Solutions to RMM–DS1/Set-1

1. (*d*)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. (*b*) A is *a* 3×2 matrix,

No. of elements = 6

B is a 3×3 matrix

No. of elements = 9

C is *a* 2×3 matrix,

No. of elements = 6

3. (*a*)

$$a_{11} = 2;\ a_{12} = -3;\ a_{13} = 5$$

$$A_{31} = -12;\ A_{32} = 22;\ A_{33} = 18$$

$$\therefore \quad a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 2 \times (-12) + (-3) \times 22 + 5 \times 18 = -24 - 66 + 90 = 0$$

4. (*c*) $f(x) = \begin{cases} x - 1, & \text{if } x < 2 \\ 2x - 3, & \text{if } x \geq 2 \end{cases}$

Note: A polynomial function is always a continuous function. And the given question is a polynomial function with degree 1. Thus the continuity is for all real value of x.

5. (*d*)

$$\cos^2 \alpha + \cos^2 \beta + \cos^2\gamma = 1$$

$$\cos^2 45° + \cos^2 60° + \cos^2\gamma = 1$$

$$\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\cos^2\gamma = 1 - \frac{3}{4}$$

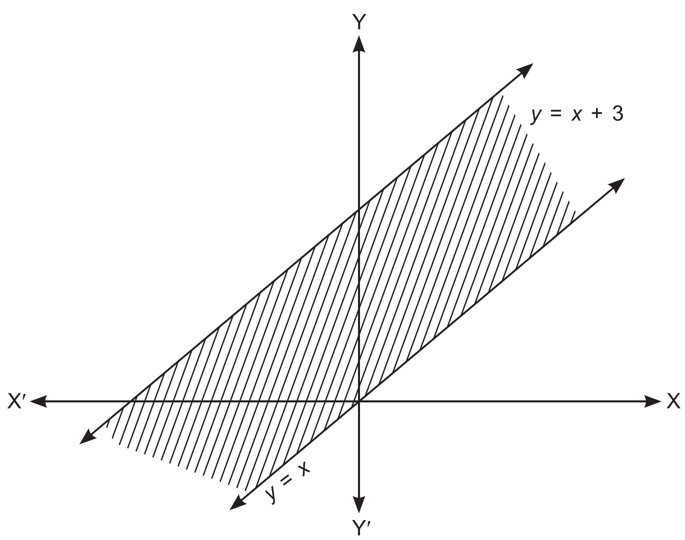
$$\cos^2\gamma = \frac{1}{4}$$

$$\cos\gamma = \pm\frac{1}{2}$$

Therefore required angle is 60° or 120°.

6. (*c*)

7. (*c*)



8. (*b*) Let
$$\vec{a} = \left(6\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$|\vec{a}| = \sqrt{6^2 + 2^2 + 3^2} = \sqrt{49} = 7$$

9. (*b*)
$$I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}}\, dx \quad ...(i)$$
$$\Rightarrow \quad I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}}\, dx \quad ...(ii)$$
Adding (*i*) and (*ii*), we get
$$2I = \int_3^6 \frac{\sqrt{9-x} + \sqrt{x}}{\sqrt{9-x} + \sqrt{x}}\, dx$$
$$\Rightarrow \quad 2I = \int_3^6 1\, dx = \Big[x\Big]_3^6$$
$$\Rightarrow \quad I = \frac{3}{2}$$

10. (*d*) Equation is $\begin{bmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{bmatrix} = 0$

On expanding : $1(-10x^2 - 10x) - 4(5x^2 - 5) + 20(2x + 2) = 0$
$$\Rightarrow \quad -10x^2 - 10x - 20x^2 + 20 + 40x + 40 = 0$$
$$\Rightarrow \quad -30x^2 + 30x + 60 = 0$$
$$\Rightarrow \quad 30(-x^2 + x + 2) = 0$$
$$\Rightarrow \quad (x - 2)(x + 1) = 0 \Rightarrow x = 2, x = -1$$

11. (*d*) $Z = px + qy$, where $p, q > 0$.

Z occurs at both the points (15, 15) and (0, 20)
$$\therefore \quad 15p + 15q = 20q$$
$$\Rightarrow \quad 15p = 5q$$
$$\Rightarrow \quad 3p = q$$

12. (*d*) Vectors $\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ and $\left(3\hat{i} + \lambda\hat{j} + \hat{k}\right)$ are perpendicular

therefore $\left(2\hat{i} - \hat{j} + 2\hat{k}\right) \cdot \left(3\hat{i} + \lambda\hat{j} + \hat{k}\right) = 0$
$$\Rightarrow \quad 6 - \lambda + 2 = 0$$
$$\Rightarrow \quad \lambda = 8$$

13. (*b*)
$$A(\text{adj. } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix},$$
$$A(\text{adj. } A) = |A|\, I$$
then
$$|A| = 10$$

14. (*c*) We have $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$.

Now,
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{7}{10} \times \frac{5}{4} = \frac{7}{8}$$

15. (*d*) $A = 5$

16. (*b*) Let $\vec{a} = \hat{i} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$.

Now, $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

Now, $$\text{area} = |\vec{a} \times \vec{b}| = \left|-\hat{i} + \hat{j} + \hat{k}\right|$$
$$= \sqrt{3} \text{ sq units}$$

17. (*c*) We have, $$y = \sec x^\circ$$

Now, $$x^\circ = \frac{\pi}{180} \times x^c$$

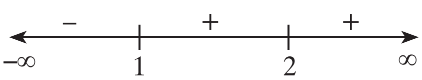
So, $$y = \sec \frac{\pi}{180}x$$

Now, $$\frac{dy}{dx} = \frac{\pi}{180^\circ} . \sec\left(\frac{\pi x}{180^\circ}\right) . \tan\left(\frac{\pi x}{180^\circ}\right)$$

$\Rightarrow$ $$\frac{dy}{dx} = \frac{\pi}{180^\circ} \sec x^\circ \tan x^\circ$$

18. (*c*) Skew lines

19. (*c*) (A) is true but (R) is false.

20. (*c*) 

Signs of $f'(x)$ for different values of x.

It is clear from figure $\frac{d}{dx}f(x)$ has no sign change at $x = 2$. Hence, $f(x)$ is neither maximum nor minimum at $x = 2$.

So (A) is true but (R) is false.

21. LHS $= \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

Let $$\sin^{-1}\frac{3}{4} = y$$

therefore, $$\sin y = \frac{3}{4} \text{ and } \cos y = \frac{\sqrt{7}}{4}$$

$\Rightarrow$ $$\tan\frac{y}{2} = \sqrt{\frac{1-\cos y}{1+\cos y}}$$

$\Rightarrow$ $$\tan\frac{y}{2} = \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}}$$
$$= \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}}$$
$$= \frac{4-\sqrt{7}}{3} = \text{RHS.}$$

Hence Proved

OR

$$5x - 3 \leq -1 \text{ or } 5x - 3 \geq 1$$

$$\Rightarrow \quad x \leq \frac{2}{5} \text{ or } x \geq \frac{4}{5}$$

$\therefore$ Domain are $\left(-\infty, \frac{2}{5}\right] \cup \left[\frac{4}{5}, \infty\right)$.

22. Let r be radius of the circle and A be its area at any instant of time 't'.

We have $\frac{dr}{dt} = 3$ cm/s; $\frac{dA}{dt} = ?$

We know that $A = \pi r^2$

Differentiating both sides. with respect to 't' we get

$$\frac{dA}{dt} = 2r\pi \frac{dr}{dt} \quad \Rightarrow \quad \frac{dA}{dt} = 6\pi r$$

$$\Rightarrow \quad \left(\frac{dA}{dt}\right)_{r=2} = 6\pi \times 2 = 12\pi \text{ cm}^2\text{/s}.$$

23. Let $f(x) = x^3 - 18x^2 + 96x$.

Differentiating both sides w.r.t. x, we get

$$\begin{aligned} f'(x) &= 3x^2 - 36x + 96 \\ &= 3(x^2 - 12x + 32) \\ &= 3(x-4)(x-8) \end{aligned}$$

For critical points, $\quad f'(x) = 0 \quad$ ...(i)

$$\Rightarrow \quad 3(x-4)(x-8) = 0$$

$$\Rightarrow \quad x = 4, 8$$

On differentiating both sides of (i), w.r.t. x, we get

$$f''(x) = 3(x-4)\,1 + 3(x-8)\,1 = 6(x-6)$$

At $x = 8$ sign of $f''(x)$ is positive.

Therefore $x = 8$ is point of minima.

Therefore minimum value $= 8^3 - 18 \times 8^2 + 96 \times 8$

$= 512 - 1152 + 768$

$= 128$

OR

We have, $\quad y = x^2 e^{-x}$

Now, $\quad \frac{dy}{dx} = -x^2 e^{-x} + 2xe^{-x}$

$$= e^{-x}(2x - x^2)$$

For 'y' to be strictly increasing

$$\frac{dy}{dx} > 0$$

$$\Rightarrow \quad \frac{2x - x^2}{e^x} > 0$$

$\Rightarrow \quad 2x - x^2 > 0 \quad [\because e^{-x} > 0]$

$\Rightarrow \quad x(2 - x) > 0$

$\Rightarrow \quad x(x - 2) < 0$

$\Rightarrow \quad x \in (0, 2)$

So, $f(x)$ is strictly increasing on (0, 2).

24. $$I = \int_0^4 \frac{dx}{\sqrt{x^2 + 2x + 3}}$$

$$I = \int_0^4 \frac{dx}{\sqrt{(x+1)^2 + (\sqrt{2})^2}}$$

$$= \left[\log\left| x + 1 + \sqrt{(x+1)^2 + (\sqrt{2})^2} \right|\right]_0^4$$

$$= \log\left(5 + \sqrt{25 + 2}\right) - \log\left(1 + \sqrt{1 + 2}\right)$$

$$= \log\left(5 + \sqrt{27}\right) - \log\left(1 + \sqrt{3}\right) = \log\left(\frac{5 + \sqrt{27}}{1 + \sqrt{3}}\right)$$

25. $f(x) = x^3 - 3x^2 + 6x - 100$

Differentiating w.r.t. x, we get

$$\frac{d}{dx} f(x) = 3x^2 - 6x + 6$$

$$= 3(x^2 - 2x + 2)$$

$$= 3[(x - 1)^2 + 1]$$

But $(x - 1)^2 \geq \forall x \in R$

Therefore $\frac{d}{dx} f(x) > 0$.

Therefore $f(x)$ is strictly increasing for all real values of x.

26. $$I = \int \frac{x^2 + x + 1}{x^2 - 1} dx$$

$$\Rightarrow \quad \int \frac{x^2 + x + 1}{x^2 - 1} dx = \int 1\, dx + \int \frac{x + 2}{(x+1)(x-1)} dx \quad ...(i)$$

Let $$\frac{x + 2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

$\Rightarrow \quad x + 2 = A(x - 1) + B(x + 1)$

On comparing the coefficients of x and constant term, we get $A + B = 1$ and $B - A = 2$

On solving, we get $B = 3/2$, $A = -\frac{1}{2}$

$$\int \frac{x + 2}{(x+1)(x-1)} dx = \frac{-1}{2} \int \frac{1}{x+1} dx + \frac{3}{2} \int \frac{1}{x-1} dx = \frac{-1}{2} \log|x + 1| + \frac{3}{2} \log|x - 1| + C$$

Therefore $$I = \int \frac{x^2 + x + 1}{x^2 - 1} dx = x - \frac{1}{2} \log|x + 1| + \frac{3}{2} \log|x - 1| + C$$

27. Let X = number of defective bulbs

$X = 0, 1, 2, 3, 4$ where X is random variable.

$$P(X = 0) = \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} = \frac{256}{625}$$

$$P(X = 1) = \frac{6}{30} \times \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} \times 4 = \frac{256}{625}$$

$$P(X = 2) = \frac{6}{30} \times \frac{6}{30} \times \frac{24}{30} \times \frac{24}{30} \times 6 = \frac{96}{625}$$

$$P(X = 3) = \frac{6}{30} \times \frac{6}{30} \times \frac{6}{30} \times \frac{24}{30} \times 4 = \frac{16}{625}$$

$$P(X = 4) = \frac{6}{30} \times \frac{6}{30} \times \frac{6}{30} \times \frac{6}{30} = \frac{1}{625}$$

So, probability distribution is:

X	0	1	2	3	4
$P(X)$	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

28. $$I = \int \frac{e^x (x-3)}{(x-1)^3} dx \Rightarrow I = \int \frac{e^x [(x-1)-2]}{(x-1)^3} dx$$

$\Rightarrow$ $$I = \int \frac{e^x (x-1)}{(x-1)^3} dx - \int \frac{2e^x}{(x-1)^3} dx$$

$\Rightarrow$ $$I = \int \frac{e^x}{(x-1)^2} dx - \int e^x \cdot \frac{2}{(x-1)^3} dx$$

$\Rightarrow$ $$I = \int e^x \left[\frac{1}{(x-1)^2} + \frac{(-2)}{(x-3)^3} \right] dx$$

$\Rightarrow$ $$I = \frac{e^x}{(x-1)^2} + C$$

Note: $\int e^x [f(x) + f'(x)] dx = e^x \cdot f(x) + C$

OR

$$I = \int_0^{\pi} \frac{x + \sin x}{1 + \cos^2 x} dx \quad \text{...}(i)$$

Now, $$I = \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \quad \text{...}(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx$$

$$2I = -\int_1^{-1} \frac{\pi \, dt}{1 + t^2}$$ [Let $\cos x = t \Rightarrow -\sin x \, dx = dt$, if $x = 0$ then $t = 1$ and $x = \pi$ then $t = -1$]

$\Rightarrow$ $$2I = \int_{-1}^{1} \frac{\pi dt}{1 + t^2}$$

$\Rightarrow$ $$2I = \pi \, [\tan^{-1} t]_{-1}^{1}$$

$\Rightarrow$ $$I = \frac{\pi}{4}$$

29. $x(x^2-1)\dfrac{dy}{dx} = 1$

$\Rightarrow \quad dy = \dfrac{dx}{x(x^2-1)}$

Integrating both sides, we get

$\Rightarrow \quad y = \displaystyle\int \frac{1}{x^3\left(1-\frac{1}{x^2}\right)}dx$ [put $\left(1-\frac{1}{x^2}\right) = t \Rightarrow \frac{2}{x^3}dx = dt$]

So, $y = \dfrac{1}{2}\displaystyle\int \frac{1}{t}\,dt$

$\Rightarrow \quad y = \dfrac{1}{2}\log|\,t\,| + C$

$\Rightarrow \quad y = \dfrac{1}{2}\log\left|\left(1-\dfrac{1}{x^2}\right)\right| + C$

On putting $y = 0$ and $x = 2$, in the above equation we get $C = -\dfrac{1}{2}\log\dfrac{3}{4}$

So, particular solution is $y = \dfrac{1}{2}\log\left|\left(1-\dfrac{1}{x^2}\right)\right| - \dfrac{1}{2}\log\dfrac{3}{4}$.

OR

$\dfrac{dy}{dx} - 3y\cot x = \sin 2x$

Comparing with $\dfrac{dy}{dx} + Py = Q$, we get $P = -3\cot x$, and $Q = \sin 2x$

$\Rightarrow \quad I.F = e^{\int P\,dx}$

$\Rightarrow \quad I.F = e^{\int -3\cot x\,dx}$

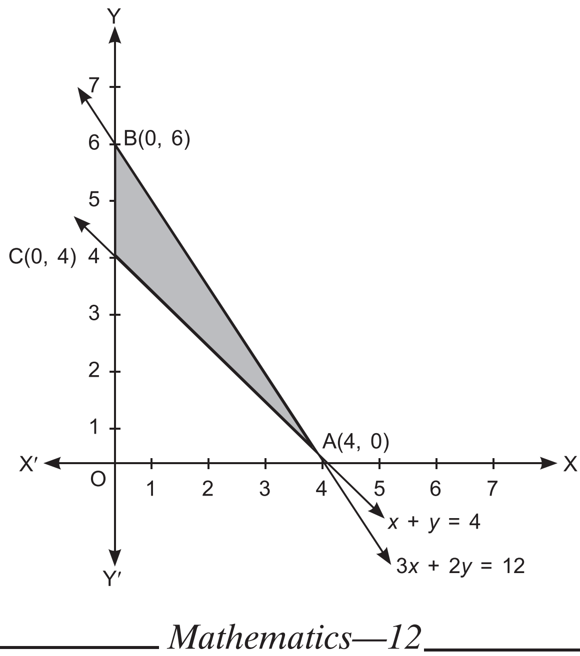
$\Rightarrow \quad I.F = e^{-3\log\sin x} = \operatorname{cosec}^3 x$

Therefore general solution is: $y.\,IF = \displaystyle\int Q\cdot IF\,dx$

$\Rightarrow \quad y.\operatorname{cosec}^3 x = \displaystyle\int 2\sin x\cos x.\frac{1}{\sin^3 x}dx$

$\Rightarrow \quad y.\operatorname{cosec}^3 x = -2\operatorname{cosec} x + C \Rightarrow y\operatorname{cosec}^3 x + 2\operatorname{cosec} x = C$

30. We have to maximise $Z = 4x + 6y$, subject to $3x + 2y \le 12$, $x + y \ge 4$, $x, y \ge 0$



Shaded portion ABC is the feasible region, where $A(4, 0)$, $C(0, 4)$ and $B(0, 6)$.

Corner points	Values of $Z = 4x + 6y$	
$A(4, 0)$	16	
$B(0, 6)$	36	←Maximum
$C(0, 4)$	24	

Thus, Z is maximised at $B(0, 6)$ and its maximum value is 36.

OR

We have to minimise $Z = 11x - 10y$
Subject to the constraints:

$$x + y \leq 20 \quad ...(i)$$
$$3x + 2y \leq 48 \quad ...(ii)$$
$$x, y \geq 0 \quad ...(iii)$$

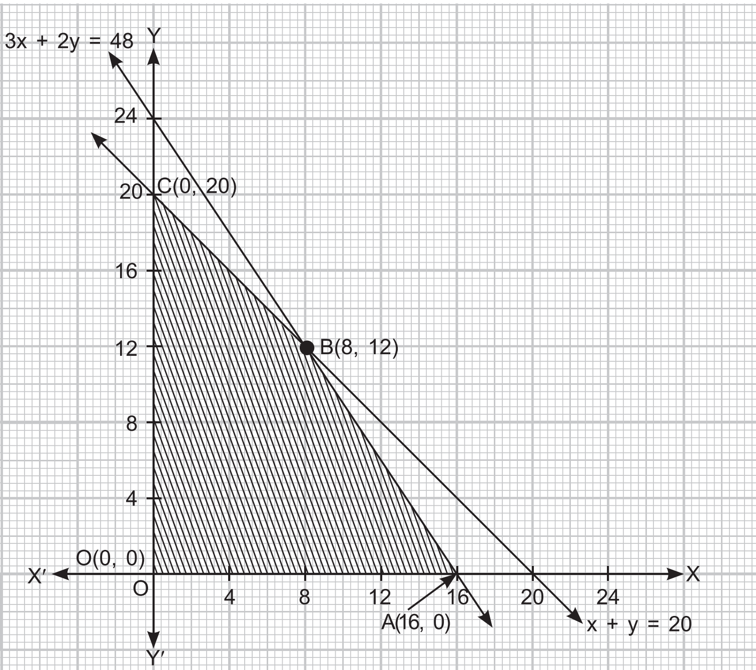
Converting (i) and (ii) to equations we get

$$x + y = 20 \quad ...(iv)$$
$$3x + 2y = 48 \quad ...(v)$$

Solving (iv) and (v), we get

$$x = 8, \quad y = 12$$

Let us graph the feasible region of the system of linear inequalities (i) to (iii). The feasible region is shaded, where $O(0, 0)$, $A(16, 0)$, $B(8, 12)$ and $C(0, 20)$



Corner points	Values of $Z = 11x - 10y$	
$O(0, 0)$	0	
$A(16, 0)$	176	
$B(8, 12)$	–32	
$C(0, 20)$	–200	←Minimum

$\therefore$ Minimum value = –200 at $x = 0$ and $y = 20$.

31. $y = e^{ax} \sin bx$

Differentiating w.r.t. 'x', we get

$$\frac{dy}{dx} = e^{ax} \cos bx \,.\, b + e^{ax} \,.\, a \sin bx$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{ax} \cos bx \,.\, b + ay$$

$$\Rightarrow \quad \frac{dy}{dx} - ay = e^{ax} \cos bx \,.\, b$$

Differentiating w.r.t. 'x', we get

$$\frac{d^2y}{dx^2} - a\frac{dy}{dx} = b(e^{ax} \cos bx\ a. - e^{ax} \,.\, b \sin bx)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - a\frac{dy}{dx} = b\ a\ e^{ax} \cos bx - b^2\ e^{ax} \sin bx$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - a\frac{dy}{dx} = a\left(\frac{dy}{dx} - ay\right) - b^2\ y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - a\frac{dy}{dx} = a\frac{dy}{dx} - a^2\ y - b^2\ y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)\ y = 0$$

Hence Proved.

32. Given curves are $y = \sqrt{x}$ .... (i) and $2y - x + 3 = 0$ .... (ii)

On solving equations (i) and (ii), we get

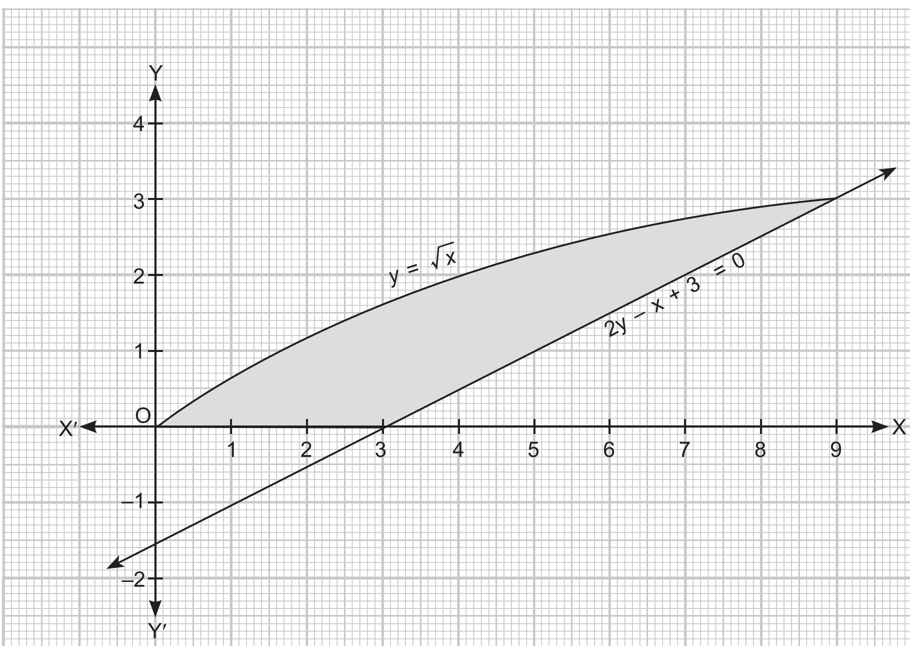
$$2y - y^2 + 3 = 0$$

$$\Rightarrow \quad y^2 - 2y - 3 = 0$$

$$\Rightarrow \quad (y - 3)\ (y + 1) = 0$$

$\Rightarrow$ $y = 3$ and $y = -1$(rejected because $\sqrt{x} = -1$ not possible)

$\Rightarrow$ $y = 3$



$\therefore$ Required area $= \int_0^3 (2y+3) - \int_0^3 y^2\,dx$

$$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3$$

$= 9 + 9 - 9$

$= 9$ sq units

33. Given relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$

$R = \{(1,2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

Reflexive:

Let $a \in A$, we have, $a \neq a + 1 \Rightarrow (a, a) \notin \text{R}$ as $(1, 1)$, $(2, 2)$, ... $(6, 6) \notin R$.

$\therefore$ It is not reflexive.

Symmetric: Let $a = 1$ and $b = 2$ i.e. $a, b \in A$, $\therefore$ $b = a + 1 \Rightarrow 2 = 1 + 1 \Rightarrow (a, b) \in \text{R}$ but $a \neq b + 1$ as $1 \neq 2 + 1 \Rightarrow (b, a) \notin R$. So, $(1, 2) \in R$ but $(2, 1) \notin R$.

$\therefore$ It is not symmetric.

Transitive: Let $a, b, c \in A$. Now, if $(a, b) \in R \Rightarrow b = a + 1$ ...(*i*) and $(b, c) \in R \Rightarrow c = b + 1$ ...(*ii*)

From (*i*) and (*ii*), we have $c = (a + 1) + 1 = a + 2 \Rightarrow c = a + 2 \Rightarrow (a, c) \notin R$

For example, $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$.

$\therefore$ It is not transitive.

Hence, relation R is neither reflexive, nor symmetric, nor transitive.

OR

Given that, $A = R - \{3\}$, $B = R - \{1\}$. $f : A \to B$ is defined by $f(x) = \dfrac{x-2}{x-3}$

For injectivity : Let $x_1, x_2 \in \text{A}$. Then,

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 3x_2 - 2x_1 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1 \Rightarrow x_1 = x_2$$

So, $f(x)$ is an injective function.

For surjectivity:

Let $$y = \frac{x-2}{x-3}$$

$$\Rightarrow xy - 3y = x - 2$$

$$\Rightarrow xy - x = 3y - 2$$

$$\Rightarrow x(y - 1) = 3y - 2$$

$$\Rightarrow x = \frac{3y-2}{y-1} \in A \text{ for every } y \in B.$$

So, $f(x)$ is a surjective function. Hence, $f(x)$ is a bijective function i.e. one-one and onto.

34. $$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & -2 \\ 1 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 & -7 & -1 \\ -5 & 10 & 5 \\ -2 & -1 & 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & -2 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 11 & -7 & -1 \\ -5 & 10 & 5 \\ -2 & -1 & 7 \end{bmatrix}$$

$$\Rightarrow \qquad AB = \begin{bmatrix} 33-10+2 & -21+20+1 & -3+10-7 \\ 11-15+4 & -7+30+2 & -1+15-14 \\ 11-5-6 & -7+10-3 & -1+5+21 \end{bmatrix}$$

$$\Rightarrow \qquad AB = 25I \qquad ...(i)$$

The given system of equations is:

$$3x + 2y - z = 4, \quad x + 3y - 2z = 2, \quad x + y + 3z = 5$$

Matrix equation is $\begin{bmatrix} 3 & 2 & -1 \\ 1 & 3 & -2 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$

We have $CX = D$ and $X = C^{-1}D$

Here $C = A \Rightarrow C^{-1} = A^{-1}$

We have $AB = 25I$

Therefore $A^{-1} = \frac{1}{25}B \Rightarrow C^{-1} = \frac{1}{25}B$

Then
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 11 & -7 & -1 \\ -5 & 10 & 5 \\ -2 & -1 & 7 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 44-14-5 \\ -20+20+25 \\ -8-2+35 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 25 \\ 25 \\ 25 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ therefore $x = 1, y = 1, z = 1$.

35. Lines are : $\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$ and $\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$

In vector form:

$$\vec{r} = (-2\hat{j} + \hat{k}) + \lambda(6\hat{i} + 6\hat{j} + \hat{k})$$

and
$$\vec{r} = (-\hat{i} + 0\hat{j} + 0\hat{k}) + \mu(12\hat{i} + 6\hat{j} - \hat{k})$$

Comparing with

$$\vec{r} = \vec{a_1} + \lambda\vec{b_1} \text{ and } \vec{r} = \vec{a_2} + \mu\vec{b_2}$$

we get

$$\vec{a_1} = -2\hat{j} + \hat{k}, \quad \vec{a_2} = -\hat{i} + 0\hat{j} + 0\hat{k}, \quad \vec{b_1} = 6\hat{i} + 6\hat{j} + \hat{k}, \quad \vec{b_2} = 12\hat{i} + 6\hat{j} - \hat{k}$$

$$\vec{a_2} - \vec{a_1} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-12)^2 + (18)^2 + (-36)^2} = \sqrt{1764} = 42$$

$$\left(\vec{b_1} \times \vec{b_2}\right) \cdot \left(\vec{a_2} - \vec{a_1}\right) = 12 + 36 + 36 = 84$$

Shortest distance = $\left|\dfrac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$

$= \dfrac{84}{42} = 2$ units

OR

Equation of line passing through (1, 2, –4) having DR's <a, b, c> is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c} \quad ...(i)$$

Line (i) is perpendicular to lines $\dfrac{x-8}{3} = \dfrac{y+19}{-16} = \dfrac{z-10}{7}$ and $\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$

$\therefore \quad 3a - 16b + 7c = 0 \quad ...(ii)$

$3a + 8b - 5c = 0 \quad ...(iii)$

Solving (ii) and (iii) by cross-multiplication,

$$\frac{a}{80-56} = \frac{-b}{-15-21} = \frac{c}{24+48}$$

$$\frac{a}{24} = \frac{-b}{-36} = \frac{c}{72}$$

Therefore $a = 2$, $b = 3$ and $c = 6$

Therefore equation of line $= \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+4}{6}$

$\therefore$ Vector equation of the line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

36. Let E_1 : Event that Annu gets a prime number when a die is thrown.

A : Event that she gets exact one head.

E_2 : Event that Annu gets non prime number when a die is thrown.

(i) $P(E_1) = \dfrac{1}{2}$

(ii) $P(E_2) = \dfrac{1}{2}$

(iii) $P\left(\dfrac{A}{E_1}\right) = \dfrac{3}{8}$ and $P\left(\dfrac{A}{E_2}\right) = \dfrac{1}{2}$

Now, $P\left(\dfrac{E_1}{A}\right) = \dfrac{P(E_1)\times P\left(\dfrac{A}{E_1}\right)}{P(E_1)\times P\left(\dfrac{A}{E_1}\right) + P(E_2)\times P\left(\dfrac{A}{E_2}\right)} = \dfrac{\frac{1}{2}\times\frac{3}{8}}{\frac{1}{2}\times\frac{3}{8}+\frac{1}{2}\times\frac{1}{2}} = \dfrac{3}{7}$

OR

(iii) $P\left(\dfrac{E_2}{A}\right) = \dfrac{P(E_2)\times P\left(\dfrac{A}{E_2}\right)}{P(E_1)\times P\left(\dfrac{A}{E_1}\right) + P(E_2)\times P\left(\dfrac{A}{E_2}\right)}$

$= \dfrac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{3}{8}+\frac{1}{2}\times\frac{1}{2}} = \dfrac{4}{7}$

37. (*i*) Length of box = (18 – 2x) cm

Breadth of box = (18 – 2x) cm

Height = x cm

Volume of the box = $x(18 - 2x)^2$ cm^3

(*ii*) Volume = $x(18 - 2x)^2$

Differentiating both sides w.r.t. x, we get

$$\frac{dV}{dx} = (18 - 2x)^2 + x \times 2(18 - 2x)(-2)$$

$$= (18 - 2x)(18 - 6x) \quad ...(i)$$

For maximum or minimum volume, $\frac{dV}{dx} = 0$

$\Rightarrow 18 - 2x = 0$ or $18 - 6x = 0 \;\Rightarrow\; x = 9$ (not possible), $x = 3$

$\therefore x = 3$

Again differentiating (*i*), w.r.t. x, we get

$$\therefore \quad \frac{d}{dx}\left(\frac{dV}{dx}\right) = (18 - 2x)(-6) + (18 - 6x)(-2)$$

$$\Rightarrow \quad \left(\frac{d^2V}{dx^2}\right)_{x=3} = -72 < 0$$

At $x = 3$, volume is maximum.

(*iii*) Maximum volume = 12 × 12 × 3

= 432 cm^3

OR

(*iii*) For maximum volume

$l = 12$ cm, $b = 12$ cm, $h = 3$ cm

Surface area = $lb + 2hl + 2hb$

= 12 × 12 + 2 × 3 × 12 + 2 × 3 × 12

= 144 + 72 + 72

= 288 cm^2

38. $A(0, 0, 0)$, $B(4, 0, 0)$, $C(4, 4, 0)$, $D(0, 4, 0)$, $E(0, 0, 4)$, $F(4, 0, 4)$, $G(4, 4, 4)$ and $H(0, 4, 4)$.

(*i*) DR's of EC = (4, 4, –4)

DR's of AG = (4, 4, 4)

(*ii*) DR's of HB = (4, –4, –4)

DR's of DF = (4, –4, 4)

Angle between HB and DF

$$\cos\theta = \frac{4 \times 4 + \{(-4) \times (-4)\} + 4 \times \{-4\}}{\sqrt{4^2 + (-4)^2 + (-4)^2}\,\sqrt{4^2 + (-4)^2 + 4^2}} = \frac{1}{3}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$