Solutions to RMM–DS1/Set-2

1. (*a*) $$(A + B) + C = A + (B + C)$$

2. (*a*) $$A = [a_{ij}]_{m \times n}$$

3. (*b*) $$[1 \quad x \quad 1]\begin{bmatrix}1 & 3 & 2\\0 & 5 & 1\\0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix} = O$$

$$\Rightarrow \quad [1 \quad x \quad 1]\begin{bmatrix}x+3-4\\5-2\\3-4\end{bmatrix} = O$$

$$\Rightarrow \quad [1 \quad x \quad 1]\begin{bmatrix}x-1\\3\\-1\end{bmatrix} = O$$

$$\Rightarrow \quad x - 1 + 3x - 1 = 0$$

$$\Rightarrow \quad 4x = 2$$

$$\Rightarrow \quad x = \frac{1}{2}$$

4. (*a*) $$x\frac{dy}{dx} + y = e^x$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{x}\cdot y = e^x$$

Comparing with $\frac{dy}{dx} + Py = Q,$

we get $P = \frac{1}{x}, \ Q = e^x$

$$IF = e^{\int Pdx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

5. (*b*) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$= 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma - 3 = 2(1) - 3 = -1 \quad \{\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = l^2 + m^2 + n^2 = 1\}$

6. (*a*) $$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 3x - \frac{dy}{dx}$$

On squaring both sides, we get

$$1 + \left(\frac{dy}{dx}\right)^2 = 9x^2 + \left(\frac{dy}{dx}\right)^2 - 6x\frac{dy}{dx} \quad \Rightarrow \quad 6x\left(\frac{dy}{dx}\right) + 1 = 9x^2$$

So, order = 1, degree = 1

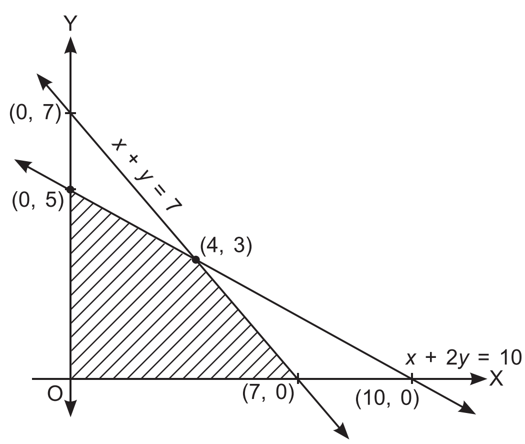
Sum = 1 + 1 = 2

7. (*c*) Solving equations $x + y = 7$

and $x + 2y = 10$, we get

$y = 3, x = 4$

Points	Values of $Z = 5x + 2y$
(0, 5)	10
(7, 0)	35 → Maximum
(0, 0)	0
(4, 3)	26



8. (*d*)

$$\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{a}\cdot\vec{b} = 2 \times (-1) + (-3) \times 2 + 2 \times 1$$

$$= -2 - 6 + 2$$

$$= -6$$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \left|\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right|$$

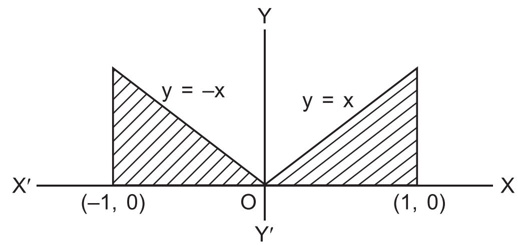
$$= \left|\frac{-6}{\sqrt{1+4+1}}\right| = \sqrt{6}$$

9. (*a*)

$$\int_{-1}^{1} |x| dx = \int_{-1}^{0} -x\, dx + \int_{0}^{1} x\, dx$$

$$= -\left(\frac{x^2}{2}\right)_{-1}^{0} + \left(\frac{x^2}{2}\right)_{0}^{1}$$

$$= -\left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right) = 1$$



10. (*a*) Points $P(3, -2)$, $Q(8, 8)$, $R(k, 2)$ are collinear.

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 3(8 - 2) + 2(8 - k) + 1(16 - 8k) = 0$$

$$\Rightarrow \quad 18 + 16 - 2k + 16 - 8k = 0$$

$$\Rightarrow \quad k = 5$$

11. (*b*) Objective function of a linear programming problem is a function to be optimised.

12. (*b*) $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$$

$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$$

$$= 2(\vec{a} \times \vec{b})$$

13. (*d*) We have,

$$A = \begin{bmatrix} 3 & 4 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 15 \\ 0 & -10 \end{bmatrix}$$

$$|AB| = |A||B|$$

$$= (-12) \times (-10) = 120$$

14. (*d*) We have,

$$P(A) = \frac{1}{5} \quad ; \quad P(B) = 0$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A \cap B)}{0} = \text{Not defined}$$

15. (*b*) $\int \frac{dx}{\cos^2 x \sin^2 x}$

$= \int \left(\frac{\cos^2 x}{\cos^2 x \sin^2 x} + \frac{\sin^2 x}{\cos^2 x \sin^2 x} \right) dx$

$= \int \left(\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \right) dx$

$= \int \text{cosec}^2 x \, dx + \int \sec^2 x \, dx$

$= -\cot x + \tan x + C$

$= \tan x - \cot x + C$

16. (*d*) $(\vec{x} + \vec{a}) \cdot (\vec{x} - \vec{a}) = 5$

$\Rightarrow \quad |\vec{x}|^2 - \vec{x} \cdot \vec{a} + \vec{a} \cdot \vec{x} - |\vec{a}|^2 = 5$

$\Rightarrow \quad |\vec{x}|^2 = 6$

$\Rightarrow \quad |x| = \sqrt{6}$

17. (*a*) $y = e^{1 + \log x} = e^{\log e + \log x}$

$\Rightarrow \quad y = e^{\log(ex)}$

$\Rightarrow \quad y = ex$

Differentiating w.r.t x, we get

$$\frac{dy}{dx} = e$$

18. (*c*) We have, $\frac{1}{\sqrt{2}}, \frac{1}{2}, k$ are direction cosines of the given line.

Let $\quad l = \frac{1}{\sqrt{2}}, m = \frac{1}{2}, n = k$

$\therefore \quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} + k^2 = 1$

$\Rightarrow \quad k^2 = \frac{1}{4}$

$\Rightarrow \quad k = \pm\frac{1}{2}$

19. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

20. (*b*) Both (A) and (R) are true but (R) is not the correct explanation of (A).

21. Let $\quad \cos^{-1}\frac{1}{2} = x \quad$ and $\quad \sin^{-1}\frac{3}{5} = y$

$\cos x = \frac{1}{2} \quad$ and $\quad \sin y = \frac{3}{5}$

$\therefore \quad \tan x = \sqrt{3} \quad$ and $\quad \tan y = \frac{3}{4}$

$\therefore \quad \tan\left(\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5}\right) = \tan(x + y)$

$$= \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

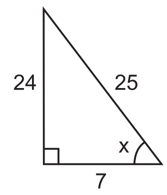
$$= \frac{\sqrt{3} + \frac{3}{4}}{1 - \sqrt{3} \times \frac{3}{4}}$$

$$= \left(\frac{4\sqrt{3} + 3}{4 - 3\sqrt{3}} \right)$$

OR

$\cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right)$

Let $\cos^{-1}\frac{7}{25} = x$



$\Rightarrow$ $\cos x = \frac{7}{25}$

We have

$$\cot x = \frac{7}{24} \Rightarrow \cot\left(\cos^{-1}\left(\frac{7}{25}\right)\right) = \frac{7}{24}$$

22. Let r be the base radius and h be the height of the cylinder at a particular instant of time 't'. Let V be its volume at that instant.

Given: $\frac{dr}{dt} = 2$ cm/s; $\frac{dh}{dt} = -3$ cm/s

We know that:

Volume of cylinder, $V = \pi r^2 h$

On differentiating both sides w.r.t 't', we get

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + h \times 2r\frac{dr}{dt}\right)$$

Now, $\left[\frac{dV}{dt}\right]_{r=3,\, h=4} = \pi[9 \times (-3) + 4 \times 2 \times 3 \times 2]$

$= \pi[-27 + 48]$

$= 21\pi$ cm^3/s

So, volume is increasing at the rate of 21π cm^3/s.

23. We have, $f(x) = x^4 - 32x^2 + ax + 10$

Differentiating w.r.t. 'x', we get

$$f'(x) = 4x^3 - 64x + a$$

A.T.Q at $x = 1$, $f'(x) = 0$, so

$0 = 4(1)^3 - 64(1) + a$

$\Rightarrow$ $0 = 4 - 64 + a$

$\Rightarrow$ $a = 60$

OR

$$y = 2x^3 + 9x^2 + 12x - 1$$

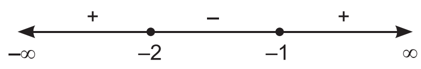
Differentiating w.r.t 'x', we get

$$\frac{dy}{dx} = 6x^2 + 18x + 12$$

$= 6(x^2 + 3x + 2)$

$= 6(x + 2)(x + 1)$

Intervals	Sign of $f'(x)$	Nature of f
$(-\infty, -2)$	+ve	Strictly increasing
$(-2, -1)$	–ve	Strictly decreasing
$(-1, \infty)$	+ve	Strictly increasing



$\therefore$ In $(-2, -1)$, 'f' is strictly decreasing.

24. If $0 \le x \le \frac{\pi}{4}$

$\Rightarrow$ $\sin x < \cos x$

$\therefore$ $|\sin x - \cos x| = \cos x - \sin x$

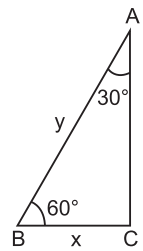
$$I = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$$

$$= \Big[\sin x + \cos x\Big]_0^{\frac{\pi}{4}}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0)$$

$$= \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

25. Let AB be ladder resting with wall at an angle of 30°. Let AC be the wall. Suppose at any instant of time t, man is at point B. Let BC = x and AB = y.



Given: $\frac{dy}{dt} = 3$ m/min

Now, $\frac{x}{y} = \cos 60°$

$\Rightarrow$ $x = y \cos 60° \Rightarrow x = \frac{y}{2}$

Differentiating w.r.t. 't', we get

$$\frac{dx}{dt} = \frac{dy}{dt} \times \frac{1}{2}$$

$$= 3 \times \frac{1}{2} = \frac{3}{2} \text{ m/min}$$

26. Let $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ ...(*i*)

$\Rightarrow$ $I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} dx$ $\left(\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

$\Rightarrow$ $I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x} dx$

$\Rightarrow$ $I = \int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx - \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$2I = \pi \int_0^{\pi} \left(\frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x}\right) dx$$

$$= \pi \int_0^{\pi} (\tan x \sec x - \tan^2 x)\,dx = \pi \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1)\,dx$$

$$= \pi[\sec x - \tan x + x]_0^{\pi}$$

$$= \pi[\sec \pi - \tan \pi + \pi - \sec 0 + \tan 0 - 0] = \pi(\pi - 2)$$

$\therefore$ $I = \frac{\pi}{2}(\pi - 2)$

27. X = Number of defective pens.

$X = 0, 1, 2, 3, 4$

$$P(X = 0) = \frac{15}{20} \times \frac{15}{20} \times \frac{15}{20} \times \frac{15}{20} = \frac{81}{256}$$

$$P(X = 1) = \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{4!}{3!} = \frac{108}{256}$$

$$P(X = 2) = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} \times 6 = \frac{54}{256}$$

$$P(X = 3) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \times 4 = \frac{12}{256}$$

$$P(X = 4) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256}$$

The probability distribution is :

X	0	1	2	3	4
$P(X)$	$\frac{81}{256}$	$\frac{108}{256}$	$\frac{54}{256}$	$\frac{12}{256}$	$\frac{1}{256}$

28. Let $I = \int \underset{②}{x}(\log \underset{①}{x})^2 dx$

Integrating by parts, we get

$$I = (\log x)^2 \cdot \frac{x^2}{2} - \int 2\log x \times \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2}(\log x)^2 - \int \underset{②}{x} \log \underset{①}{x}\, dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2} dx\right]$$

$$I = \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$

OR

$$I = \int \frac{dx}{x(x^4 - 1)}$$

$$= \int \frac{dx}{x^5\left(1 - \frac{1}{x^4}\right)}$$

Let $1 - \frac{1}{x^4} = y$

$\Rightarrow \frac{4}{x^5} dx = dy$

$$\therefore \quad I = \frac{1}{4}\int \frac{dy}{y}$$

$$= \frac{1}{4}\log|y| + C$$

$$= \frac{1}{4}\log\left|1 - \frac{1}{x^4}\right| + C$$

$$I = \frac{1}{4}\log\left|\frac{x^4 - 1}{x^4}\right| + C$$

29. $$2(y+3) - xy\frac{dy}{dx} = 0$$

$\Rightarrow$ $$xy\frac{dy}{dx} = 2(y+3)$$

$\Rightarrow$ $$\frac{y}{y+3}dy = \frac{2}{x}dx$$

$\Rightarrow$ $$\left(\frac{y+3}{y+3} - \frac{3}{y+3}\right)dy = \frac{2}{x}dx$$

$\Rightarrow$ $$\left(1 - \frac{3}{y+3}\right)dy = \frac{2}{x}dx$$

Integrating both sides, we get

$$\int\left(1 - \frac{3}{y+3}\right)dy = \int\frac{2}{x}dx$$

$\Rightarrow$ $$y - 3\log|y+3| = 2\log|x| + C$$

Putting $x = 1$, $y = -2$, we get

$$-2 - 3\log 1 = 2\log 1 + C$$

$\therefore$ $$C = -2$$

$\therefore$ Particular solution is : $y - 3\log|y+3| = 2\log|x| - 2$

OR

$$y\,dx - (x + 2y^2)dy = 0$$

$\Rightarrow$ $$y\,dx = (x + 2y^2)dy$$

$\Rightarrow$ $$y\frac{dx}{dy} = x + 2y^2$$

$\Rightarrow$ $$\frac{dx}{dy} = \frac{x}{y} + 2y$$

$\Rightarrow$ $$\frac{dx}{dy} - \frac{x}{y} = 2y$$

Comparing with $\frac{dx}{dy} + Px = Q$, we get

$P = -\frac{1}{y}$ and $Q = 2y$

$$I.F. = e^{\int P dy}$$
$$= e^{-\int \frac{1}{y}dy}$$
$$= e^{-\log y}$$
$$= \frac{1}{y}$$

Solution is $x \times I.F. = \int Q \times IF\,dy \Rightarrow x\cdot\frac{1}{y} = \int 2y \times \frac{1}{y}dy$

$\Rightarrow$ $$\frac{x}{y} = 2y + C$$

$\Rightarrow$ $$x = 2y^2 + Cy$$

30. We have to maximise, $Z = 3x + 5y$

The given constraints:

$$x + 4y \le 24 \quad ...(i)$$
$$3x + y \le 21 \quad ...(ii)$$
$$x + y \le 9 \quad ...(iii)$$
$$x \ge 0, \quad y \ge 0 \quad ...(iv)$$

Converting (i) and (iii) inequations to equations and solving, we get

$$\begin{array}{r} x + 4y = 24 \\ \underline{\underset{-}{x} + \underset{-}{y} = \underset{-}{9}} \\ \underline{3y = 15} \end{array}$$

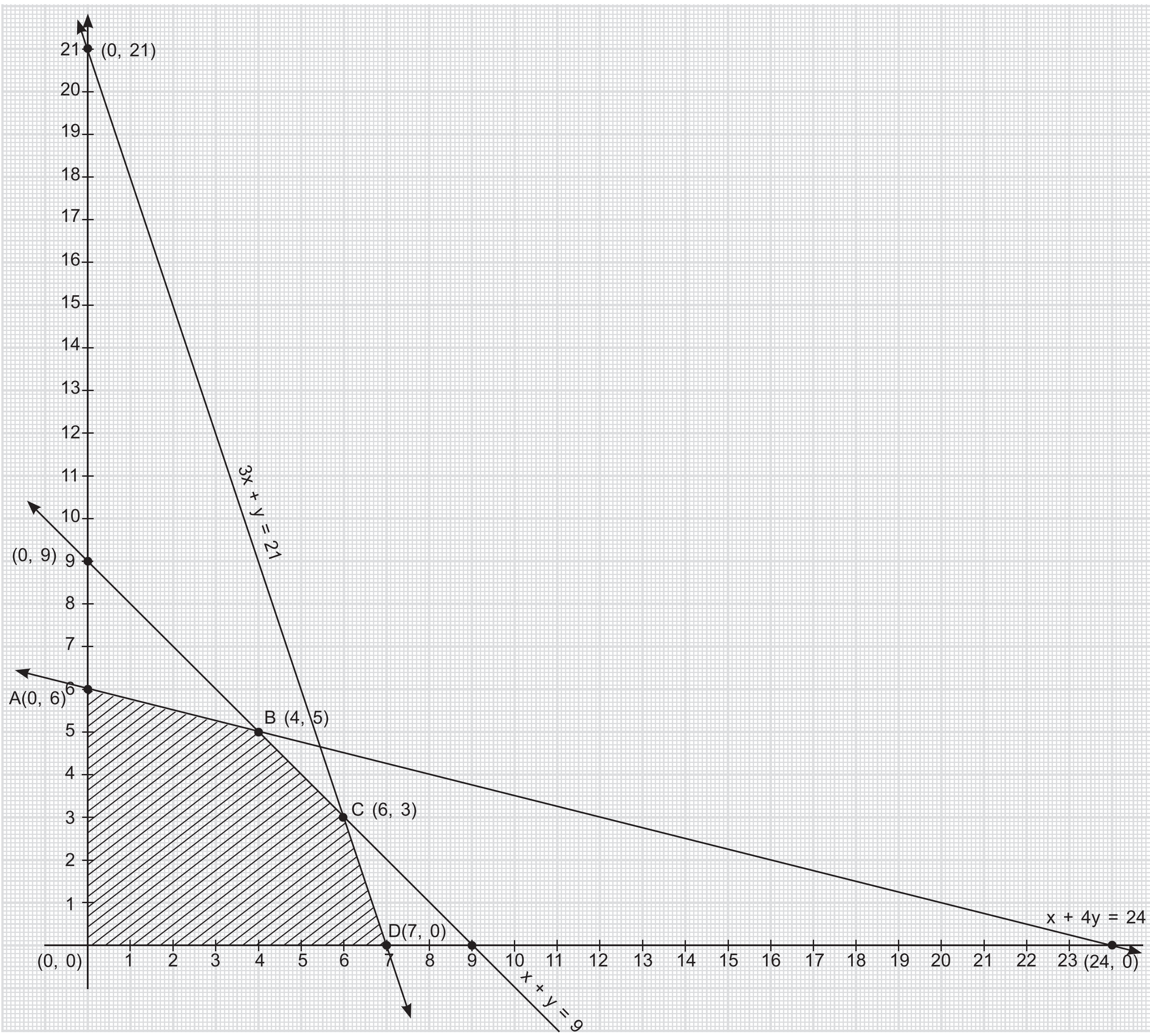
$\Rightarrow$ $y = 5$, and $x = 4$

Converting (ii) and (iii) inequations to equations and solving, we get

$$\begin{array}{r} x + y = 9 \\ \underline{\underset{-}{3x} + \underset{-}{y} = \underset{-}{21}} \\ \underline{-2x = -12} \end{array}$$

$\Rightarrow$ $x = 6$, and $y = 3$

Converting (i) and (ii) inequations to equations and solving we get $x = \frac{60}{11}$ and $y = \frac{51}{11}$.



Corner Points	Values of $Z = 3x + 5y$
$O(0, 0)$	0
$A(0, 6)$	30
$B(4, 5)$	37 ← Maximum
$C(6, 3)$	33
$D(7, 0)$	21

$\therefore$ Maximum value = 37 at $x = 4, y = 5$

OR

We have to minimise, $Z = 9x - 11y$

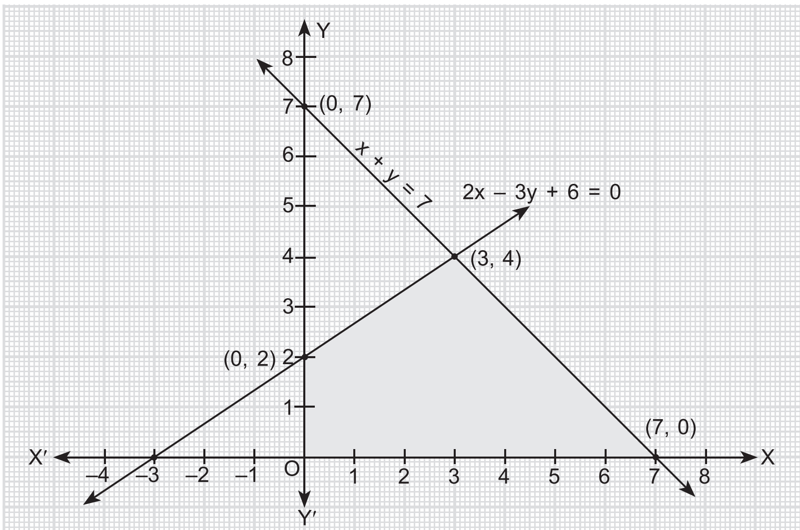
The given constraints are:

$$x + y \leq 7 \quad ...(i)$$

$$2x - 3y + 6 \geq 0 \quad ...(ii)$$

$$x, y \geq 0 \quad ...(iii)$$

Let us graph the feasible region of the system of linear inequalities (*i*) to (*iii*). The shaded region is the feasible region.



Corner Points	Values of $Z = 9x - 11y$
(0, 0)	0
(7, 0)	63
(3, 4)	–17
(0, 2)	–22 ← Minimum

$\therefore$ Minimum value = –22 at $x = 0, y = 2$.

31. $$y = x^x \quad ...(i)$$

Taking log on both sides, we get

$$\log y = x \log x \quad ...(ii)$$

Differentiating w.r.t x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = x\frac{1}{x} + \log x$$

$\Rightarrow \qquad \dfrac{dy}{dx} = y + y\log x \qquad ...(iii)$

Differentiating (*iii*) w.r.t x, we get

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + y\frac{1}{x} + \log x\frac{dy}{dx} \qquad ...(iv)$$

From (*iii*), we get $\qquad \log x = \dfrac{\frac{dy}{dx} - y}{y} \qquad ...(v)$

Putting this value in (*iv*), we get

$$\frac{d^2y}{dx^2} = \left[\frac{\frac{dy}{dx} - y}{y}\right] \times \frac{dy}{dx} + \frac{dy}{dx} + \frac{y}{x}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx} + \frac{dy}{dx} + \frac{y}{x}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 + \frac{y}{x}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

32. The given curves are: $\qquad x = y + 4$ and $y^2 = 2x$

$\Rightarrow \qquad y^2 = 2y + 8$

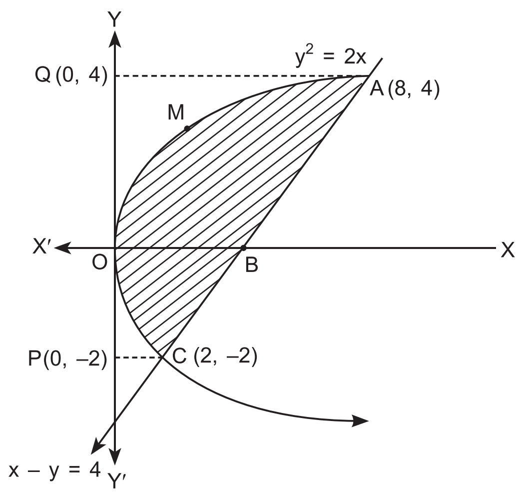
$\Rightarrow \qquad y^2 - 2y - 8 = 0$

$\Rightarrow \qquad (y - 4)(y + 2) = 0$

$\Rightarrow \qquad y = 4, y = -2$

For $y = 4, x = 8$

For $y = -2, x = 2$



Area of shaded region = ar(*QACPQ*) – ar(*QAMOCP*)

$$= \int_{-2}^{4} x_L dy - \int_{-2}^{4} x_P dy$$

$$= \int_{-2}^{4} (4 + y)dy - \int_{-2}^{4} \frac{y^2}{2}dy$$

$$= \left[4y + \frac{y^2}{2}\right]_{-2}^{4} - \frac{1}{2 \times 3}\left[y^3\right]_{-2}^{4}$$

$$= [(16 + 8) - (-8 + 2)] - \frac{1}{6}[64 + 8]$$

$= 30 - 12 = 18$ sq units

33. $\qquad R = \{(a, b) : a \le b^2\}$

For reflexive:

Let $\qquad \left(\dfrac{1}{2}, \dfrac{1}{2}\right) \in R$

$\Rightarrow \qquad \dfrac{1}{2} \le \dfrac{1}{4}$, which is false

$\therefore$ R is not reflexive

For symmetric:

We have, $(1, 2) \in R$ as $1 \leq 4$

But $2 \leq 1^2$, is false

i.e. $(2, 1) \notin R$

$\therefore$ R is not symmetric

For transitive:

We have, $(7, 3) \in R$ as $7 \leq 9$

$(3, 2) \in R$ as $3 \leq 4$

Now, $7 \leq 2^2$ is false

i.e. $(7, 2) \notin R$

$\therefore$ R is not transitive.

$\therefore$ Given relation is neither reflexive, nor symmetric, nor transitive.

OR

We have, $f(x) = \frac{2x - 7}{4}$

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$\Rightarrow$ $\frac{2x_1 - 7}{4} = \frac{2x_2 - 7}{4}$

$\Rightarrow$ $2x_1 = 2x_2$

$\Rightarrow$ $x_1 = x_2$

$\therefore$ f is one-one.

Let $f(x) = y$ {Where $y \in$R co-domain}

$\Rightarrow$ $\frac{2x - 7}{4} = y$

$\Rightarrow$ $2x - 7 = 4y$

$\Rightarrow$ $x = \frac{4y + 7}{2} \in R$

Thus for all $y \in R$ (co-domain), there exists $x = \frac{4y + 7}{2} \in R$ (domain).

$$f(x) = \frac{\frac{2(4y + 7)}{2} - 7}{4}$$

$$= \frac{4y + 7 - 7}{4}$$

$$= y$$

$\therefore$ f is onto.

$\therefore$ f is one-one and onto function.

34. $$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$|A| = 1(0-2) - 1(0) + 3 \times (2-0) = 4 \neq 0, \ \therefore A^{-1} \text{ exists}$$

$$C_{11} = -2, \quad C_{12} = 0, \quad C_{13} = 2$$

$$C_{21} = 5, \quad C_{22} = -2, \quad C_{23} = -1$$

$$C_{31} = 1, \quad C_{32} = 2, \quad C_{33} = -1$$

$$\text{adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(A)} \text{ adj } A$$

$$= \frac{1}{4}\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$$

$$x + y + z = 6, \quad x + 2z = 7, \quad 3x + y + z = 12$$

Matrix equation is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$BX = C$$

$$X = B^{-1}C$$

Here, $B = A^1$

$\therefore$ $B^{-1} = (A^{-1})^1$

$\therefore$ $$B^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Now $X = B^{-1}C$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -12 + 24 \\ 30 - 14 - 12 \\ 6 + 14 - 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}$$

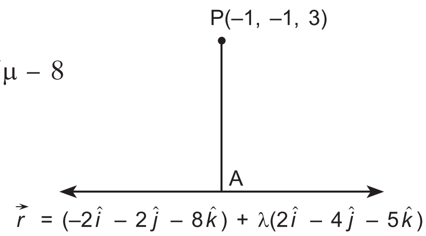
$$= \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore \quad x = 3, \quad y = 1, \quad z = 2$

35. Cartesian form a line is

$$\frac{x+2}{2} = \frac{y+2}{-4} = \frac{z+8}{-5} \quad ...(i)$$

Let A be foot of perpendicular drawn from point P on the given line.



Suppose, $$\frac{x+2}{2} = \frac{y+2}{-4} = \frac{z+8}{-5} = \mu \text{ (say)}$$

$\Rightarrow$ $$x = 2\mu - 2, \quad y = -4\mu - 2, \quad z = -5\mu - 8$$

$\therefore$ Co-ordinates of A be $(2\mu - 2, -4\mu - 2, -5\mu - 8)$

$$\text{DR's of } PA = (2\mu - 1, -4\mu - 1, -5\mu - 11)$$

PA is perpendicular to given line (i),

$\therefore$ $$2(2\mu - 1) - 4(-4\mu - 1) - 5(-5\mu - 11) = 0$$

$\Rightarrow$ $$4\mu - 2 + 16\mu + 4 + 25\mu + 55 = 0$$

$\Rightarrow$ $$45\mu + 57 = 0$$

$\Rightarrow$ $$\mu = \frac{-57}{45}$$

$$\mu = \frac{-19}{15}$$

$\therefore$ Co-ordinates of foot of perpendicular are:

$$\left(2\times\left(\frac{-19}{15}\right)-2, -4\times\left(\frac{-19}{15}\right)-2, -5\left(\frac{-19}{15}\right)-8\right) \text{ i.e., } \left(\frac{-68}{15}, \frac{46}{15}, \frac{-5}{3}\right)$$

OR

The given lines are:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad ...(i)$$

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} \quad ...(ii)$$

Now, $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \text{ (say)}$$

$\Rightarrow$ $$x = 2\lambda + 1, \quad y = 3\lambda + 2, \quad z = 4\lambda + 3$$

So, coordinates of any general point on line (i) are $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

Now, $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$ (say) coordinates of any general point on line (ii) are $(5\mu + 1, 2\mu + 1, \mu)$. If lines intersect, they must have a common point. So, for some values of λ and μ we must have,

$$2\lambda + 1 = 5\mu + 4 \quad ...(iii)$$
$$3\lambda + 2 = 2\mu + 1 \quad ...(iv)$$
$$4\lambda + 3 = \mu \quad ...(v)$$

Solving (iii) and (iv), we get

$$\lambda = \mu = -1$$

Also, $\lambda = \mu = -1$ Satisfies (v).

$\therefore$ these lines intersect each other.

Coordinates of point of intersection $= (2 \times (-1) + 1, 3 \times (-1) + 2, 4 \times (-1) + 3)$

i.e., $(-1, -1, -1)$

36. (*i*) Volume of tank $= x^2y$

$$V = x^2y$$

(*ii*) $$y = \frac{V}{x^2} \quad [\because V = x^2y]$$

Surface area of tank $= x^2 + 4xy$

$$= x^2 + 4x \cdot \frac{V}{x^2}$$

$$= x^2 + \frac{4V}{x}$$

(*iii*) We have, $$S = x^2 + \frac{4V}{x}$$

Differentiating w.r.t. x, we get

$$\frac{dS}{dx} = 2x - \frac{4V}{x^2}$$

For maximum or minimum surface area,

$$\frac{dS}{dx} = 0 \quad \Rightarrow \quad 2x = \frac{4V}{x^2}$$

$$\Rightarrow \quad x^3 = 2V$$

$$\Rightarrow \quad x = (2V)^{\frac{1}{3}}$$

Now, $$\frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$$

$$\left(\frac{d^2S}{dx^2}\right)_{x=(2V)^{1/3}} = 2 + \frac{8V}{2V} > 0$$

$\therefore$ Surface area is minimum at $x = (2V)^{1/3}$

Now, $$x^3 = 2V \Rightarrow V = \frac{x^3}{2}$$

$$\Rightarrow \quad \frac{x^3}{2} = x^2y$$

$$\Rightarrow \quad x = 2y$$

OR

(*iii*) We have, $$x^2y = V$$

$$\Rightarrow \quad 8 = x^2 \times 2$$

$$\Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = 2 \text{ (–2 rejected)}$$

$$S = x^2 + \frac{4V}{x} = 4 + \frac{4 \times 8}{2} = 20 \text{ m}^2$$

37. (*i*) $$P(E_3) = 1 - \left(\frac{1}{4} + \frac{1}{2}\right)$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

(*ii*) $$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right) + P(E_3) \times P\left(\frac{A}{E_3}\right)$$

$$= \frac{1}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{1}{12} + \frac{1}{4} + \frac{1}{16}$$

$$= \left(\frac{4 + 12 + 3}{48}\right)$$

$$= \frac{19}{48}$$

(*iii*) $$P(E_3/A) = \frac{P(E_3) \times P\left(\frac{A}{E_3}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right) + P(E_3) \times P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{4} \times \frac{1}{4}}{\frac{19}{48}} = \frac{1}{16} \times \frac{48}{19}$$

$$= \frac{3}{19}$$

OR

(*iii*) $$P(E_1/A) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right) + P(E_3) \times P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{4} \times \frac{1}{3}}{\frac{1}{4} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4}} = \frac{4}{19}$$

38. (*i*) $$\overrightarrow{AB} = 2\hat{i} + 5\hat{j} - 5\hat{k}$$

$$\overrightarrow{AC} = -3\hat{i} + 4\hat{j} - 8\hat{k}$$

Vector perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{AC}$ $= \overrightarrow{AB} \times \overrightarrow{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -5 \\ -3 & 4 & -8 \end{vmatrix}$$

$$= \hat{i}(-40 + 20) - \hat{j}(-16 - 15) + \hat{k}(8 + 15)$$

$$= -20\hat{i} + 31\hat{j} + 23\hat{k}$$

(*ii*)

$$\text{Ar } (\Delta ABC) = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$= \frac{1}{2}|-20\hat{i} + 31\hat{j} + 23\hat{k}|$$

$$= \frac{1}{2}\sqrt{400 + 961 + 529}$$

$$= \frac{1}{2}\sqrt{1890}$$

$$= \frac{3}{2}\sqrt{210} \text{ sq units.}$$