Solutions to RMM/Set-3

1. (*a*) $A = [a_{ij}]_{m \times n}$

2. (*c*) $|A| = 3(0 - 1) + 1(0 - 1) + 2(0 - 4)$

$= -3 - 1 - 8 = -12$

Now, $|A^{-1}| = \frac{1}{|A|} = \frac{-1}{12}$

3. (*d*) $A + B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

Let $D = A + B$, then $|D| = \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} = -1 - 2 = -3 \neq 0$

So, D^{-1} exists as $|D| \neq 0$.

Let D_{ij} be the cofactors of each element in $|D|$.

Now, $D_{11} = (-1)^2\,(1) = 1,\ D_{12} = (-1)^3\,(2) = -2$

$D_{21} = (-1)^3\,(1) = -1,\ D_{22} = (-1)^4\,(-1) = -1$

$\therefore$ $\text{Adj}(D) = \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$

$\therefore$ $D^{-1} = \frac{\text{Adj}(D)}{|D|} = \frac{-1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$

$\therefore$ $(A + B)^{-1} = \frac{-1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$

4. (*a*) As for (0, 0), $0 + 0 - 2 \leq 0$, true, but feasible region does not contain (0, 0).

5. (*d*) Line through the points (3, 1, –2) and (0, 2, 4) is

$$\frac{x - 0}{-3} = \frac{y - 2}{1} = \frac{z - 4}{6}$$

DR's are –3, 1, 6

If line with DR's a, b, c makes acute angle with y-axis then $b > 0$

$\therefore$ DR's are $<-3, 1, 6>$

6. (*c*) $x\frac{dy}{dx} - y = x^4 - 3x$

$\Rightarrow$ $\frac{dy}{dx} - \frac{y}{x} = x^3 - 3$

Comparing with standard form of linear differential equation, i.e. $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{-1}{x};\ Q = x^3 - 3$$

Now, $IF = e^{\int P dx} = e^{\int -\frac{1}{x} dx}$

$= e^{-\log|x|} = \frac{1}{x}$

7. (*b*) As (1, 2) does not satisfy inequation $3x \geq 5$.

8. (*c*) as $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \dfrac{1}{\sqrt{2}\cdot\sqrt{2}} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{6}$

9. (*c*) as $I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(*i*)

Using property: $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad (ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\pi/2} 1\cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

10. (*a*) Points $P(3, -2)$, $Q(8, 8)$, $R(k, 2)$ are collinear.

$$\therefore \qquad \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 3(8-2) + 2(8-k) + 1(16-8k) = 0$$

$$\Rightarrow \quad 18 + 16 - 2k + 16 - 8k = 0$$

$$\Rightarrow \quad k = 5$$

11. (*a*) Since f is continuous at $x = 1$, then $\displaystyle\lim_{x\to 1} f(x) = f(1) \Rightarrow \lim_{x\to 1}\frac{x^2-1}{x-1} = k$

$\Rightarrow \displaystyle\lim_{x\to 1}(x+1) = k \Rightarrow k = 2$

12. (*b*) Let

$$\vec{a} = \left(6\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$|\vec{a}| = \sqrt{6^2+2^2+3^2} = \sqrt{49} = 7$$

13. (*d*) if matrix is singular, then

$$\begin{vmatrix} 4+3k & 3 \\ 1+2k & 2 \end{vmatrix} = 0$$

$$\Rightarrow \quad 8 + 6k - 3 - 6k = 0$$

$$\Rightarrow \quad 5 = 0, \text{ false.}$$

$\therefore$ matrix is not singular for any k.

14. (*a*)

15. (*c*) $\displaystyle\int \frac{\tan x - 1}{\tan x + 1}dx = \int -\tan\left(\frac{\pi}{4}-x\right)dx$

$$= -\frac{\log\left|\sec\left(\frac{\pi}{4}-x\right)\right|}{-1} + C$$

$$= \log\left|\sec\left(\frac{\pi}{4}-x\right)\right| + C$$

16. (*a*) On evaluating the determinant,

$$\begin{aligned}\Delta &= |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2\\ &= |\vec{a}|^2|\vec{b}|^2 - \{|\vec{a}|\,|\vec{b}|\cos\theta\}^2\\ &= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta\\ &= |\vec{a}|^2\,|\vec{b}|^2\sin^2\theta\\ &= \{|\vec{a}|\,|\vec{b}|\sin\theta\}^2\\ &= |\vec{a}\times\vec{b}|^2\\ &= (\vec{a}\times\vec{b})^2\end{aligned}$$

17. (*a*)

$$y = e^{1 + \log x} = e^{\log e + \log x}$$

$\Rightarrow$ $$y = e^{\log(ex)}$$

$\Rightarrow$ $$y = ex$$

Differentiating both sides w.r.t x, we get

$$\frac{dy}{dx} = e$$

18. (*a*)

$$\text{Vector along } \overrightarrow{AD} = \frac{3\hat{i} + 0\hat{j} + 5\hat{k}}{2}$$

$$|\overrightarrow{AD}| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{34}{4}} = \frac{\sqrt{34}}{2} \text{ units}$$

19. (*b*) Assertion

$$R'(x) = 6x + 36$$

$$R'(5) = 30 + 36 = 66, \text{ true}$$

Both A and R are true and R is not the correct explanation of A.

20. (*a*) Both A and R are true and R is the correct explanation of A.

21.

$$\sin\left[\cos^{-1}\left(\cos\frac{7\pi}{4}\right)\right] = \sin\left[\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$= \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

OR

For domain $$-1 \le 3x + 4 \le 1$$

$\Rightarrow$ $$-5 \le 3x \le -3$$

$\Rightarrow$ $$-\frac{5}{3} \le x \le -1$$

Domain $= \left[-\frac{5}{3}, -1\right]$

22. Let r be the base radius and h be the height of the cylinder at a particular instant of time 't'. Let V be its volume at that instant.

Given: $$\frac{dr}{dt} = 2 \text{ cm/s}; \ \frac{dh}{dt} = -3 \text{ cm/s}$$

We know that:

Volume of cylinder, $$V = \pi r^2 h$$

On differentiating both sides w.r.t 't', we get

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + h \times 2r\frac{dr}{dt}\right)$$

Now, $$\left[\frac{dV}{dt}\right]_{r=3,\, h=4} = \pi[9 \times (-3) + 4 \times 2 \times 3 \times 2] = \pi[-27 + 48]$$

$$= 21\pi \text{ cm}^3/\text{s}$$

So, volume is increasing at the rate of 21π cm^3/s.

23. Let the required point on curve be (h, k). Now, point (h, k) must satisfy $x^2 = 2y$ as it lies on curve.

$\therefore$ $$h^2 = 2k \qquad ...(i)$$

Distance between (h, k) and $(0, 5)$, $D = \sqrt{(h-0)^2 + (k-5)^2}$

$\Rightarrow$ $$D^2 = h^2 + (k-5)^2$$

$\Rightarrow$ $$D^2 = 2k + (k-5)^2 \qquad \text{[using } (i)\text{]}$$

Let $D^2 = Z$, then $Z = 2k + (k-5)^2$

Differentiating both sides w.r.t. k, we get

$$\frac{dZ}{dk} = 2 + 2(k-5) = 2k - 8$$

For maxima or minima, $$\frac{dZ}{dk} = 0 \Rightarrow 2k - 8 = 0 \Rightarrow k = 4$$

$$\frac{d^2Z}{dk^2} = 2 \Rightarrow \left[\frac{d^2Z}{dk^2}\right]_{k=4} = 2 > 0$$

So, Z is minimum at $k = 4$ or D^2 is minimum at $k = 4$ i.e. D is minimum at $k = 4$.

If $k = 4$, then $h = \pm 2\sqrt{2}$ [using (i)]

$\therefore$ Coordinates of required points are $(\pm 2\sqrt{2}, 4)$.

OR

We have, $$f(x) = 2x^2 - 3x$$

Differentiating w.r.t. x, we get

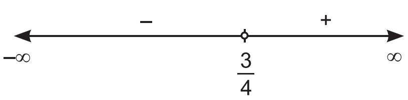
$$f'(x) = 4x - 3$$

For critical points, $f'(x) = 0$

$\Rightarrow$ $$4x = 3$$

$\Rightarrow$ $$x = \frac{3}{4}$$

Plotting on number line, we get



Sign of $f'(x)$ for different values of x

Intervals	Sign of $f'(x)$	Nature of 'f'
$\left(-\infty, \frac{3}{4}\right)$	– ve	Strictly decreasing
$\left(\frac{3}{4}, \infty\right)$	+ ve	Strictly increasing

Hence, f is strictly decreasing on $\left(-\infty, \frac{3}{4}\right)$ and strictly increasing on $\left(\frac{3}{4}, \infty\right)$.

24. $$\int_{-1}^{5} |x-3|\,dx$$

Let $f(x) = |x-3|$

Now, $$f(x) = \begin{cases} -(x-3), \text{ when } x < 3 \\ (x-3), \text{ when } x \geq 3 \end{cases}$$

So, $$\int_{-1}^{5} |x-3|\,dx = -\int_{-1}^{3} (x-3)\,dx + \int_{3}^{5} (x-3)\,dx = -\left[\frac{x^2}{2} - 3x\right]_{-1}^{3} + \left[\frac{x^2}{2} - 3x\right]_{3}^{5}$$

$$= -\left[\left(\frac{9}{2} - 9\right) - \left(\frac{1}{2} + 3\right)\right] + \left[\left(\frac{25}{2} - 15\right) - \left(\frac{9}{2} - 9\right)\right]$$

$$= \left(\frac{9}{2} + \frac{7}{2}\right) + \left(-\frac{5}{2} + \frac{9}{2}\right) = 8 + 2 = 10$$

25. $$y = \log_2(\sqrt{x-a} + \sqrt{x-b}) = \frac{\log(\sqrt{x-a} + \sqrt{x-b})}{\log 2}$$

$$\frac{dy}{dx} = \frac{1}{\log 2} \cdot \frac{1}{(\sqrt{x-a} + \sqrt{x-b})} \cdot \left\{\frac{1}{2\sqrt{x-a}} \cdot 1 + \frac{1}{2\sqrt{x-b}} \cdot 1\right\}$$

$$= \frac{1}{\log 2} \cdot \frac{1}{(\sqrt{x-a} + \sqrt{x-b})} \cdot \frac{1}{2}\left\{\frac{\sqrt{x-b} + \sqrt{x-a}}{\sqrt{x-a}\,\sqrt{x-b}}\right\}$$

$$= \frac{1}{2\log 2 \cdot (\sqrt{x-a} \cdot \sqrt{x-b})}$$

26. $$\int \underset{②}{x} \cdot \log\underset{①}{(1+2x)}\,dx = \log(1+2x) \cdot \frac{x^2}{2} - \int \frac{2}{1+2x} \cdot \frac{x^2}{2}\,dx$$

$$= \frac{x^2}{2}\log(1+2x) - \int \frac{x^2}{2x+1}\,dx = \frac{x^2}{2}\log(2x+1) - \frac{1}{4}\int \frac{(4x^2-1)+1}{2x+1}\,dx$$

$$= \frac{x^2}{2}\log(2x+1) - \frac{1}{4}\int\left(2x - 1 + \frac{1}{2x+1}\right)dx$$

$$= \frac{x^2}{2}\log(2x+1) - \frac{1}{4}x^2 + \frac{x}{4} - \frac{1}{8}\log|2x+1|$$

$$\int_0^1 x\log(1+2x)\,dx = \left[\frac{x^2}{2}\log(2x+1) - \frac{x^2}{4} + \frac{x}{4} - \frac{1}{8}\log|2x+1|\right]_0^1$$

$$= \left[\frac{1}{2}\log 3 - \frac{1}{4} + \frac{1}{4} - \frac{1}{8}\log 3\right] - [0 - 0 + 0 - 0] = \frac{3}{8}\log 3$$

27. $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}, P(C) = \frac{1}{4}$

Problem is solved by exactly two students.

$\therefore$ P(Problem solved by exactly two students) $= P(AB\overline{C})$ or $P(A\overline{B}C)$ or $P(\overline{A}BC)$

$$= \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{24}(3+1+2) = \frac{1}{4}$$

28. Let $I = \int \sqrt{(x-3)(5-x)}\,dx$. Then,

$$I = \int \sqrt{\{-x^2 + 8x - 15\}}\,dx$$

$$I = \int \sqrt{-\{x^2 - 8x + 16 - 16 + 15\}}\,dx$$

$$I = \int \sqrt{-\{(x-4)^2 - 1^2\}}\,dx = \int \sqrt{1^2 - (x-4)^2}\,dx$$

$$I = \frac{1}{2}(x-4)\sqrt{1^2 - (x-4)^2} + \frac{1}{2}(1)^2\sin^{-1}\left(\frac{x-4}{1}\right) + C$$

$$I = \frac{1}{2}(x-4)\sqrt{(x-3)(5-x)} + \frac{1}{2}\sin^{-1}(x-4) + C$$

OR

Let $I = \int \frac{1}{\sqrt{1-e^{2x}}}dx = \int \frac{1}{\sqrt{1-\frac{1}{e^{-2x}}}}dx = \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}}dx = \int \frac{e^{-x}}{\sqrt{(e^{-x})^2-1^2}}dx$

Let $e^{-x} = t$. Then, $d(e^{-x}) = dt \Rightarrow -e^{-x}dx = dt \Rightarrow e^{-x}dx = -dt$

$$\therefore \qquad I = -\int \frac{dt}{\sqrt{t^2-1^2}} = -\log\left|t+\sqrt{t^2-1}\right| + C$$

$$= -\log\left|e^{-x}+\sqrt{e^{-2x}-1}\right| + C$$

29. $y = \log(1 + \sin x)$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{\cos x}{1+\sin x}$$

Differentiating both sides again w.r.t. 'x', we get

$$\frac{d^2y}{dx^2} = \frac{(1+\sin x)(-\sin x) - \cos x \cdot \cos x}{(1+\sin x)^2}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{-1}{1+\sin x}$$

OR

We have, $\qquad \log\left(\frac{dy}{dx}\right) = 3x + 4y$

$$\Rightarrow \qquad \frac{dy}{dx} = e^{3x+4y} = e^{3x} \cdot e^{4y}$$

$$\Rightarrow \qquad \frac{dy}{e^{4y}} = e^{3x}\,dx$$

Integrating both sides, we get

$$\int e^{-4y}dy = \int e^{3x}dx$$

$$\Rightarrow \qquad -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C$$

30. We have to maximise, $Z = 3x + 5y$

The given constraints: $\qquad x + 4y \le 24 \qquad$ ...(*i*)

$3x + y \le 21 \qquad$ ...(*ii*)

$x + y \le 9 \qquad$ ...(*iii*)

$x \ge 0, \quad y \ge 0 \qquad$ ...(*iv*)

Converting (*i*) and (*iii*) inequations to equations and solving, we get

$$x + 4y = 24$$

$$\underline{\begin{array}{c} x + y = 9 \\ - \quad - \quad - \end{array}}$$

$$3y = 15$$

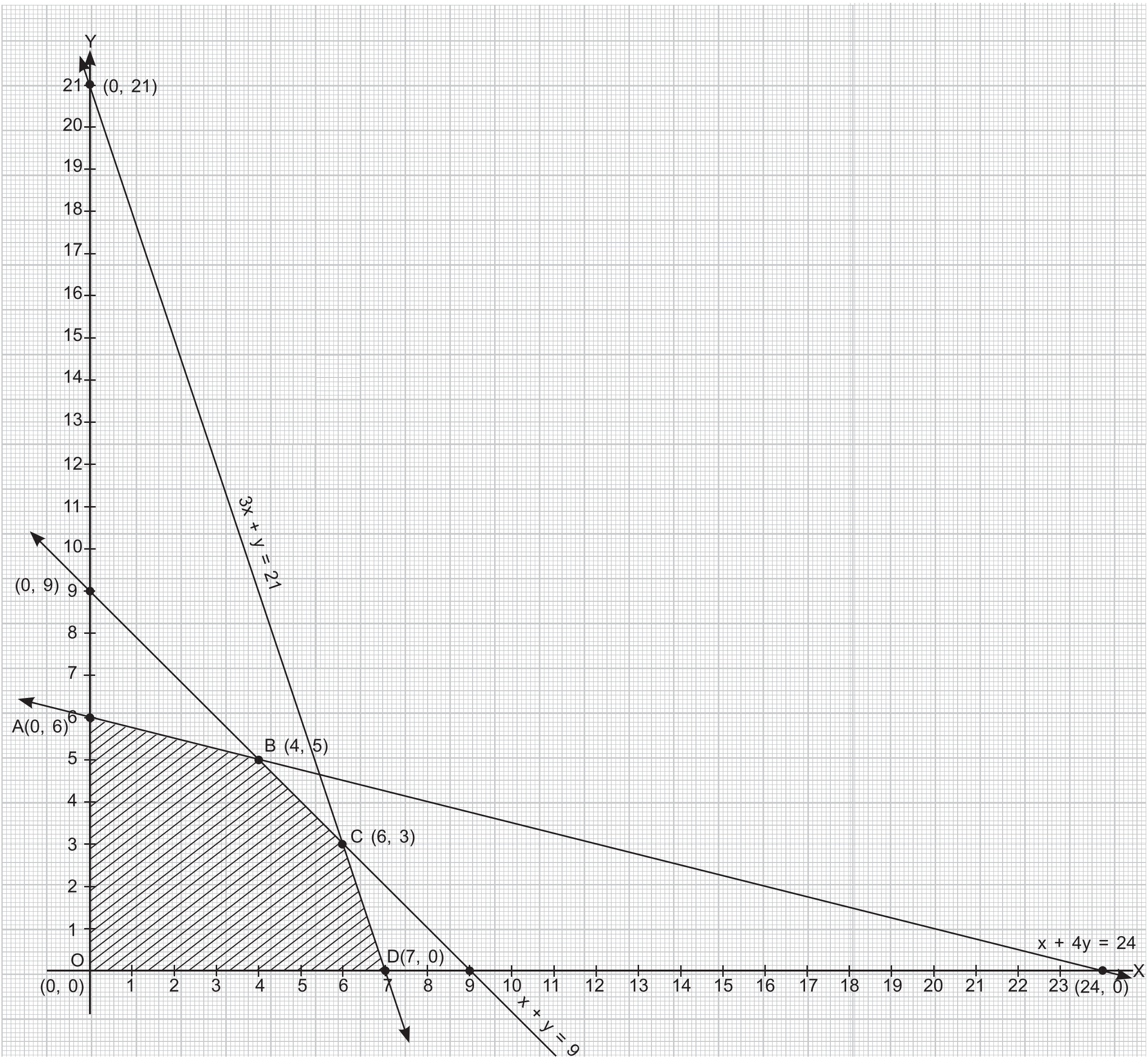
$$\Rightarrow \qquad y = 5, \text{ and } x = 4$$

Converting (*ii*) and (*iii*) inequations to equations and solving, we get

$$\begin{array}{r} x + y = 9 \\ 3x + y = 21 \\ - \quad - \quad - \\ \hline -2x = -12 \\ \hline \end{array}$$

$\Rightarrow$ $x = 6$, and $y = 3$

Converting (*i*) and (*ii*) inequations to equations and solving we get $x = \frac{60}{11}$ and $y = \frac{51}{11}$.



Corner Points	Values of $Z = 3x + 5y$
$O(0, 0)$	0
$A(0, 6)$	30
$B(4, 5)$	37 ← Maximum
$C(6, 3)$	33
$D(7, 0)$	21

$\therefore$ Maximum value $= 37$ at $x = 4, y = 5$

OR

We have to minimise, $Z = 9x - 11y$

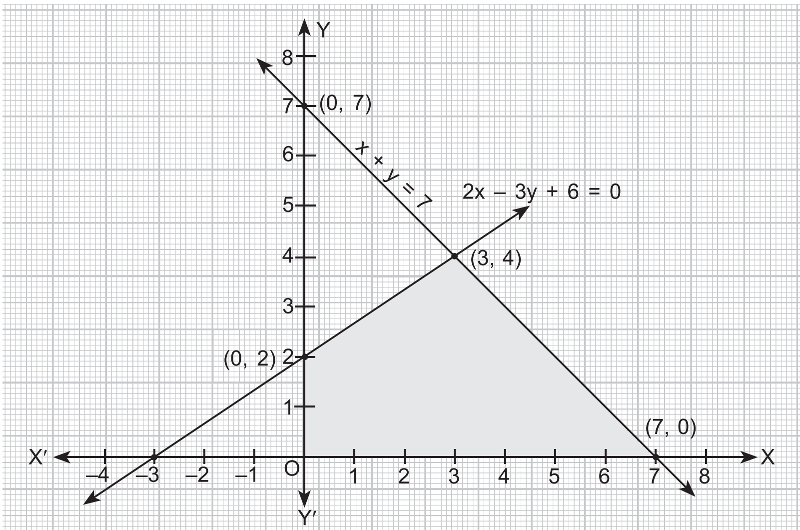
The given constraints are:

$$x + y \leq 7 \quad ...(i)$$

$$2x - 3y + 6 \geq 0 \quad ...(ii)$$

$$x, y \geq 0 \quad ...(iii)$$

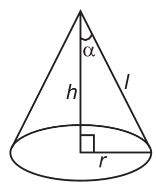
Let us graph the feasible region of the system of linear inequalities (*i*) to (*iii*). The shaded region is the feasible region.



Corner Points	Values of $Z = 9x - 11y$
(0, 0)	0
(7, 0)	63
(3, 4)	–17
(0, 2)	–22 ← Minimum

$\therefore$ Minimum value $= -22$ at $x = 0, y = 2$.

31. Let r be radius of the base, h the height and l the slant height of a cone.



$\therefore \quad l^2 = h^2 + r^2 \quad ...(i)$

Given, total surface area, $S = \pi rl + \pi r^2 \quad ...(ii)$

Volume of the cone,

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\sqrt{l^2 - r^2} \quad \text{[from } (i)]$$

$$= \frac{1}{3}\pi r^2\sqrt{\left(\frac{S - \pi r^2}{\pi r}\right)^2 - r^2} \quad \text{[from } (ii)]$$

If V is maximum, then $V^2 = W$(say) is maximum.

$$\therefore \quad V^2 = W = \frac{1}{9}\pi^2 r^4\left[\frac{S^2 + \pi^2 r^4 - 2S\pi r^2 - \pi^2 r^4}{\pi^2 r^2}\right] = \frac{1}{9}r^2[S^2 - 2S\pi r^2]$$

$$W = \frac{S}{9}[Sr^2 - 2\pi r^4]; \ \frac{dW}{dr} = \frac{S}{9}[2Sr - 8\pi r^3]$$

For maximum volume, $\frac{dW}{dr} = 0$

$$\Rightarrow \qquad 8\pi r^3 = 2Sr$$

$$\Rightarrow \qquad r^2 = \frac{S}{4\pi} \qquad \text{...}(iii)$$

$$\frac{d^2W}{dr^2} = \frac{S}{9}[2S - 24\pi r^2]$$

$$\Rightarrow \qquad \left.\frac{d^2W}{dr^2}\right]_{r=\sqrt{\frac{S}{4\pi}}} = \frac{S}{9}[2S - 6S] < 0$$

Hence, volume is maximum at $r = \sqrt{\frac{S}{4\pi}}$

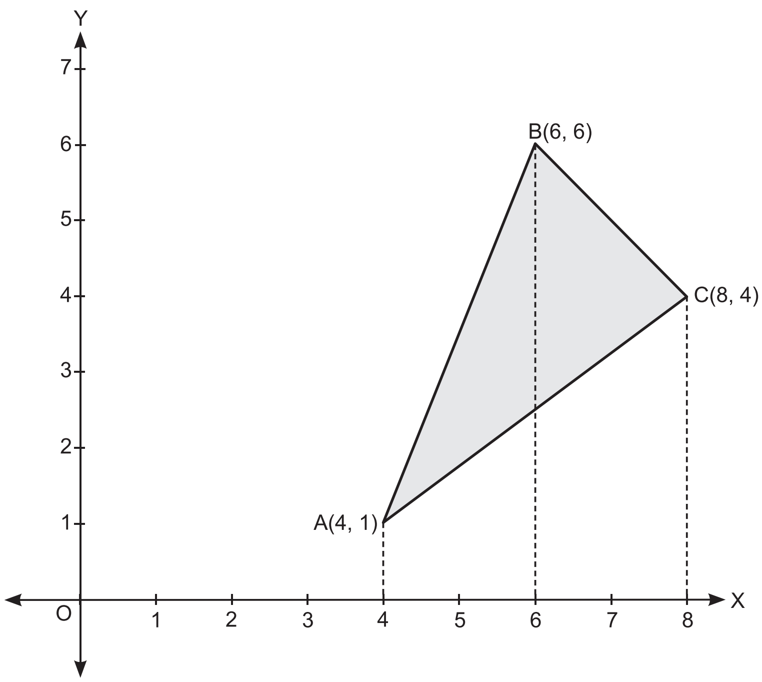
$\therefore$ from (iii), we have

$$4\pi r^2 = S \Rightarrow 4\pi r^2 = \pi rl + \pi r^2 \qquad \text{[from } (ii)\text{]}$$

$$\Rightarrow \qquad 3\pi r^2 = \pi rl \quad \Rightarrow \quad \frac{r}{l} = \frac{1}{3}$$

$$\Rightarrow \qquad \sin\alpha = \frac{1}{3} \quad \Rightarrow \quad \alpha = \sin^{-1}\frac{1}{3} \qquad [\alpha = \text{semi-vertical angle of cone}]$$

32. Plotting the points $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$ on graph, we notice, we have to find shaded area.



$$\text{Area } (\Delta ABC) = \int_4^6 y_{AB}\,dx + \int_6^8 y_{BC}\,dx - \int_4^8 y_{AC}\,dx$$

Equation of AB : $y - 1 = \frac{6-1}{6-4}(x-4) \Rightarrow y - 1 = \frac{5}{2}(x-4)$

$$\Rightarrow \qquad y = \frac{5}{2}x - 10 + 1 \Rightarrow y = \frac{5}{2}x - 9$$

Equation of BC : $y - 6 = \frac{4-6}{8-6}(x-6) \Rightarrow y - 6 = -1(x-6)$

$$\Rightarrow \qquad y - 6 = -x + 6 \Rightarrow y = -x + 12$$

Equation of AC : $y - 1 = \frac{4-1}{8-4}(x-4) \Rightarrow y - 1 = \frac{3}{4}(x-4)$

$\Rightarrow \quad y = \frac{3}{4}x - 3 + 1 \Rightarrow y = \frac{3}{4}x - 2$

$\therefore \quad \text{Area} = \int_4^6 \left(\frac{5}{2}x - 9\right)dx + \int_6^8 (-x+12)\,dx - \int_4^8 \left(\frac{3}{4}x - 2\right)dx$

$= \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[-\frac{x^2}{2} + 12x\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$

$= [(45 - 54) - (20 - 36)] + [(-32 + 96) - (-18 + 72)] - [(24 - 16) - (6 - 8)]$

$= [-9 + 16] + [64 - 54] - [8 + 2]$

$= 7 + 10 - 10$

$= 7$ sq units

33. Given function $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd.} \\ x-1, & \text{if } x \text{ is even.} \end{cases}$

For one-one:

(*i*) Let $x_1, x_2 \in N$ and x_1, x_2 are both even. Then,

$f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$

(*ii*) Let $x_1, x_2 \in N$ and x_1, x_2 are both odd. Then,

$f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$

(*iii*) Let $x_1, x_2 \in N$ and x_1 is even and x_2 is odd. Then $x_1 \neq x_2$.

Also, $f(x_1) = x_1 - 1$ (odd) and $f(x_2) = x_2 + 1$ (even) $\Rightarrow f(x_1) \neq f(x_2)$

So, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

(*iv*) We can also take x_1 as odd and x_2 as even and show as above $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

Hence, function 'f' is one-one.

For onto: Let $y \in N$ (co-domain)

If y is even

$\Rightarrow y = x + 1 \Rightarrow x = y - 1 \in N$ (domain)

$f(y - 1) = y - 1 + 1 = y$

If y is odd

$\Rightarrow y = x - 1 \Rightarrow x = y + 1 \in N$ (domain)

$f(y + 1) = y + 1 - 1 = y$

$\therefore$ For every $y \in N$ (co-domain), there exists $x \in N$ (domain) such that $y = f(x)$. Hence, function is onto.

$\therefore$ f is both one-one and onto and hence bijective.

OR

For reflexive: Let for $(a, b) \in A \times A$, $(a, b)\, R\, (a, b)$.

Now, $(a, b)\, R\, (a, b) \Rightarrow a + b = b + a$, true. Hence, R is reflexive.

For symmetric: Let for $(a, b), (c, d) \in A \times A$, $(a, b)\, R\, (c, d)$

Now, $(a, b)\, R\, (c, d) \Rightarrow a + d = b + c \Rightarrow b + c = a + d$

$\Rightarrow c + b = d + a \Rightarrow (c, d)\, R\, (a, b)$

$(a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$ for $(a, b), (c, d) \in A \times A$

Hence, R is symmetric.

For transitive: Let for $(a, b), (c, d), (e, f) \in A \times A$, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$.

Now, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow\ a + d = b + c$ and $c + f = d + e$

$\Rightarrow\ a + d + c + f = b + c + d + e$

$\Rightarrow\ a + f = b + e\ \Rightarrow\ (a, b)\ R\ (e, f)$

As for $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f) \Rightarrow\ (a, b)\ R\ (e, f)$ for $(a, b), (c, d), (e, f) \in A \times A$

Hence, R is transitive.

As R is reflexive, symmetric and transitive, so R is an equivalence relation.

If $(a, b) \in$ equivalence class $[(3, 7)]$ then

$(a, b)\ R\ (3, 7) \Rightarrow a + 7 = b + 3$

e.g. $(1, 5) \in [(3, 7)]$

$\therefore$ Equivalence class $[(3, 7)] = \{(1, 5), (2, 6), (3, 7), (4, 8), (5, 9)\}$

34. $$|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1 \qquad \text{(expanding along } C_3\text{)}$$

Let A_{ij} be the cofactors of elements in $|A|$. Then,

$$\begin{array}{lll} A_{11} = \cos\alpha, & A_{12} = -\sin\alpha, & A_{13} = 0 \\ A_{21} = \sin\alpha, & A_{22} = \cos\alpha, & A_{23} = 0 \\ A_{31} = 0, & A_{32} = 0, & A_{33} = 1 \end{array}$$

$$\therefore \qquad \text{adj}\,A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}' = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Consider } A(\text{adj}\,A) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha + 0 & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha + 0 & 0+0+0 \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha + 0 & \sin^2\alpha + \cos^2\alpha + 0 & 0+0+0 \\ 0-0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 = |A|\ I_3$$

$$(\text{adj}\,A)A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha + 0 & -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha + 0 & 0+0+0 \\ -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha + 0 & \sin^2\alpha + \cos^2\alpha + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 = |A|\ I_3$$

$\therefore \qquad A(\text{adj}\,A) = (\text{adj}\,A)A = |A|\ I_3.$

OR

Consider $$A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

and $$B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I \qquad ...(i)$$

Consider equations

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

Corresponding matrix equation is

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$\Rightarrow$ $BX = C$ is matrix equation.

Its solution is $X = B^{-1}C$ ...(*ii*)

Now we can use result (*i*) as

$$AB = 8I \Rightarrow \left(\frac{1}{8}A\right)B = I$$

$$\Rightarrow \quad B^{-1} = \frac{1}{8}A$$

Now we can substitute B^{-1} in (*ii*) and proceed further by substituting for A and finding X and then x, y, z.

$$X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow \quad x = 3; y = -2; z = -1$$

35. Lines are $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$

Here, $\vec{a}_1 = 5\hat{i} + 7\hat{j} - 2\hat{k}$, $\vec{b}_1 = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = -3\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{b}_2 = -3\hat{i} + 2\hat{j} + 4\hat{k}$

$$\vec{a}_2 - \vec{a}_1 = -8\hat{i} - 4\hat{j} + 8\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}(-4-2) - \hat{j}(12+3) + \hat{k}(6-3)$$

$$= -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{36+225+9}$$

$$= \sqrt{270} = 3\sqrt{30}$$

$$\text{Shortest distance between the lines} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{(-8\hat{i} - 4\hat{j} + 8\hat{k}) \cdot (-6\hat{i} - 15\hat{j} + 3\hat{k})}{\sqrt{270}} \right|$$

$$= \left| \frac{48+60+24}{\sqrt{270}} \right| = \frac{132}{\sqrt{270}}$$

$$= \frac{132}{3\sqrt{30}} \text{ units} = \frac{44}{\sqrt{30}} \text{ units}$$

36. (*i*) Length = $(18 - 2x)$ cm, breadth = $(18 - 2x)$ cm, height = x cm

(*ii*) Volume V = length × breadth × height = $x(18 - 2x)^2$ cm^3

(*iii*) As, $V = x(18 - 2x)^2$

$$\Rightarrow \quad \frac{dV}{dx} = x \cdot 2(18-2x)(-2) + (18-2x)^2$$

$$= (18-2x)(18-6x)$$

For maximum or minimum volume, $\dfrac{dV}{dx} = 0$

$$\Rightarrow \quad (18-6x)(18-2x) = 0$$

$$\Rightarrow \quad x = 3 \text{ or } x = 9$$

Now, $x = 9$ is rejected as length and breadth becomes 0 for $x = 9$.

Now, $$\frac{d^2V}{dx^2} = (18-2x)(-6) + (18-6x)(-2)$$

$$\left.\frac{d^2V}{dx^2}\right]_{x=3} = (18 - 2 \times 3)(-6) + (-2)(18 - 6 \times 3) = -72 < 0$$

$\therefore$ Volume is maximum at $x = 3$.

OR

(*iii*) Maximum Volume = $3(18 - 6)^2 = 432$ cm^3

37. Let E_1 : Event that Annu gets a prime number when a die is thrown.

A : Event that she gets exact one head.

E_2 : Event that Annu gets non prime number when a die is thrown.

(*i*) $P(E_1) = \dfrac{1}{2}$

(*ii*) $P(E_2) = \dfrac{1}{2}$

(*iii*) $P\left(\dfrac{A}{E_1}\right) = \dfrac{3}{8}$ and $P\left(\dfrac{A}{E_2}\right) = \dfrac{1}{2}$

Now, $P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2} \times \frac{3}{8}}{\frac{1}{2} \times \frac{3}{8} + \frac{1}{2} \times \frac{1}{2}} = \frac{3}{7}$

OR

(*iii*) $$P\left(\frac{E_2}{A}\right) = \frac{P(E_2) \times P\left(\frac{A}{E_2}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{3}{8} + \frac{1}{2} \times \frac{1}{2}} = \frac{4}{7}$$

38. (*i*) $$\overrightarrow{AB} = 2\hat{i} + 5\hat{j} - 5\hat{k}$$

$$\overrightarrow{AC} = -3\hat{i} + 4\hat{j} - 8\hat{k}$$

Vector perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{AC}$ $= \overrightarrow{AB} \times \overrightarrow{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -5 \\ -3 & 4 & -8 \end{vmatrix}$$

$$= \hat{i}(-40 + 20) - \hat{j}(-16 - 15) + \hat{k}(8 + 15)$$

$$= -20\hat{i} + 31\hat{j} + 23\hat{k}$$

(*ii*) $$\text{Ar}(\Delta ABC) = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$= \frac{1}{2}|-20\hat{i} + 31\hat{j} + 23\hat{k}|$$

$$= \frac{1}{2}\sqrt{400 + 961 + 529}$$

$$= \frac{1}{2}\sqrt{1890}$$

$$= \frac{3}{2}\sqrt{210} \text{ sq units.}$$