^2$ and N = p^7q^5r

Therefore HCF (M, N) = p^5q^3r

LCM (M, N) = $p^7q^5r^2$

22. (*a*) Number of days in a leap year = 366

Number of weeks in a year = 52

Total days in 52 weeks = 7 × 52 = 364

Remaining days = 2

Sample space for remaining days = {(S,M), (M,T), (T,W), (W, TH), (TH, F), (F, S), (S, S)}

Favourable outcomes = {(S, S), (S,M)}

P(getting 53 Sundays) = $\frac{2}{7}$.

OR

(*b*) Sample space = {3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

Favourable outcomes = {4, 6, 8, 10, 12, 14, 16, 18, 20}

P(getting an even number) = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{9}{18} = \frac{1}{2}$

23.

$$\frac{\sin^2 30° - \tan^2 60°}{2\cos^2 60°} = \frac{\frac{1}{2} \times \frac{1}{2} - \sqrt{3} \times \sqrt{3}}{2 \times \frac{1}{2} \times \frac{1}{2}}$$

$$= \frac{\frac{1}{4} - 3}{\frac{1}{2}} = \frac{-11}{4} \times \frac{2}{1} = \frac{-11}{2}$$

24. Let the point on the x-axis be $P(x, 0)$.

According to question, $PA = 10$

Or $(PA)^2 = 100$

$\Rightarrow \quad (x - 11)^2 + (0 + 8)^2 = 100$ (Using distance formula)

$\Rightarrow \quad (x - 11)^2 = 100 - 64$

$\Rightarrow \quad (x - 11)^2 = 36$

$\Rightarrow \quad (x - 11) = \pm 6$

Either $x - 11 = 6$ or $x - 11 = -6$

$\Rightarrow \quad x = 17$ or $x = 5$

Therefore points on the x-axis are (17, 0) or (5, 0).

25. Let the y-axis divides line segment joining the points (5, –6) and (–1, –4) in the ratio $k : 1$.

Therefore point $(0, y)$ divides line joining the points (5, –6) and (–1, –4) in the ratio $k : 1$.

Using section formula: $x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$\Rightarrow \quad 0 = \dfrac{k \times (-1) + 1 \times 5}{k + 1}$

$\Rightarrow \quad \dfrac{5 - k}{k + 1} = 0$

$\Rightarrow \quad 5 - k = 0 \Rightarrow k = 5$

Therefore the y-axis divides line segment joining the points (5, –6) and (–1, –4) in the ratio 5 : 1.

26. (*a*) (*i*) In ΔABC and ΔAMP

$\angle ABC = \angle AMP$ (90° Each)

$\angle CAB = \angle MAP$ (Common)

Therefore $\Delta ABC \sim \Delta AMP$ (AA Criterion)

(*ii*) Since $\Delta ABC \sim \Delta AMP$

$\therefore \quad \dfrac{CA}{PA} = \dfrac{BC}{MP}$ (Corresponding sides of similar triangles)

OR

(*b*) In ΔABE, DF || AE,

$\therefore \quad \dfrac{BD}{AD} = \dfrac{BF}{EF}$ (Basic proportionality theorem)...(*i*)

In ΔABC, DE || AC,

$\therefore \quad \dfrac{BD}{AD} = \dfrac{BE}{EC}$ (Basic proportionality theorem)...(*ii*)

Using (*i*) and (*ii*), we get

$\dfrac{BF}{EF} = \dfrac{BE}{EC}.$ Hence proved

27. Let length and breadth of the rectangular field be x m and y m respectively.

According to the first condition,

$2(x + y) = 82$

$\Rightarrow \quad x + y = 41$

$\Rightarrow \quad y = 41 - x$ ...(*i*)

According to the second condition,

$xy = 400$ ...(*ii*)

Using equations (*i*) and (*ii*), we get

$x(41 - x) = 400$

$\Rightarrow \quad x^2 - 41x + 400 = 0$

$\Rightarrow \qquad x^2 - 25x - 16x + 400 = 0$

$\Rightarrow \qquad x(x - 25) - 16(x - 25) = 0$

$\Rightarrow \qquad (x - 25)(x - 16) = 0$

$\Rightarrow x = 16$ or 25, if $x = 16$ then $y = 41 - 16 = 25$

if $x = 25$ then $y = 16$

Therefore dimensions are 25 m and 16 m.

28. Let α and β are zeroes of the polynomial $P(x) = x^2 + x + 5$

Therefore, $\alpha + \beta = -1$ and $\alpha\beta = 5$ ...(*i*)

Now, zeroes are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

$$\text{Sum of zeroes} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(-1)^2 - 2 \times (5)}{5} = \frac{-9}{5}$$

$$\text{Product of zeroes} = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$$

Therefore required polynomial $= k\{x^2 - (\text{sum of zeroes})x + \text{product of zeroes}\}$, where k = constant

$$= k\left(x^2 + \frac{9}{5}x + 1\right)$$

$$= 5x^2 + 9x + 5$$

29. $\text{LHS} = \sec^4 x - \sec^2 x$

$= \sec^2 x \cdot \sec^2 x - \sec^2 x$

$= \sec^2 x\ (\sec^2 x - 1)$ (Using identity $\sec^2 x = 1 + \tan^2 x$)

$= (1 + \tan^2 x) \tan^2 x$

$= \tan^2 x + \tan^4 x = \text{RHS}$

30. (*a*) Let radius of circular playground be r m.

According to the question, $\pi r^2 = 22176$

$\Rightarrow \qquad \frac{22}{7} r^2 = 22176$

$\Rightarrow \qquad r^2 = \frac{22176 \times 7}{22}$

$\Rightarrow \qquad r^2 = 7056$

$\Rightarrow \qquad r = 84$ m

For fencing we have to calculate circumference of the circle.

Therefore circumference, $\quad C = 2\pi r$

$= 2 \times \frac{22}{7} \times 84$

$= 528$ m

$\therefore \qquad$ Cost of fencing = rate × circumference

= ₹ 50 × 528

= ₹ 26400

OR

(*b*) Given: $r = 21$ cm, central angle $= 60°$

$$\text{Area of sector} = \pi r^2 \frac{\theta}{360°}$$
$$= \frac{22}{7} \times 21 \times 21 \times \frac{60°}{360°}$$
$$= 231 \text{ cm}^2$$

We know that area of sector $= \frac{1}{2} \times l \times r$

$\Rightarrow$ $$231 = \frac{1}{2} \times l \times 21$$

$\Rightarrow$ $$l = 22 \text{ cm}$$

31. Let $\sqrt{5}$ be a rational number.

Therefore $\sqrt{5} = \frac{x}{y}$ where x, y are co-prime and y is not equal to zero.

Squaring on both sides and rearranging, we get

$$5y^2 = x^2$$

Therefore, 5 divides x^2.
$\Rightarrow$ 5 divides x.
So, we can write $x = 5c$ for some integer c.
Substituting for x, we get $5y^2 = 25c^2$, $\Rightarrow y^2 = 5c^2$
This means that 5 divides y^2, and so 5 divides y.
Therefore, x and y have at least 5 as a common factor.
But this contradicts the fact that x and y have no common factors other than 1.
This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is rational.
Hence $\sqrt{5}$ is an irrational number.

32. (*a*) $37x + 43y = 123$ ...(*i*)

$43x + 37y = 117$ ...(*ii*)

Adding equations (*i*) and (*ii*), we get

$$80x + 80y = 240$$

Or $x + y = 3$ ...(*iii*)

Subtracting equation (*ii*) from equation (*i*), we get

$$-6x + 6y = 6$$

Or $-x + y = 1$ ...(*iv*)

Adding equations (*iii*) and (*iv*), we get

$$y = 2$$

Putting $y = 2$ in equation (*iv*), we get

$$-x + 2 = 1 \Rightarrow x = 1$$

Hence, $x = 1$, $y = 2$

OR

(*b*) Let present age of father be x years and sum of present ages of two children be y years.

According to the question, $x = 2y$ ...(*i*)

After 20 years age of father be $(x + 20)$ years

After 20 years sum of ages of two children be $(y + 40)$ years

According to the condition, $x + 20 = y + 40$

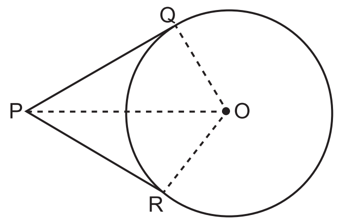
Or $x - y = 20$ ...(*ii*)

Putting the value of x from equation (*i*) in equation (*ii*), we get

$2y - y = 20 \Rightarrow y = 20$ years

$\therefore$ age of father $= 2 \times 20 = 40$ years

33. **Given:** A circle with centre O, a point P lying outside the circle and two tangents PQ and PR are drawn on the circle from point P.



To prove: PQ = PR.

Construction: Join OP, OQ and OR.

Proof: In ΔOQP and ΔORP

$\angle OQP = \angle ORP$ (Radius is perpendicular to tangent at point of contact)

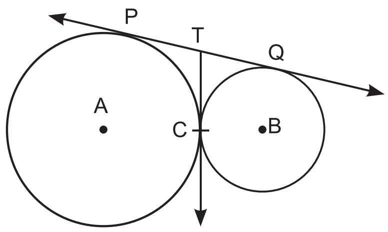
$OQ = OR$ (Radii of the same circle)

$OP = OP$ (Common)

Therefore, $\Delta OQP \cong \Delta ORP$ (RHS)

$\therefore$ $PQ = PR$ (CPCT) Hence proved

Now, in given figure,



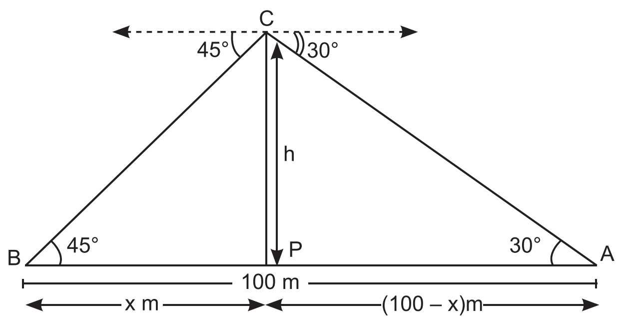
TP = TC ...(*i*) (The lengths of tangents drawn from an external point to the circle are equal)

TQ = TC ...(*ii*) (The lengths of tangents drawn from an external point to the circle are equal)

From (*i*) and (*ii*), TP = TQ, i.e. T is mid-point of PQ.

Hence, the common tangent to the circles at C, bisects the common tangent at P and Q.

34. Let A and B be the positions of two ships and PC is lighthouse, angle of depression of two ships are 30° and 45°.



Let BP be x m.

Then $AP = (100 - x)$ m

Now, in triangle BPC, $\dfrac{CP}{BP} = \tan 45°$

$\Rightarrow$ $\dfrac{h}{x} = 1 \Rightarrow h = x$ ...(*i*)

In triangle APC, $\dfrac{CP}{AP} = \tan 30°$

$\Rightarrow$ $\dfrac{h}{100 - x} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $h\sqrt{3} = 100 - x$ ...(*ii*)

From equations (*i*) and (*ii*), we get

$$h\sqrt{3} = 100 - h$$

$$\Rightarrow \quad h\sqrt{3} + h = 100$$

$$\Rightarrow \quad h(\sqrt{3}+1) = 100$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\Rightarrow \quad h = \frac{100}{2} \times (1.732 - 1)$$

$$\Rightarrow \quad h = 50 \times 0.732$$

$$\Rightarrow \quad h = 36.600 \text{ m}$$

Hence, height of the lighthouse is 36.6 m.

35. (*a*) Let $A = 35$, here $h = 10$

Class intervals	**Frequency (f_i)**	x_i	$u_i = \dfrac{x_i - A}{h}$	$f_i u_i$
10-20	15	15	–2	–30
20-30	10	25	–1	–10
30-40	12	35	0	0
40-50	17	45	1	17
50-60	4	55	2	8
Total	$\Sigma f_i = 58$			$\Sigma f_i u_i = -15$

From formula, $\bar{x} = A + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times h$, we get

$$\bar{x} = 35 + \frac{(-15)}{58} \times 10$$

$$\bar{x} = 35 - 2.59 = 32.41$$

Hence, mean = 32.41

Modal class = 40-50

Here, $l = 40, f_1 = 17, f_2 = 4, f_0 = 12$

$$\text{Mode} = l + \left(\frac{f_1 - f_o}{2f_1 - f_o - f_2}\right) \times h$$

$$\text{Mode} = 40 + \left(\frac{17-12}{2 \times 17 - 12 - 4}\right) \times 10$$

$$= 40 + \frac{5}{18} \times 10$$

$$= 40 + 2.78 = 42.78$$

Hence, mode 42.78

OR

(*b*)

Class intervals	Frequency	*cf*
0-10	3	3
10-20	5	8
20-30	11	19
30-40	10	29
40-50	x	$29 + x$
50-60	3	$32 + x$
60-70	2	$34 + x$

Median class = 30-40

Here, $l = 30$, $cf = 19$, $f = 10$, $h = 10$, $n = 34 + x$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 34.5 = 30 + \left(\frac{\frac{34 + x}{2} - 19}{10}\right) \times 10$$

$$\Rightarrow \quad 34.5 = 30 + \left(\frac{34 + x}{2} - 19\right)$$

$$\Rightarrow \quad 4.5 = \left(\frac{34 + x}{2} - 19\right)$$

$$\Rightarrow \quad 23.5 = \frac{34 + x}{2}$$

$$\Rightarrow \quad 47 = x + 34$$

$$\Rightarrow \quad x = 13$$

36. (*i*) Radius of sphere = 30 cm

$$\text{Volume of spherical part} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 30 \times 30 \times 30$$

$$= 113142.86 \text{ cm}^3$$

(*ii*) The height and the diameter of the conical section of the tent are equal.

$$\text{Therefore } r = \frac{h}{2}$$

$$\text{Volume of conical part} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times \frac{h}{2} \times \frac{h}{2} \times h$$

$$= \frac{\pi h^3}{12} \text{ m}^3$$

(*iii*) (*a*) The height of cylindrical part = $\frac{3}{2}h$

$$\text{Volume of cylinder} = \pi \times \frac{h}{2} \times \frac{h}{2} \times \frac{3}{2}h$$

$$= 3\pi\frac{h^3}{8}$$

$$\therefore \quad \frac{\text{Volume of conical part}}{\text{Volume of cylindrical part}} = \frac{\pi\frac{h^3}{12}}{3\pi\frac{h^3}{8}} = \frac{2}{9}$$

Volume of conical part : Volume of cylindrical part = 2 : 9

OR

(*iii*) (*b*) There are two horizontal planks (top and bottom).

Length of horizontal plank = 105 cm

Width of horizontal plank = 45 cm

Height of horizontal plank = 2 cm

Volume of planks = 2 × length × width × height

Volume of planks = 2 × 105 × 45 × 2 = 18900 cm^3

37. (*i*) According to the question

For AP, $a = 5$, $d = 3$ and $n = 10$

Using formula $a_n = a + (n - 1)d$, we get

$$a_n = 5 + (10 - 1) \times 3$$

$$= 5 + 27$$

$$= 32$$

Hence, there are 32 trees in the 10th row.

(*ii*) Reverse AP = 32, ..., 8, 5

Here $a = 32$, $d = -3$ and $n = 3$

Using formula $a_n = a + (n - 1)d$, we get

$$a_{\text{second last row}} = 32 + (3 - 1)(-3)$$

$$= 32 - 6 = 26$$

Hence, there are 26 trees in the second last row.

(*iii*) (*a*) Using formula $S_n = \frac{n}{2}(a + a_n)$

$$S_{10} = \frac{10}{2}(5 + 32)$$

$$= 5 \times 37 = 185$$

Hence, there are 185 trees in the 10 rows of the orchord.

OR

(*iii*) (*b*) Using formula $a_n = a + (n-1)d$

$$a_5 = 5 + (5-1)(3)$$
$$= 5 + 12 = 17$$
$$a_6 = 5 + (6-1)(3)$$
$$= 5 + 15 = 20$$

Total = 17 trees + 20 trees = 37 trees

38. (*i*) AA criterion

(*ii*) Distance travelled in 2 sec. = 2×1.2 m = 2.4 m

(*iii*) (*a*) Distance travelled in 2 sec. = $1.2 \times 2 = 2.4$ m.

In ΔABE and ΔCDE

$\angle ABE = \angle CDE$ (90° each)

$\angle E = \angle E$ (Common)

Therefore $\Delta ABE \sim \Delta CDE$ (AA criterion)

$$\therefore \quad \frac{AB}{CD} = \frac{BE}{DE}$$

$$\Rightarrow \quad \frac{3.6}{CD} = \frac{2.4+1}{1}$$

$$\Rightarrow \quad CD = \frac{3.6}{3.4} \text{ metres} = 1.06 \text{ metres}$$

OR

(*iii*) (*b*) Distance travelled in 4 sec. = $1.2 \times 4 = 4.8$ m

In ΔABE and ΔCDE

$\angle ABE = \angle CDE$ (90° each)

$\angle E = \angle E$ (Common)

Therefore $\Delta ABE \sim \Delta CDE$ (AA criterion)

$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\Rightarrow \quad \frac{3.6}{1.06} = \frac{4.8+DE}{DE}$$

$$\Rightarrow \quad 3.6\,DE = 5.088 + 1.06\,DE$$

$$\Rightarrow \quad 2.54\,DE = 5.088$$

$$\Rightarrow \quad DE = 2.0 \text{ m (approx.)}$$

Hence, the length of her shadow after 4 seconds is 2.0 m (approx.)