Solutions to RMM/Set-2

1. (*b*) $A \cdot \text{Adj } A = |A| \cdot I$ and $A^{-1} = \frac{1}{|A|}\text{Adj } A$

$\Rightarrow \quad |A| = -4$

$\Rightarrow \quad K = -4$

$\Rightarrow \quad 16K = -64$

2. (*d*) 3×1

3. (*b*) As, $(A + A')' = A' + (A')' = A' + A = A + A'$ [$\because$ Matrix addition is commutative]

$\therefore A + A'$ is symmetric matrix.

4. (*d*) as '*f*' is not defined at $x = 0$. i.e. $f(0)$ does not exist.

5. (*d*) $3x + 1 = 6y - 2 = 1 - z$

$$3\left(x + \frac{1}{3}\right) = 6\left(y - \frac{1}{3}\right) = -(z - 1)$$

$$\frac{x + \frac{1}{3}}{\frac{1}{3}} = \frac{y - \frac{1}{3}}{\frac{1}{6}} = \frac{z - 1}{-1}$$

$$\frac{x + \frac{1}{3}}{2} = \frac{y - \frac{1}{3}}{1} = \frac{z - 1}{-6}.$$

6. (*b*) $(y - px)^2 = a^2p^2 + b^2$

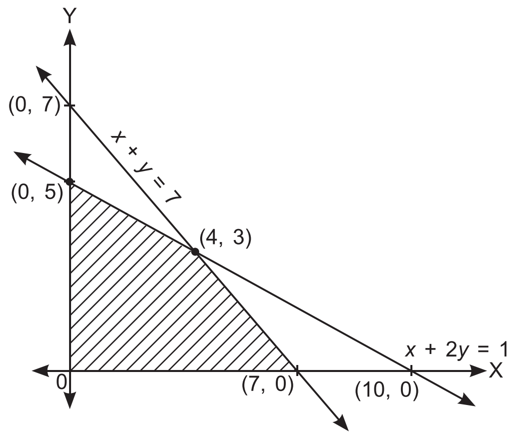
$\Rightarrow (x^2 - a^2)p^2 - 2xyp - b^2 + y^2 = 0$

Degree = 2, as $p^2 = \left(\frac{dy}{dx}\right)^2$

7. (*c*) Solving equations $x + y = 7$ and $x + 2y = 10$, we get

$y = 3, x = 4$

Plotting the graph of inequations, we see that shaded region is the feasible solution. The coordinates of corner points of shaded feasible region are (0, 0), (7, 0), (4, 3) and (0, 5).



Points	Values of $Z = 5x + 2y$	
(0, 5)	10	
(7, 0)	35	← Maximum
(0, 0)	0	
(4, 3)	26	

8. (*b*) $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$

$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$ $\quad [\because \vec{a} \times \vec{a} = \vec{b} \times \vec{b} = \vec{0}]$

$= 2(\vec{a} \times \vec{b})$

9. (*b*) Let $$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2023} x}{\cos^{2023} x + \sin^{2023} x} dx \quad ...(i)$$

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2023}\left(\frac{\pi}{2} - x\right)}{\cos^{2023}\left(\frac{\pi}{2} - x\right) + \sin^{2023}\left(\frac{\pi}{2} - x\right)} dx$$

[Using property: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$]

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{2}} \frac{\cos^{2023} x}{\sin^{2023} x + \cos^{2023} x} dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2023} x + \cos^{2023} x}{\cos^{2023} x + \sin^{2023} x} dx$$

$\Rightarrow$ $$2I = \int_0^{\frac{\pi}{2}} dx$$

$\Rightarrow$ $$2I = \left[x\right]_0^{\frac{\pi}{2}} \Rightarrow 2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

10. (*b*) $$[1 \quad x \quad 1]\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = O$$

$\Rightarrow$ $$[1 \quad x \quad 1]\begin{bmatrix} x+3-4 \\ 5-2 \\ 3-4 \end{bmatrix} = O$$

$\Rightarrow$ $$[1 \quad x \quad 1]\begin{bmatrix} x-1 \\ 3 \\ -1 \end{bmatrix} = O$$

$\Rightarrow$ $$x - 1 + 3x - 1 = 0$$

$\Rightarrow$ $$4x = 2$$

$\Rightarrow$ $$x = \frac{1}{2}$$

11. (*b*) For (3, 5), $x - y \leq 0$ is true

12. (*d*) as $|k\,\vec{a}| = |k||\vec{a}| = 2|k|$

Now, $-3 \leq k \leq 2$

$\Rightarrow$ $0 \leq |k| \leq 3$

$\Rightarrow$ $0 \leq 2|k| \leq 6$

$\Rightarrow$ $0 \leq |k\,\vec{a}| \leq 6$

So, $|k\,\vec{a}| \in [0, 6]$.

13. (*d*) $$|A| = 0 - 1(-3) + 2(-9) = -15$$

$$|A^{-1}| = \frac{1}{|A|} = -\frac{1}{15}$$

14. (*c*)

15. (*c*) $$\frac{dy}{dx} - x \cdot y = e^{x^2/2}$$

So, $$P(x) = -x; Q(x) = e^{x^2/2}$$

$$\text{I.F.} = e^{\int P\,dx} = e^{\int -x\,dx} = e^{-x^2/2}$$

Solution is given by,

$$y \times \text{I.F.} = \int (Q \times \text{I.F.})dx$$

$$\Rightarrow \quad ye^{-x^2/2} = \int e^{x^2/2} \cdot e^{\frac{-x^2}{2}} dx$$

$$\Rightarrow \quad \frac{y}{e^{x^2/2}} = x + C$$

$$\Rightarrow \quad y = (x + C) \cdot e^{x^2/2}$$

16. (*a*) Given $|\vec{a}|=1$, $|\vec{b}|=1$ and $|\vec{a}+\vec{b}| = 1$

Since $|\vec{a}+\vec{b}| = 1 \Rightarrow |\vec{a}+\vec{b}|^2 = 1$

$$\Rightarrow \quad (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = 1$$

$$\Rightarrow \quad \vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a} = 1$$

$$\Rightarrow \quad |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b} = 1$$

$$\Rightarrow \quad 1 + 1 + 2\vec{a}\cdot\vec{b} = 1$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} = -\frac{1}{2}$$

$$\therefore \quad \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\frac{-1}{2}}{1\times 1} = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120° \text{ or } \frac{2\pi}{3}$$

17. (*c*) $f'(x) = 1 + \sin x > 0$ for $x \in R$. $\left\{1+\sin x = \left(\cos\frac{x}{2}+\frac{\sin x}{2}\right)^2 \geq 0\right\}$

As $0 \leq 1 + \sin x \leq 2$

$\therefore$ Always increasing.

18. (*d*) As, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \quad \cos^2 45° + \cos^2 60° + \cos^2\gamma = 1 \qquad [\alpha = 45°\ ;\ \beta = 60°;\ \gamma = ?]$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\Rightarrow \quad \cos^2\gamma = 1 - \frac{3}{4}$$

$$\Rightarrow \quad \cos^2\gamma = \frac{1}{4}$$

$$\Rightarrow \quad \cos\gamma = \pm\frac{1}{2}$$

Therefore required angle is 60° or 120°.

19. (*a*) Both A and R are true and R is the correct explanation of A.

20. (*d*) A is false but R is true.

21.
$$\tan^{-1}\left[2\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left[2\sin\left(\frac{\pi}{6}\right)\right]$$
$$= \tan^{-1}\left[2\times\frac{1}{2}\right]$$
$$= \tan^{-1}(1)$$
$$= \frac{\pi}{4}$$

OR

We have, $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = 3\pi$ ...(i)

Now, range of $\cos^{-1} x$ is $[0, \pi]$.

$\therefore$ Equation (i) is satisfied when $\cos^{-1} p = \pi, \quad \cos^{-1} q = \pi, \quad \cos^{-1} r = \pi$

$\Rightarrow \quad p = \cos \pi, \quad q = \cos \pi, \quad r = \cos \pi$

$\Rightarrow \quad p = -1, \quad q = -1 \quad r = -1$

$$\therefore \quad pq + qr + rp = (-1) \times (-1) + (-1) \times (-1) + (-1) \times (-1)$$
$$= 1 + 1 + 1 = 3$$

22. Let 'r' be the radius and 'V' be the volume of balloon at any instant 't'.

Given $\quad \dfrac{dV}{dt} = 900 \text{ cm}^3\text{/s},$

We have: $\quad V = \dfrac{4}{3}\pi r^3$

Differentiating w.r.t. t, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$$\Rightarrow \quad 900 = \frac{4}{3}\pi \times 3 \times r^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2}$$

$$\Rightarrow \quad \left.\frac{dr}{dt}\right]_{r=15} = \frac{900}{4\pi \times 15 \times 15} = \frac{1}{\pi} \text{ cm/s}$$

23. We have, $\quad f(x) = x^4 - 32x^2 + ax + 10$

Differentiating w.r.t. 'x', we get

$$f'(x) = 4x^3 - 64x + a$$

A.T.Q $\quad$ at $\quad x = 1, f'(x) = 0$, so

$$0 = 4(1)^3 - 64(1) + a$$

$$\Rightarrow \quad 0 = 4 - 64 + a$$

$$\Rightarrow \quad a = 60$$

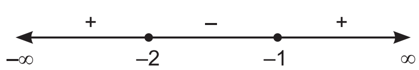
OR

$$y = 2x^3 + 9x^2 + 12x - 1$$

Differentiating w.r.t 'x', we get

$$\frac{dy}{dx} = 6x^2 + 18x + 12 = 6(x^2 + 3x + 2)$$
$$= 6(x + 2)(x + 1)$$

Intervals	Sign of $f'(x)$	Nature of f
$(-\infty, -2)$	+ve	Strictly increasing
$(-2, -1)$	–ve	Strictly decreasing
$(-1, \infty)$	+ve	Strictly increasing



Sign of $f'(x)$ for different values of x

$\therefore$ In $(-2, -1)$, 'f' is strictly decreasing.

24. Let $\quad I = \displaystyle\int \frac{dx}{1+\tan x} = \int \frac{\cos x}{\sin x + \cos x}\, dx = \frac{1}{2}\int \frac{2\cos x}{\sin x + \cos x}\, dx = \frac{1}{2}\left[\int \frac{\cos x + \cos x + \sin x - \sin x}{\sin x + \cos x} dx\right]$

$$= \frac{1}{2}\left[\int \frac{\cos x + \sin x}{\sin x + \cos x} dx + \int \frac{\cos x - \sin x}{\sin x + \cos x} dx\right] = \frac{1}{2}\left[x + \int \frac{\cos x - \sin x}{\sin x + \cos x} dx\right]$$

$$= \frac{1}{2}\left[x + \int \frac{dt}{t}\right]$$

$$= \frac{1}{2}[x + \log|\cos x + \sin x|] + C$$

Let $\cos x + \sin x = t$
$\Rightarrow (-\sin x + \cos x)dx = dt$
$\Rightarrow (\cos x - \sin x)dx = dt$

25.

$$y = \log(\log x^2)$$

Differentiating w.r.t. x both sides,

$$y_1 = \frac{1}{\log x^2} \cdot \frac{1}{x^2} \cdot 2x = \frac{2}{x \log x^2}$$

$$\Rightarrow \quad y_1 = \frac{2}{x \cdot 2 \log x} = \frac{1}{x \log x}$$

Differentiating again w.r.t. x both sides,

$$y_2 = \frac{x \log x \times 0 - 1\left\{x \times \frac{1}{x} + \log x\right\}}{(x \log x)^2} = \frac{-(1 + \log x)}{(x \log x)^2}$$

26. $\int_1^4 (|x-1|) + |x-2|)\, dx$

When $x < 1$:

$\therefore \quad |x-1| + |x-2| = -(x-1) - (x-2)$

$= -2x + 3$ (Rejected as no interval given for integrals)

When $1 \le x < 2$:

$\therefore \quad |x-1| + |x-2| = x - 1 - x + 2$

$= 1$

When $x \ge 2$:

$\therefore \quad |x-1| + |x-2| = x - 1 + x - 2$

$= 2x - 3$

$$\therefore \quad \int_1^4 (|x-1| + |x-2|)\, dx = \int_1^2 1\, dx + \int_2^4 (2x-3)\, dx$$

$$= [x]_1^2 + \left[\frac{2x^2}{2} - 3x\right]_2^4$$

$$= 2 - 1 + (16 - 12) - (4 - 6)$$

$$= 1 + 4 + 2$$

$$= 7$$

27. Let green signals be on D_1, D_2 and D_3.

$P(D_1) = P(D_2) = P(D_3) = \frac{30}{100} = \frac{3}{10}$

$\therefore$ Probability of green signal on two consecutive days $= P(D_1\, D_2\, \overline{D}_3) + P(\overline{D}_1\, D_2\, D_3)$

$$= \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{126}{1000} = 0.126$$

28.

$$I = \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx \qquad ...(i)$$

Using property $\int_0^a f(x) dx = \int_0^a f(a - x)\, dx$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \sin(\pi - x)}\, dx$$

$$= \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1 + \sin x} dx \qquad ...(ii)$$

$$\therefore \quad 2I = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x}\, dx \qquad \text{[On adding } (i) \text{ and } (ii)]$$

$$= \pi\int_0^{\pi} \frac{\sin x\,(1-\sin x)}{(1+\sin x)\,(1-\sin x)}dx$$

$$= \pi\int_0^{\pi} (\sec x \tan x - \tan^2 x)\,dx$$

$$= \pi\int_0^{\pi} (\sec x \tan x - \sec^2 x + 1)\ dx$$

$$= \pi\Big[\sec x - \tan x + x\Big]_0^{\pi}$$

$$= \pi[(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0)]$$

$$2I = \pi(-2 + \pi)$$

$$\Rightarrow \quad I = \frac{\pi}{2}\ (\pi - 2) = \pi\left(\frac{\pi}{2} - 1\right)$$

OR

Let $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$ ...(*i*)

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4} - x\right)dx\right]$ $\quad [\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx]$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right)dx$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\frac{\pi}{4}} \left[\log(1+\tan x) + \log\left(\frac{2}{1+\tan x}\right)\right]dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{4}} [\log\,(1+\tan x) + \log 2 - \log(1+\tan x)]\,dx$$

$$\Rightarrow \quad 2I = \log 2\int_0^{\frac{\pi}{4}} dx$$

$$\Rightarrow \quad 2I = \log 2 \times \Big[x\Big]_0^{\frac{\pi}{4}}$$

$$\Rightarrow \quad 2I = \frac{\pi}{4}\log 2$$

$$\Rightarrow \quad I = \frac{\pi}{8}\log 2$$

29. $$2(y + 3) - xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad xy\frac{dy}{dx} = 2(y + 3)$$

$$\Rightarrow \quad \frac{y}{y+3}dy = \frac{2}{x}dx$$

$$\Rightarrow \quad \left(\frac{y+3}{y+3} - \frac{3}{y+3}\right)dy = \frac{2}{x}dx$$

$$\Rightarrow \quad \left(1 - \frac{3}{y+3}\right)dy = \frac{2}{x}dx$$

Integrating both sides, we get

$$\int\left(1-\frac{3}{y+3}\right)dy = \int\frac{2}{x}dx$$

$\Rightarrow$ $$y - 3\log|y+3| = 2\log|x| + C$$

Putting $x = 1, y = -2$, we get

$$-2 - 3\log 1 = 2\log 1 + C$$

$\therefore$ $$C = -2$$

$\therefore$ Particular solution is : $y - 3\log|y+3| = 2\log|x| - 2$

OR

Given $\quad xy = c^2 \quad$ ...(*i*)

Let $\quad Z = ax + by$

$\Rightarrow \quad Z = ax + \dfrac{bc^2}{x} \quad$ ...(*ii*)

Differentiating both sides w.r.t. *x,* we get

$$\frac{dZ}{dx} = a - \frac{bc^2}{x^2}$$

For minimum Z, $\dfrac{dZ}{dx} = 0 \Rightarrow a - \dfrac{bc^2}{x^2} = 0$

$\Rightarrow \quad x^2 = \dfrac{bc^2}{a} \Rightarrow x = \pm\sqrt{\dfrac{bc^2}{a}}$

Now, $\quad \dfrac{d^2Z}{dx^2} = 0 + \dfrac{2bc^2}{x^3} = \dfrac{2bc^2}{x^3}$

$$\left.\frac{d^2Z}{dx^2}\right]_{x=\sqrt{\frac{bc^2}{a}}} = \frac{2bc^2}{\left(\frac{bc^2}{a}\right)^{3/2}} > 0 \text{ and } \left.\frac{d^2Z}{dx^2}\right]_{x=-\sqrt{\frac{bc^2}{a}}} = \frac{2bc^2}{-\left(\frac{bc^2}{a}\right)^{\frac{3}{2}}} < 0$$

Hence, for $x = \sqrt{\dfrac{bc^2}{a}}$, Z is minimum.

Substituting in (*ii*), we get

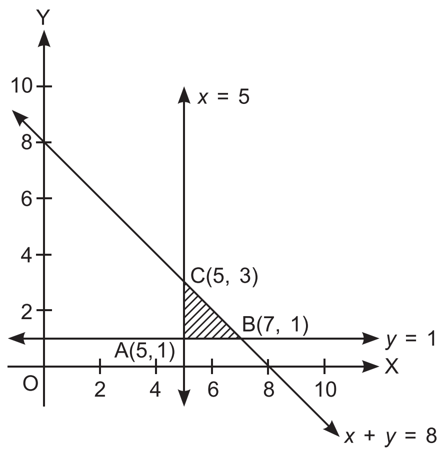
$$\text{Minimum } Z = a\cdot\sqrt{\frac{bc^2}{a}} + \frac{bc^2}{\sqrt{\frac{bc^2}{a}}} = 2\times\sqrt{abc^2}$$

30. To maximise, $Z = 3x + y$

subject to constraints,

$$x \geq 5, y \geq 1, x + y - 8 \leq 0, x \geq 0, y \geq 0$$

On plotting the inequations on graph, we notice shaded portion is feasible solution.



Possible points for maximum Z are $A(5, 1)$, $B(7, 1)$, $C(5, 3)$

Corner Points	$Z = 3x + y$	Values	
$A(5, 1)$	15 + 1	16	
$B(7, 1)$	21 + 1	22	← Maximum
$C(5, 3)$	15 + 3	18	

$\therefore$ Z is maximum at $B(7, 1)$ i.e. $x = 7, y = 1$.

Maximum value of $Z = 22$.

OR

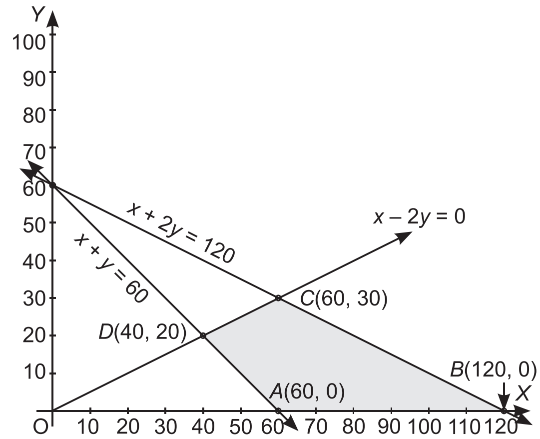
To minimise

$$Z = 5x + 10y$$

subject to the constraints

$$x \geq 0, y \geq 0, x - 2y \geq 0, x + y \geq 60, x + 2y \leq 120$$

Plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for minimum Z are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$



Corner Points	$Z = 5x + 10y$	Values	
$A(60, 0)$	300 + 0	300	← Minimum
$B(120, 0)$	600 + 0	600	
$C(60, 30)$	300 + 300	600	
$D(40, 20)$	200 + 200	400	

$\therefore$ Z is minimum at $A(60, 0)$. Hence, for $x = 60$ and $y = 0$, Z is minimum.

Minimum value of $Z = 300$.

31. We have $$y = (\sin^{-1} x)^2$$

Differentiating both sides, w.r.t. x, we get

$$\frac{dy}{dx} = 2(\sin^{-1} x) \cdot \frac{1}{\sqrt{1 - x^2}}$$

$$\Rightarrow \quad \sqrt{1 - x^2}\frac{dy}{dx} = 2(\sin^{-1} x)$$

Differentiating again w.r.t. x, on both sides,

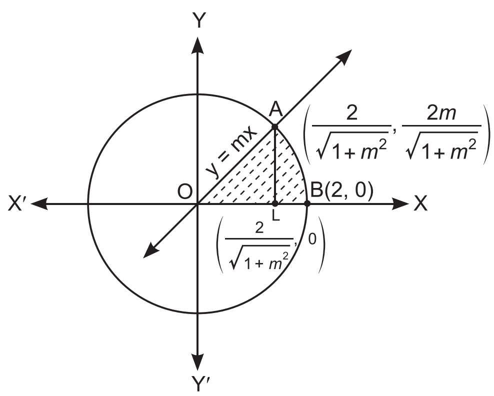
$$\sqrt{1 - x^2}\frac{d^2y}{dx^2} + \frac{1}{2}\frac{(-2x)}{\sqrt{1 - x^2}} \cdot \frac{dy}{dx} = \frac{2}{\sqrt{1 - x^2}}$$

$$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} - x \cdot \frac{dy}{dx} = 2$$

$$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

32. Given curves are : $$y = mx (m > 0) \quad ...(i)$$

and $$x^2 + y^2 = 4 \quad ...(ii)$$



Put $y = mx$ from (i) in (ii), we get

$$x^2 + m^2x^2 = 4$$

$$\Rightarrow \quad x^2(1 + m^2) = 4$$

$$\Rightarrow \quad x = \frac{2}{\sqrt{1 + m^2}}$$

From (i), $$y = \frac{2m}{\sqrt{1 + m^2}}$$

So, point of intersection of (i) and (ii) in 1st quadrant is $A\left(\frac{2}{\sqrt{1 + m^2}}, \frac{2m}{\sqrt{1 + m^2}}\right)$.

On plotting the given curves on graph, we notice that area of shaded region is to be found.

$\therefore$ Area(OAB) = ar(OAL) + ar(LAB)

$$\Rightarrow \int_0^{\frac{2}{\sqrt{1+m^2}}} mx\,dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} \sqrt{4-x^2}\,dx = \frac{\pi}{2}$$

$$\Rightarrow \left[\frac{mx^2}{2}\right]_0^{\frac{2}{\sqrt{1+m^2}}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{2}{\sqrt{1+m^2}}}^{2} = \frac{\pi}{2}$$

$$\Rightarrow \frac{m}{2}\cdot\frac{4}{1+m^2} + \{0 + 2\sin^{-1}(1)\} - \left\{\frac{1}{\sqrt{1+m^2}}\sqrt{4-\frac{4}{1+m^2}} + 2\cdot\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right)\right\} = \frac{\pi}{2}$$

$$\Rightarrow \frac{2m}{1+m^2} + 2\times\frac{\pi}{2} - \frac{2m}{1+m^2} - 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{1}{\sqrt{1+m^2}} = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{1+m^2} = \sqrt{2}$$

$$\Rightarrow 1 + m^2 = 2 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$$

As, $m > 0$, so we take $m = 1$.

33. $(a, b)R\,(c, d) \Rightarrow \frac{1}{a}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c}$

For reflexive: Let $(a, b) \in A \times A$, *s.t.* $(a, b)\,R\,(a, b)$.

Now, $(a, b)\,R\,(a, b) \Rightarrow \frac{1}{a}+\frac{1}{b} = \frac{1}{b}+\frac{1}{a}$

which is true because addition is commutative.

So, R is reflexive.

For symmetric: Let $(a, b), (c, d) \in A \times A$, *s.t.* $(a, b)\,R\,(c, d)$.

Now, $(a, b)\,R\,(c, d)$

$$\Rightarrow \frac{1}{a}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c} \quad ...(i) \qquad \Rightarrow \frac{1}{c}+\frac{1}{b} = \frac{1}{d}+\frac{1}{a} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$(a, b)\,R\,(c, d) \Rightarrow (c, d)\,R\,(a, b)$$

$\therefore$ R is symmetric relation.

For transitive: Let $(a, b), (c, d), (e, f) \in A \times A$, *s.t.* $(a, b)\,R\,(c, d)$ and $(c, d)\,R\,(e, f)$.

Now, $(a, b)\,R\,(c, d)$

$$\Rightarrow \frac{1}{a}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c} \quad ...(iii)$$

Now, $(c, d)\,R\,(e, f)$

$$\Rightarrow \frac{1}{c}+\frac{1}{f} = \frac{1}{d}+\frac{1}{e} \quad ...(iv)$$

From (*iii*) and (*iv*), we get

$$\frac{1}{a}+\frac{1}{c}+\frac{1}{f}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}$$

$$\Rightarrow \frac{1}{a}+\frac{1}{f} = \frac{1}{b}+\frac{1}{e}$$

$$\Rightarrow (a, b)\,R\,(e, f)$$

$\therefore$ R is transitive relation.

Hence, given relation is an equivalence relation.

OR

$$f(x) = \begin{cases} \frac{x+1}{2}, \text{ if } x \text{ is odd} \\ \frac{x}{2}, \text{ if } x \text{ is even} \end{cases}$$

For one-one:

We observe that,

$$f(1) = \frac{1+1}{2} = 1; f(2) = \frac{2}{2} = 1$$

So, we get $f(1) = f(2)$, but $1 \neq 2$.

Also $1, 2 \in N$.

Hence, f is not one-one.

For onto:

Let $f(x) = y$, such that $y \in N$.

When x is odd

$$y = \frac{x+1}{2}$$

$\Rightarrow$ $x = 2y - 1$

So, for each $y \in N$, $x \in N$.

When x is even

$$y = \frac{x}{2}$$

$\Rightarrow$ $x = 2y$

So for each $y \in N$, $x \in N$

So, for every $y \in N$, there exists $x \in N$ such that $f(x) = y$

Hence f is onto.

$\therefore$ f is not bijective function.

34. Given matrix is

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj } A$$

$$|A| = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} = 1(-3) - 2(-2) + 0 = 1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}' = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad ...(i)$$

Given equations are

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

Matrix equation is

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$A'X = B$$

Solution is $X = (A')^{-1}B$

$$= (A^{-1})'B$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}' \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 0, y = -5, z = -3$ is the solution.

35. Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$

In vector form, these equations are written as

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

and $$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})$$

Here $\vec{a_1} = \hat{i} - \hat{j}$ and $\vec{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$

$$\vec{a_2} = -\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b_2} = 5\hat{i} + \hat{j}$$

$$\vec{a_2} - \vec{a_1} = -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j} = -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(0-5) + \hat{k}(2-15) = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$\therefore \quad (\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (-2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 5\hat{j} - 13\hat{k})$$

$$= (2 + 15 - 26) = 17 - 26 = -9$$

$$\text{Shortest distance} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| = \left| \frac{-9}{\sqrt{1+25+169}} \right| = \left| \frac{-9}{\sqrt{195}} \right| = \frac{9}{\sqrt{195}} \text{ units}$$

Here, shortest distance is not zero, so lines are not intersecting.

OR

Let $\vec{d}$ be $x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\vec{d} \perp \vec{c}$, then

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + \hat{j} - \hat{k}) = 0$

$\Rightarrow 3x + y - z = 0$ ...(*i*)

Also, $\vec{d} \perp \vec{b}$, then

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} - 4\hat{j} + 5\hat{k}) = 0$

$\Rightarrow x - 4y + 5z = 0$ ...(*ii*)

Also, $\vec{d} \cdot \vec{a} = 21$

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (4\hat{i} + 5\hat{j} - \hat{k}) = 21$

$\Rightarrow 4x + 5y - z = 21$ ...(*iii*)

Subtracting (*i*) from (*iii*), we get

$x + 4y = 21$ ...(*iv*)

Multiply (*i*) by '5' and then adding (*ii*) to it, we get $15x + 5y - 5z + x - 4y + 5z = 0$

$\Rightarrow 16x + y = 0$ ...(*v*)

Solving (*iv*) and (*v*), we get

$x = \frac{-1}{3}$ and $y = \frac{16}{3}$

Put the values of '*x*' and '*y*' in (*i*),

we get $z = \frac{13}{3}$

$\therefore \vec{d} = \frac{1}{3}(-\hat{i} + 16\hat{j} + 13\hat{k})$

36. (*i*) As, $\Sigma P(X) = 1$

$\Rightarrow p + 2p + 2p + p + 2p + p^2 + 2p^2 + 7p^2 + p = 1$

$\Rightarrow 10p^2 + 9p - 1 = 0 \Rightarrow (10p - 1)(p + 1) = 0$

$\Rightarrow 10p - 1 = 0$ or $p + 1 = 0$

$\Rightarrow p = \frac{1}{10}$ or $p = -1$ (rejected)

(*ii*) $P(X > 6) = P(7) + P(8) = 2p^2 + 7p^2 + p$

$= 9p^2 + p = \frac{9}{100} + \frac{1}{10} = \frac{19}{100} = 0.19$

(*iii*) Required probability $= P(3) + P(6)$

$= 2p + p^2 = \frac{2}{10} + \frac{1}{100}$

$= \frac{21}{100} = 0.21$

OR

(*iii*) Required probability $= P(2) + P(4) + P(6) + P(8)$

$= 2p + p + p^2 + 7p^2 + p = 8p^2 + 4p$

$= \frac{8}{100} + \frac{4}{10} = \frac{48}{100} = 0.48$

37. Given $x = \frac{600 - p}{8}$

$\Rightarrow 8x = 600 - p$

$\Rightarrow p = 600 - 8x$

$R(x) = p \cdot x$

$= 600x - 8x^2$

Cost function, $C(x) = x^2 + 78x + 2500$

(*i*) $p = 600 - 8x$

(*ii*) $P(x) = R(x) - C(x)$

$= 600x - 8x^2 - x^2 - 78x - 2500$

$= -9x^2 + 522x - 2500$

(*iii*) $P(x) = -9x^2 + 522x - 2500$

Differentiating both sides w.r.t. *x*, we get

$P'(x) = -18x + 522$

For maximum or minimum profit,

put $P'(x) = 0$

$\Rightarrow 18x = 522$

$\Rightarrow x = \frac{522}{18} = 29$

$$P''(x) = -18 < 0$$

$\therefore$ $P(x)$ is maximum when $x = 29$.

OR

(*iii*) $$P(x) = -9x^2 + 522x - 2500$$

Differentiating both sides w.r.t. x, we get

$$P'(x) = -18x + 522$$

Put $P'(x) = 0$, for critical point

$\therefore$ $-18x = -522 \Rightarrow x = 29$

Interval	sign of $P'(x)$
(0, 29)	+ve
(29, ∞)	–ve

$\therefore$ $P(x)$ is increasing in the interval (0, 29)

38. (*i*) $\overrightarrow{AB}$ = position vector of B – position vector of A = $(2-1)\hat{i} + (1-1)\hat{j} + (3-1)\hat{k} = \hat{i} + 2\hat{k}$

(*ii*) $\overrightarrow{CD}$ = position vector of D – position vector of C = $(3-3)\hat{i} + (3-2)\hat{j} + (4-2)\hat{k} = \hat{j} + 2\hat{k}$

Now, $\overrightarrow{AB} \cdot \overrightarrow{CD} = (\hat{i} + 2\hat{k}) \cdot (\hat{j} + 2\hat{k}) = 4$