Solutions to RMM/Set-1

1. (*a*) Here

$$\theta = \sin^{-1}\{\sin(-600°)\}$$
$$= \sin^{-1}\{-\sin(720° - 120°)\}$$
$$= \sin^{-1}\{\sin 120°\} \neq 120°$$

$\begin{bmatrix} \text{as } \sin(-600°) = -\sin 600° \\ \text{as } \sin(4\pi - \theta) = -\sin\theta \end{bmatrix}$

So,
$$\sin^{-1}(\sin 120°) = \sin^{-1}[\sin(180° - 60°)] = \sin^{-1}(\sin 60°)$$
$$= 60° = \frac{\pi}{3} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$\therefore$
$$\theta = \frac{\pi}{3}$$

2. (*d*) Let $y = \sin^{-1}\sqrt{x-1}$

$\therefore$ $-1 \le \sqrt{x-1} \le 1$

$\Rightarrow$ $0 \le \sqrt{x-1} \le 1$

$\Rightarrow$ $0 \le x - 1 \le 1$

$\Rightarrow$ $1 \le x \le 2$

$x \in [1, 2]$

3. (*c*)

4. (*b*) Direction ratios of line through the points (1, – 1, 2) and (3, 4, – 2) are <2, 5, –4>.

Direction ratios of line through the points (0, 3, 2) and (3, 5, 6) are <3, 2, 4>.

Now $a_1a_2 + b_1b_2 + c_1c_2 = 3 \times 2 + 2 \times 5 + 4 \times (-4) = 0$

$\therefore$ Lines are perpendicular.

5. (*b*)

$$A = \begin{bmatrix} a & b \\ c & \dfrac{1+bc}{a} \end{bmatrix} \Rightarrow |A| = \frac{a(1+bc)}{a} - bc$$
$$= 1 + bc - bc = 1$$

$\Rightarrow$ A^{-1} exists.

Now,
$$\text{adj}\, A = \begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

$\Rightarrow$
$$A^{-1} = \frac{1}{|A|}(\text{adj}\, A) = 1\begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$
$$= \begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

6. (*b*) by definition.

7. (*d*) as
$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t}$$
$$= -\tan t$$

$\Rightarrow$
$$\left.\frac{dy}{dx}\right]_{t=\frac{3\pi}{4}} = 1$$

8. (c) $[x]$ represents greatest integer function less than or equal to x

$\therefore x - [x]$ is defined at all integral points.

Now $f(x) = x - [x]$ is defined at all integral points

Consider 'n' be any integer.

$$\text{LHL} = \lim_{x \to n^-} f(x) = \lim_{x \to n^-} x - [x]$$

$$= \lim_{x \to n^-} x - \lim_{x \to n^-} [x] \qquad \left\{\begin{array}{l}\text{as } x \to n^- \\ \Rightarrow [x] \to (n-1)\end{array}\right\}$$

$$= n - (n - 1) = 1$$

$$\text{RHL} = \lim_{x \to n^+} f(x) = \lim_{x \to n^+} x - [x] \qquad (\text{as } x \to n^+ \Rightarrow [x] \to n)$$

$$= \lim_{x \to n^+} x - \lim_{x \to n^+} [x]$$

$$= n - n = 0$$

$$\lim_{x \to n^-} f(x) \neq \lim_{x \to n^+} f(x)$$

$\Rightarrow$ $f(x)$ is not continuous at $x = n$

9. (a) Projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$= \frac{(2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1 + 4 + 4}}$$

$$= \frac{2 - 2 + 2}{3} = \frac{2}{3}$$

10. (c) as $y = x^2$ ...(i)

$$\Rightarrow \quad \frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$\Rightarrow \quad x = \frac{1}{2} \qquad \left[\because \frac{dx}{dt} = \frac{dy}{dt}\right]$$

So, from (i), we get

$$y = \frac{1}{4}$$

$\therefore$ Point is $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$.

11. (c)

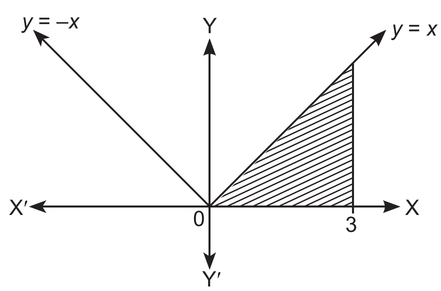
12. (a) as we have to find shaded area

$$\text{Area} = \int_0^3 |x| dx$$

$$= \int_0^3 x \, dx$$

$$= \left[\frac{x^2}{2}\right]_0^3$$

$$= \frac{9}{2} \text{ sq units}$$



13. (*d*) let $2^x = t \Rightarrow 2^x \cdot \log_e 2\, dx = dt$

$\therefore$

$$\int \frac{2^x}{\sqrt{1-4^x}}dx = \frac{1}{\log_e 2} \cdot \int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{\log_e 2}.\sin^{-1}t + C$$

$$= \frac{1}{\log_e 2} \cdot \sin^{-1}(2^x) + C \Rightarrow p = \frac{1}{\log_e 2}$$

14. (*d*) We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow$ $k^2 + k^2 + k^2 = 1$

$\Rightarrow$ $3k^2 = 1$

$\Rightarrow$ $k = \pm \frac{1}{\sqrt{3}}$

15. (*b*)

$$I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx \qquad ...(i)$$

$$\Rightarrow \quad I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x}+\sqrt{9-x}}\,dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_3^6 \frac{\sqrt{9-x}+\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx$$

$$\Rightarrow \quad 2I = \int_3^6 1\,dx = \Big[x\Big]_3^6$$

$$\Rightarrow \quad I = \frac{3}{2}$$

16. (*a*) Degree = 2, order = 2

17. (*c*)

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.4 + 0.3 - 0.5 = 0.2$$

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

$$= 0.3 - 0.2 = 0.1$$

18. (*d*) as, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

19. (*d*) $f(x) = \lambda x^2 - 5x + 5 \Rightarrow f'(x) = 2\lambda x - 5$

Now, $f'(1) = 5 \Rightarrow 2\lambda - 5 = 5 \Rightarrow \lambda = 5$

So, A is false.

But, R is true.

20. (*c*) Assertion is true but the reason is false.

21. As, $(1 + \sin^2 x)dy + (1 + y^2)\cos x\, dx = 0$

$$\Rightarrow \quad \frac{1}{1+y^2}dy = -\left(\frac{\cos x}{1+\sin^2 x}\right)dx$$

$$\Rightarrow \quad \int \frac{dy}{1+y^2} = \int \frac{-\cos x}{1+\sin^2 x}dx$$

$$\Rightarrow \quad \tan^{-1}y = -\tan^{-1}(\sin x) + C \qquad ...(i)$$

When $x = \frac{\pi}{2}, y = 0$ then from (i), $0 = -\frac{\pi}{4} + C \Rightarrow C = \frac{\pi}{4}$

Substituting $C = \frac{\pi}{4}$ in (i), we get

$\tan^{-1}y + \tan^{-1}(\sin x) = \frac{\pi}{4}$ as the required solution.

22. Let A be the event, when sum of 9 appears on both dice.

$A = \{(3, 6), (4, 5), (5, 4) (6, 3)\}$

Let B be the event, such that second die exhibits prime number.

$B = \{(1, 2), (2, 2), (2, 3), (2, 5), (1, 3), (1, 5)$
$(3, 2), (3, 3), (3, 5), (4, 2), (4, 5), (5, 2)$
$(4, 3), (5, 3), (5, 5), (6, 2), (6, 3), (6, 5)\}$

Now, $A \cap B = \{(4, 5), (6, 3)\}$

$\therefore$ Probability of getting 9 as the sum when second die exhibits prime number is given by,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}.$$

OR

As, sum of all probabilities in a probability distribution is 1, then

$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$

$\Rightarrow \quad 0.1 + k + 2k + 2k + k = 1$

$\Rightarrow \quad 6k = 0.9$

$\Rightarrow \quad k = \frac{9}{60} = 0.15$

23. Given $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ defined on $R : \{1, 2, 3\} \to \{1, 2, 3\}$

For reflexive: As $(1, 1), (2,2), (3, 3) \in R$.

Hence, reflexive

For symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$.

Hence, not symmetric.

For transitive: $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$. Hence, not transitive.

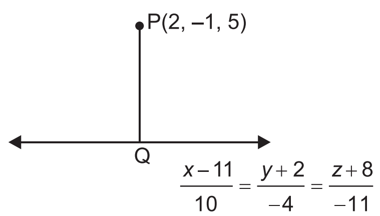
24. General point on the line

$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$ (say) is $Q\,(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$ ...(i)

Direction ratios of PQ are

$10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5$

i.e. $10\lambda + 9, -4\lambda - 1, -11\lambda - 13$



If PQ is perpendicular to the given line, then

$10\,(10\lambda + 9) - 4\,(-4\lambda - 1) - 11\,(-11\lambda - 13) = 0$

$\Rightarrow \quad 237\,\lambda = -237 \Rightarrow \lambda = -1$

Substituting in (i), we get the foot of perpendicular as $Q(1, 2, 3)$.

Length of perpendicular, $PQ = \sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2}$

$= \sqrt{1+9+4} = \sqrt{14}.$

25. Consider $(A + A')' = A' + (A')'$

$= A' + A = (A + A')$

As $(A + A')' = A' + A$, Hence, $A + A'$ is a symmetric matrix.

Consider $(A - A')' = A' - (A')' = A' - A$

$\Rightarrow \quad (A - A')' = -(A - A')$

Hence, $A - A'$ is a skew symmetric matrix.

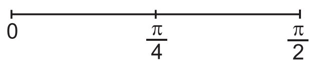
OR

Consider $[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$

$\Rightarrow \quad [x-2 \quad 0] = [0 \quad 0]$

$\Rightarrow \quad x - 2 = 0 \quad \Rightarrow \quad x = 2$

26. $f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x = -\sin 4x$ ...(*i*)



$f'(x) = 0 \Rightarrow \sin 4x = 0 \Rightarrow 4x = 0, \pi, 2\pi$

$\Rightarrow x = 0, \frac{\pi}{4}, \frac{\pi}{2}$

(*i*) $0 < x < \frac{\pi}{4} \Rightarrow 0 < 4x < \pi$

$\Rightarrow 4x \in$ I, IInd quadrant

From (*i*), $f'(x) < 0 \Rightarrow f$ is decreasing

(*ii*) $\frac{\pi}{4} < x < \frac{\pi}{2} \Rightarrow \pi < 4x < 2\pi$

$\Rightarrow 4x \in$ III, IVth quadrant.

From (*i*), $f'(x) > 0 \Rightarrow f$ is increasing.

$\therefore f$ is decreasing for $\left(0, \frac{\pi}{4}\right)$, increasing for $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

27. $$x = \frac{1}{y} \Rightarrow \frac{dx}{dy} = -\frac{1}{y^2} = -\frac{1}{y^2}\sqrt{\frac{1+y^4}{1+y^4}}$$

$$= -\sqrt{\frac{1}{y^4}\cdot\left(\frac{1+y^4}{1+y^4}\right)} = -\sqrt{\frac{\frac{1}{y^4}+1}{1+y^4}}$$

$$= -\sqrt{\frac{x^4+1}{1+y^4}} = -\sqrt{\frac{1+x^4}{1+y^4}}$$

28. We have, $f(x) = \dfrac{2x-7}{4}$

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$\Rightarrow \quad \dfrac{2x_1 - 7}{4} = \dfrac{2x_2 - 7}{4}$

$\Rightarrow \quad 2x_1 = 2x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one.

Let $f(x) = y$ {Where $y \in$R (co-domain)}

$\Rightarrow \quad \dfrac{2x-7}{4} = y$

$\Rightarrow \quad 2x - 7 = 4y$

$\Rightarrow \quad x = \dfrac{4y+7}{2} \in R$

Thus for all $y \in R$ (co-domain), there exists $x = \dfrac{4y+7}{2} \in R$ (domain).

$$f(x) = \frac{\frac{2(4y+7)}{2} - 7}{4}$$

$$= \frac{4y + 7 - 7}{4}$$

$$= y$$

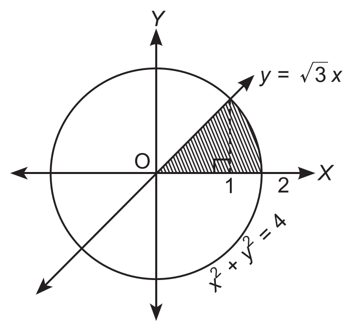
$\therefore$ f is onto.

$\therefore$ f is one-one and onto function.

29. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get $x^2 + 3x^2 = 4$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = \pm 1$



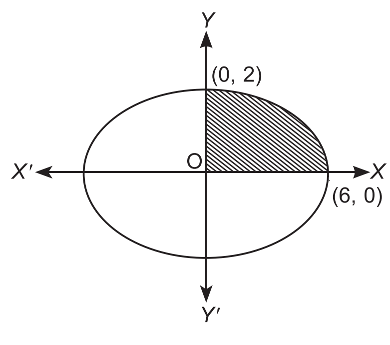
Required area $= \sqrt{3}\int_0^1 x\, dx + \int_1^2 \sqrt{2^2 - x^2}\, dx$

$= \sqrt{3}\left[\dfrac{x^2}{2}\right]_0^1 + \left[\dfrac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\dfrac{x}{2}\right)\right]_1^2$

$= \dfrac{\sqrt{3}}{2} + \left[2 \times \dfrac{\pi}{2} - \dfrac{\sqrt{3}}{2} - 2 \times \dfrac{\pi}{6}\right] = \dfrac{2\pi}{3}$ sq units.

OR

Curve is symmetrical to both the axes.



Required area $= 4\int_0^6 y\, dx = 4 \times \dfrac{1}{3}\int_0^6 \sqrt{36 - x^2}\, dx$

$= \dfrac{4}{3}\left[\dfrac{x}{2}\sqrt{6^2 - x^2} + 18\sin^{-1}\left(\dfrac{x}{6}\right)\right]_0^6$

$= \dfrac{4}{3}\left[18 \times \dfrac{\pi}{2} - 0\right] = 12\pi$ sq units

30. Let $y = x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{x\cos x}\right) + \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) \quad ...(i)$$

Consider $u = x^{x\cos x} \quad \Rightarrow \log u = x\cos x \cdot \log x$

$$\Rightarrow \quad \frac{1}{u}\cdot\frac{du}{dx} = x\cos x\cdot\frac{1}{x} + x\log x\,(-\sin x) + \log x\cdot 1\cdot\cos x$$

$$\Rightarrow \quad \frac{d}{dx}(x^{x\cos x}) = x^{x\cos x}\,[\cos x - x\log x\cdot\sin x + \log x\cos x] \quad ...(ii)$$

Consider,

$$\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) = \frac{(x^2-1)\cdot\frac{d}{dx}(x^2+1) - (x^2+1)\cdot\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\,2x - (x^2+1)2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2} \quad ...(iii)$$

Substituting from (*ii*) and (*iii*) in (*i*), we get

$$\frac{dy}{dx} = x^{x\cos x}\,[\cos x - x\log x\cdot\sin x + \log x\cdot\cos x] - \frac{4x}{(x^2-1)^2}.$$

OR

Given function $f(x) = |x-3| = \begin{cases} x-3, & x \geq 3 \\ -x+3, & x < 3 \end{cases}$

For continuity at $x = 3$:

$$\underset{x=3}{\text{LHL}} = \lim_{h\to 0} f(3-h) = \lim_{h\to 0}\{-(3-h)+3\} = \lim_{h\to 0} h = 0$$

$$\underset{x=3}{\text{RHL}} = \lim_{h\to 0} f(3+h) = \lim_{h\to 0}\{(3+h)-3\}$$

$$= \lim_{h\to 0} h = 0$$

$$f(3) = 3-3 = 0$$

As $\underset{x=3}{\text{LHL}} = \underset{x=3}{\text{RHL}} = f(3)$, hence, function is continuous at $x = 3$.

For differentiability at $x = 3$:

$$\underset{x=3}{\text{LHD}} = \lim_{h\to 0}\frac{f(3-h)-f(3)}{-h} = \lim_{h\to 0}\frac{(-3+h+3)-(0)}{-h}$$

$$= \lim_{h\to 0}\frac{h}{-h} = \lim_{h\to 0}(-1) = -1$$

$$\underset{x=3}{\text{RHD}} = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{(3+h-3)-(0)}{h}$$

$$= \lim_{h\to 0}\frac{h}{h} = \lim_{h\to 0}(1) = 1$$

As $\underset{x=3}{\text{LHD}} \neq \underset{x=3}{\text{RHD}}$. Hence, function is not derivable (differentiable) at $x = 3$.

31. Consider

$$\int\frac{e^x}{(e^x-1)^2(e^x+2)}dx = \int\frac{1}{(t-1)^2(t+2)}dt \qquad \left|\begin{array}{l}\text{Let } e^x = t \\ \Rightarrow e^x\,dx = dt\end{array}\right.$$

Let

$$\frac{1}{(t-1)^2(t+2)} = \frac{A}{t-1} + \frac{B}{(t-1)^2} + \frac{C}{t+2} \quad ...(i)$$

$$\Rightarrow \quad 1 = A(t-1)(t+2) + B(t+2) + C(t-1)^2$$

$$= A(t^2+t-2) + B(t+2) + C(t^2-2t+1)$$

$$= t^2(A+C) + t(A+B-2C) + (-2A+2B+C)$$

Comparing the coefficients, we get

$$A + C = 0 \Rightarrow A = -C$$

$$A + B - 2C = 0 \Rightarrow -3A = B$$

$$-2A + 2B + C = 1 \Rightarrow -2A - 6A - A = 1 \Rightarrow A = -\frac{1}{9}$$

$$\Rightarrow \quad A = -\frac{1}{9}, B = \frac{1}{3}, C = \frac{1}{9}$$

Substituting in (*i*) and integrating, we get

$$\int \frac{1}{(t-1)^2(t+2)}dt = -\frac{1}{9}\int \frac{1}{t-1}dt + \frac{1}{3}\int \frac{1}{(t-1)^2}dt + \frac{1}{9}\int \frac{1}{t+2}dt$$

$$= -\frac{1}{9}\log|t-1| - \frac{1}{3(t-1)} + \frac{1}{9}\log|t+2| + C$$

$$\int \frac{e^x}{(e^x-1)^2(e^x+2)}dx = -\frac{1}{9}\log|e^x-1| - \frac{1}{3(e^x-1)} + \frac{1}{9}\log|e^x+2| + C$$

OR

Consider the equation, $\frac{dy}{dx} + 2y = xe^{4x}$

Here $P(x) = 2$, $Q(x) = xe^{4x}$

Integrating factor $(I.F.) = e^{\int 2\,dx} = e^{2x}$

Solution is $\quad (I.F.)\,y = \int \{(I.F.)\,Q(x)\}\,dx$

$$e^{2x}.y = \int e^{2x}\cdot xe^{4x}\,dx = \int \underset{①}{x}\cdot \underset{②}{e^{6x}}\,dx$$

$$= x\cdot\frac{e^{6x}}{6} - \int 1\cdot\frac{e^{6x}}{6}\,dx$$

$$= \frac{1}{6}xe^{6x} - \frac{1}{36}e^{6x} + C$$

$\Rightarrow \quad y = \frac{1}{6}xe^{4x} - \frac{1}{36}e^{4x} + Ce^{-2x}$ is the required solution.

32. Given $\quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

and $\quad B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Also, $\quad (AB)^{-1} = B^{-1}A^{-1} \qquad ...(i)$

A^{-1} is given, so we find B^{-1}.

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= 1(3) - 2(-1) - 2(2)$$

$$= 3 + 2 - 4 = 1 \neq 0.$$

Matrix formed by cofactors of each element in $|B|$. B_{ij} is cofactor of element b_{ij} in $|B|$

$$B_{11} = +(3-0) = 3\ ; \quad B_{12} = -(-1-0) = 1;$$
$$B_{13} = +(2-0) = 2$$
$$B_{21} = -(2-4) = 2\ ; \quad B_{22} = +(1-0) = 1;$$
$$B_{23} = -(-2-0) = 2$$
$$B_{31} = +(0+6) = 6\ ; \quad B_{32} = -(0-2) = 2;$$
$$B_{33} = +(3+2) = 5$$

$\therefore$ $$\text{adj } B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$\Rightarrow$ $$B^{-1} = \frac{\text{adj } B}{|B|}$$

$$= \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$\therefore$ From (*i*)

$$(AB)^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

OR

Given $$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= 1(1-4) - 2(2-4) + 2(4-2) = -3 + 4 + 4 = 5 \neq 0.$$

Let A_{ij} be the cofactors of each element in $|A|$.

$$A_{11} = +(1-4) = -3\ ; \quad A_{12} = -(2-4) = 2\ ;$$
$$A_{13} = +(4-2) = 2$$
$$A_{21} = -(2-4) = 2\ ; \quad A_{22} = +(1-4) = -3\ ;$$
$$A_{23} = -(2-4) = 2$$
$$A_{31} = +(4-2) = 2\ ; \quad A_{32} = -(2-4) = 2;$$
$$A_{33} = +(1-4) = -3$$

$\therefore$ $$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \quad ...(i)$$

Consider $A^2 - 4A - 5I = O$

Multiplying both sides by A^{-1},

$A^{-1}(AA) - 4A^{-1}A - 5A^{-1}I = A^{-1}O$

$\Rightarrow \quad (A^{-1}A)A - 4I - 5A^{-1} = O \Rightarrow IA - 4I - 5A^{-1} = O$

$\Rightarrow \quad IA - 4I = 5A^{-1} \Rightarrow A - 4I = 5A^{-1}$

$$\text{LHS} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1-4 & 2-0 & 2-0 \\ 2-0 & 1-4 & 2-0 \\ 2-0 & 2-0 & 1-4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = 5 \times \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = 5\,A^{-1} = \text{RHS}$$

Hence $A^2 - 4A - 5I = O$

33. Consider equation $2y\, e^{x/y}\, dx + (y - 2x\, e^{x/y})\, dy = 0$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{2x\, e^{x/y} - y}{2y\, e^{x/y}} \quad ...(i)$$

Let $F(x, y) = \dfrac{2x\, e^{x/y} - y}{2y\, e^{x/y}}$

$$F(\lambda x, \lambda y) = \frac{2\lambda x \,.\, e^{\frac{\lambda x}{\lambda y}} - \lambda y}{2\lambda y\, e^{\frac{\lambda x}{\lambda y}}} = \frac{2x \cdot e^{\frac{x}{y}} - y}{2y\, e^{\frac{x}{y}}} = F(x, y)$$

Hence, function is homogeneous, so corresponding differential equation is homogeneous.

Let $\dfrac{x}{y} = v \Rightarrow x = vy \Rightarrow \dfrac{dx}{dy} = v + y\dfrac{dv}{dy}$

From (i), we have

$$v + y\frac{dv}{dy} = \frac{2vy \cdot e^v - y}{2ye^v} = \frac{2ve^v - 1}{2e^v}$$

$$\Rightarrow \quad y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v} = \frac{-1}{2e^v}$$

$$\Rightarrow \quad e^v dv = -\frac{1}{2} \cdot \frac{dy}{y} \Rightarrow \int e^v dv = -\frac{1}{2}\int \frac{dy}{y}$$

$$\Rightarrow \quad e^v = -\frac{1}{2}\log|y| + C$$

$$\Rightarrow \quad e^{\frac{x}{y}} = -\frac{1}{2}\log|y| + C \quad ...(ii)$$

Given $x = 0$, when $y = 1$, then from (ii), we get

$$e^0 = -\frac{1}{2}\log|1| + C$$

$$\Rightarrow \quad 1 = 0 + C \Rightarrow C = 1$$

Substituting $C = 1$ in (*ii*), we get

$$e^{\frac{x}{y}} = -\frac{1}{2}\log|y| + 1 \text{ is the particular solution.}$$

34. $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$, where $\vec{\beta_1} \,||\, \vec{\alpha}$ and $\vec{\beta_2} \perp \vec{\alpha}$ ... (*i*)

Let $\vec{\beta_1} = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k})$ ... (*ii*)

$\therefore$ $2\hat{i} + \hat{j} - 4\hat{k} = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{\beta_2}$ [from (*i*)]

$\Rightarrow$ $\vec{\beta_2} = (2 - 3\lambda)\hat{i} + (1 - 4\lambda)\hat{j} + (-4 - 5\lambda)\hat{k}$ ... (*iii*)

Also $\vec{\beta_2} \perp \vec{\alpha} \Rightarrow \vec{\beta_2} \cdot \vec{\alpha} = 0$

$\Rightarrow$ $3(2 - 3\lambda) + 4(1 - 4\lambda) + 5(-4 - 5\lambda) = 0$

$\Rightarrow$ $50\lambda = -10 \Rightarrow \lambda = -\frac{1}{5}$

Substituting for λ in (*ii*) and (*iii*), we get

$$\vec{\beta_1} = -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) = -\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j} - \hat{k}$$

and $\vec{\beta_2} = \left(2 + \frac{3}{5}\right)\hat{i} + \left(1 + \frac{4}{5}\right)\hat{j} + (-4 + 1)\hat{k}$

$\Rightarrow$ $\vec{\beta_2} = \frac{13}{5}\hat{i} + \frac{9}{5}\hat{j} - 3\hat{k}$

$\therefore$ $2\hat{i} + \hat{j} - 4\hat{k} = \left(-\frac{3}{5}\hat{i} - \frac{4}{5}j - \hat{k}\right) + \left(\frac{13}{5}\hat{i} + \frac{9}{5}\hat{j} - 3\hat{k}\right)$

OR

Let line through the point $A(1, 2, -4)$ be $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ ...(*i*)

If line (*i*) is perpendicular to the lines

$$\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $\frac{x-1}{1} = \frac{y+2}{-3} = \frac{z-3}{5}$

then $2a + 3b + 4c = 0$ ...(*ii*)

and $a - 3b + 5c = 0$ ...(*iii*)

Solving (*ii*) and (*iii*), we get

$$\frac{a}{15+12} = \frac{-b}{10-4} = \frac{c}{-6-3} \Rightarrow \frac{a}{27} = \frac{b}{-6} = \frac{c}{-9}$$

DR's are 9, –2, –3

From (*i*), line in Cartesian form is $\frac{x-1}{9} = \frac{y-2}{-2} = \frac{z+4}{-3}$

Let $\frac{x-1}{9} = \frac{y-2}{-2} = \frac{z+4}{-3} = \lambda$ (say)

General point on the line is $(9\lambda + 1, -2\lambda + 2, -3\lambda - 4)$

Position vector of point on the line is

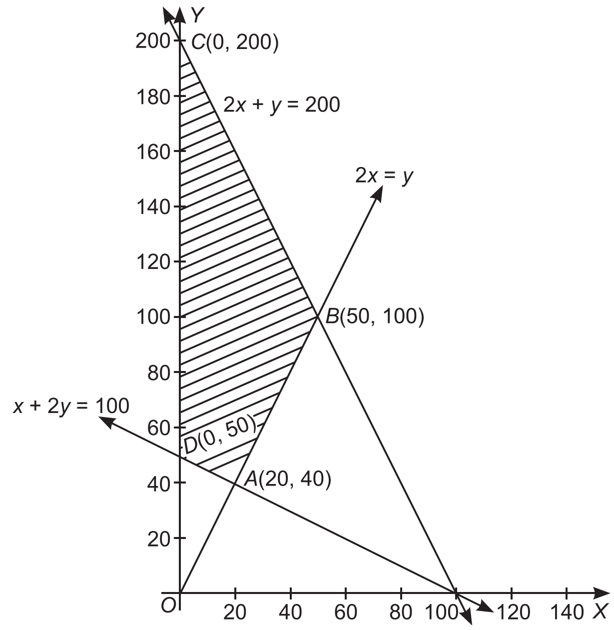
$$\vec{r} = (9\lambda + 1)\hat{i} + (-2\lambda + 2)\hat{j} + (-3\lambda - 4)\hat{k}$$

$\Rightarrow$ $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(9\hat{i} - 2\hat{j} - 3\hat{k})$ is equation of line in vector form.

35. To maximise $Z = x + 2y$

subject to the constraints

$$\begin{aligned} x + 2y &\geq 100 \\ 2x - y &\leq 0 \\ 2x + y &\leq 200 \\ x, y &\geq 0 \end{aligned}$$



On plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for maximum Z are $A(20, 40)$, $B(50, 100)$, $C(0, 200)$ and $D(0, 50)$.

Points	$Z = x + 2y$	**Values**
$A(20, 40)$	20 + 80	100
$B(50, 100)$	50 + 200	250
$C(0, 200)$	0 + 400	400 ← Maximum
$D(0, 50)$	0 + 100	100

$\therefore$ Z is maximum for $C(0, 200)$, i.e. $x = 0, y = 200$.

36. (*i*) $\pi r^2 h = 432\pi$

$\Rightarrow$ $r^2 h = 432$

(*ii*)
$$S = 2\pi rh + 2\pi r^2$$
$$= 2\pi r \frac{432}{r^2} + 2\pi r^2 = \left(\frac{864\pi}{r} + 2\pi r^2\right) \text{ sq units}$$

(*iii*) For minimum surface area, $\dfrac{dS}{dr} = 0$ and $\dfrac{d^2S}{dr^2} > 0$

$$\frac{dS}{dr} = \frac{-864\pi}{r^2} + 4\pi r$$

Now, $\dfrac{dS}{dr} = 0 \Rightarrow r^3 = 216 \Rightarrow r = 6$ units

Now, $\dfrac{d^2S}{dr^2} = \dfrac{1728\pi}{r^3} + 4\pi$

So, $\dfrac{d^2S}{dr^2} > 0$ for $r = 6$ units

OR

(*iii*) $S_{\min} = \dfrac{864\pi}{6} + 2\pi(6)^2 = 144\pi + 72\pi = 216\pi$ sq units

37. Let E_1 : scooter driver is selected

E_2 : truck driver is selected

E_3 : car driver is selected

E : insured person meets with an accident (*i*) $P(E_3) = \dfrac{9000 - 2000 - 4000}{9000} = \dfrac{1}{3}$

(*ii*) $P(E/E_3) = 0.02$

(*iii*) $P(\text{truck driver meeting with an accident}) = P(E_2)\,P(E/E_2)$

$$= \frac{4}{9} \times 0.04 = \frac{4}{225}$$

OR

(*iii*)

$$P(E) = P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2) + P(E_3)\,P(E/E_3)$$

$$= \frac{2}{9} \times 0.01 + \frac{4}{9} \times 0.04 + \frac{3}{9} \times 0.02$$

$$= \frac{2 + 16 + 6}{900} = \frac{24}{900}$$

$$= \frac{2}{75}$$

38. $A(0, 0, 0)$, $B(4, 0, 0)$, $C(4, 4, 0)$, $D(0, 4, 0)$, $E(0, 0, 4)$, $F(4, 0, 4)$, $G(4, 4, 4)$ and $H(0, 4, 4)$.

(*i*) DR's of EC = $\langle 4, 4, -4\rangle$

DR's of AG = $\langle 4, 4, 4\rangle$

(*ii*) DR's of HB = $\langle 4, -4, -4\rangle$

DR's of DF = $\langle 4, -4, 4\rangle$

Let 'θ' be the angle between HB and DF

$$\cos\theta = \frac{4 \times 4 + \{(-4) \times (-4)\} + 4 \times \{-4\}}{\sqrt{4^2 + (-4)^2 + (-4)^2}\,\sqrt{4^2 + (-4)^2 + 4^2}} = \frac{1}{3}$$

$$\therefore \qquad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$