Solutions to RMM–DS1/Set-3

1. (*c*)
$$\det(B^{-1}AB) = |B^{-1}|\cdot|AB|$$
$$= |B^{-1}|\,|A|\cdot|B|$$
$$= \frac{1}{|B|}|A|\,|B| = |A| = \det A$$

2. (*c*)
$$A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$
$\Rightarrow$
$$|A| = \begin{vmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$
$$= 3(2+1) + 2(1-0) + 4(1-0)$$
$$= 9 + 2 + 4 = 15$$
$$A^{-1} = \frac{1}{|A|}\text{adj } A$$
$\therefore$ $$k = 15$$

3. (*b*)
$$A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix}$$
$\Rightarrow$
$$|A| = \begin{vmatrix} 1 & 3 & 2 \\ -2 & 5 & 1 \\ 0 & 3 & 2 \end{vmatrix}$$
$$= 1(10-3) - 3(-4-0) + 2(-6-0)$$
$$= 7 + 12 - 12 = 7$$
$$(A\cdot\text{adj } A) = |A|I$$
$$= 7I$$

4. (*b*)
$$y = \log_3 3^{\cos x}$$
$\therefore$ $$y = \cos x \qquad (\log_a a^x = x)$$
Differentiating w.r.t. x, we get
$$\frac{dy}{dx} = -\sin x$$

5. (*b*) Direction ratios of line through the points (1, – 1, 2) and (3, 4, – 2) are (2, 5, – 4).

Direction ratios of line through the points (0, 3, 2) and (3, 5, 6) are (3, 2, 4).

Now $a_1a_2 + b_1b_2 + c_1c_2 = 3\times 2 + 2\times 5 + 4\times(-4) = 0$

$\therefore$ Lines are perpendicular.

6. (*d*) In particular solution, there is no arbitrary constant.

7. (*a*) convex polygon.

8. (*d*)
$$\vec{a} = 2\hat{i} - m\hat{j} + \hat{k}$$
$$\vec{b} = -\hat{i} + 2\hat{j} + 4\hat{k}$$

For orthogonal vector, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow \quad 2 \times (-1) - m \times 2 + 1 \times 4 = 0$

$\Rightarrow \quad -2 - 2m + 4 = 0$

$\Rightarrow \quad -2m = -2$

$\Rightarrow \quad m = 1$

9. (*c*) Let $I = \int_0^5 \log \underset{①}{x} \; \underset{②}{1} \cdot dx$

$= [x \cdot \log x]_0^5 - \int_0^5 \frac{1}{x} \times x \, dx$

$= 5 \log 5 - [x]_0^5$

$= 5 \log 5 - 5$

10. (*d*) $|A| = 5$

$|3A| = 3^3 \cdot |A|$

$= 27 \times 5 = 135$

11. (*b*) Decision variables

12. (*c*) Projection of $\vec{a}$ on $\vec{b} = 0$

$\Rightarrow$ Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$\Rightarrow \quad 0 = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$\Rightarrow \quad \vec{a} \cdot \vec{b} = 0$

$\therefore$ $\vec{a}$ is perpendicular to $\vec{b}$.

13. (*a*) Given $A' = A$

$(B'AB)' = (AB)'(B')'$

$= B'A'B$

$= B'AB$

$\therefore$ $B'AB$ is symmetric matrix.

14. (*b*) For independent events, $P(A \cap B) = P(A) \times P(B)$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P(A) \times P(B)}{P(B)} = \frac{1}{2}$

15. (*c*) Let $I = \int \frac{dx}{x + \sqrt{x}}$

$= \int \frac{dx}{\sqrt{x}(\sqrt{x} + 1)}$

Let $\sqrt{x} + 1 = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \frac{dx}{\sqrt{x}} = 2dt$

$= 2 \int \frac{dt}{t}$

$= 2 \log |t| + C$

$= 2 \log |\sqrt{x} + 1| + C$

16. (*a*) Vector $\overrightarrow{PQ} = P.V.\text{ of } Q - P.V.\text{ of } P$

$$= 3\hat{i} - 4\hat{j} + 7\hat{k} - 2\hat{i} + 3\hat{j} - 5\hat{k} = \hat{i} - \hat{j} + 2\hat{k}$$

17. (*d*) Let $u = \sin x$ and $v = \cos x$

$$\therefore \quad \frac{du}{dx} = \cos x, \text{ and } \frac{dv}{dx} = -\sin x$$

$$\frac{du}{dv} = \frac{du}{dx} \div \frac{dv}{dx}$$

$$\therefore \quad \frac{du}{dv} = -\frac{\cos x}{\sin x}$$

$$= -\cot x$$

18. (*c*) Direction ratios of the line $AB = (-1-2, 0-1, -3+2)$

$$= (-3, -1, -1)$$

$$|AB| = \sqrt{9+1+1}$$

$$\text{DC's are } = \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}$$

19. (*d*) (A) is false but (R) is true.

20. (*b*) Both (A) and (R) are true but (R) is not the correct explanation of (A).

21.

$$\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{4}\right) = \tan\frac{x}{2}$$

Let $\cos^{-1}\frac{\sqrt{5}}{4} = x$

$\Rightarrow \cos x = \frac{\sqrt{5}}{4}$

$\therefore$ We have

$$\tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\Rightarrow \quad \tan\frac{x}{2} = \sqrt{\frac{1-\frac{\sqrt{5}}{4}}{1+\frac{\sqrt{5}}{4}}}$$

$$= \sqrt{\frac{4-\sqrt{5}}{4+\sqrt{5}}}$$

$$\Rightarrow \quad \tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{4}\right) = \frac{4-\sqrt{5}}{\sqrt{11}}$$

OR

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) + \cos\left(\tan^{-1}\sqrt{3}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] + \cos\left[\tan^{-1}\left(\tan\frac{\pi}{3}\right)\right]$$

$$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{3}\right)$$

$$= \left(\frac{\pi}{6} + \frac{1}{2}\right) = \left(\frac{\pi+3}{6}\right)$$

22.

$$f(x) = \begin{cases} \dfrac{3k\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ 4, & x = \dfrac{\pi}{2} \end{cases}$$

For continuous function

$$\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{2}^-} \frac{3k\cos x}{\pi - 2x} = \lim_{x \to \frac{\pi}{2}^+} \frac{3k\cos x}{\pi - 2x} = 4$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{3k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{3k\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)} = 4$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{3k\sin h}{2h} = \lim_{h \to 0} \frac{-3k\sin h}{-2h} = 4$$

$$\Rightarrow \quad \frac{3k}{2} = \frac{3k}{2} = 4$$

$$\Rightarrow \quad 3k = 8$$

$$\Rightarrow \quad k = \frac{8}{3}$$

23. Let $f(x) = \dfrac{\log x}{x}$ ...(*i*)

Differentiating w.r.t. x, we get

$$f'(x) = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2} \qquad ...(ii)$$

For critical point $f'(x) = 0$

$$\Rightarrow \quad \frac{1 - \log x}{x^2} = 0$$

$$\Rightarrow \quad \log x = 1$$

$$\Rightarrow \quad \log x = \log e$$

$$\therefore \quad x = e$$

Differentiating (*ii*) w.r.t. x, we get

$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)2x}{x^4}$$

$$\Rightarrow \quad [f''(x)]_{x=e} = \frac{-e^2 \times \frac{1}{e}}{e^4} < 0$$

$\therefore$ $x = e$ is the point of maxima.

$$\therefore \quad \text{Maximum value} = \frac{\log e}{e} = \frac{1}{e}$$

OR

$$f(x) = \cos x$$

Differentiating w.r.t. x, we get

$$f'(x) = -\sin x$$

$$f'(x) = 0$$

$$\Rightarrow \quad -\sin x = 0$$

$$\therefore \quad x = 0, \pi$$

Sign of $f'(x)$ in interval $(0, \pi)$

Now, $f'(x) < 0$ when $x \in (0, \pi)$

$\therefore$ $f(x)$ is decreasing in the interval $(0, \pi)$.

Sign of $f'(x)$ in interval $(\pi, 2\pi)$:

Now, $f'(x) > 0$ when $x \in (\pi, 2\pi)$

$\therefore$ $f(x)$ is increasing in the interval $(\pi, 2\pi)$.

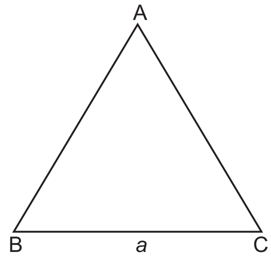
24. Let

$$I = \int \frac{dx}{1+\tan x}$$
$$= \int \frac{\cos x\, dx}{\sin x + \cos x}$$
$$= \frac{1}{2}\int \frac{2\cos x\, dx}{\sin x + \cos x}$$
$$= \frac{1}{2}\left[\int \frac{\cos x + \cos x + \sin x - \sin x}{\sin x + \cos x} dx\right]$$
$$= \frac{1}{2}\left[\int \frac{\cos x + \sin x}{\sin x + \cos x} dx + \int \frac{\cos x - \sin x}{\sin x + \cos x} dx\right]$$
$$= \frac{1}{2}\left[x + \int \frac{\cos x - \sin x}{\sin x + \cos x} dx\right]$$
$$= \frac{1}{2}\left[x + \int \frac{dt}{t}\right]$$

Let $\cos x + \sin x = t$
$\Rightarrow (-\sin x + \cos x)dx = dt$
$\Rightarrow (\cos x - \sin x)dx = dt$

$$= \frac{1}{2}[x + \log|\cos x + \sin x|] + C$$

25. Let side of an equilateral triangle be a and A be its area at any instant of time t.



$\therefore$ Area of triangle, $A = \frac{\sqrt{3}}{4}a^2$

Differentiating w.r.t. 't' we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}\cdot 2a \cdot \frac{da}{dt}$$

$\Rightarrow$ $$\left.\frac{dA}{dt}\right]_{a=10} = \frac{\sqrt{3}}{4}\times 2 \times 10 \times 4 \qquad \left[\frac{da}{dt} = 4 \text{ cm/ min}, a = 10 \text{ cm}\right]$$

$$= 20\sqrt{3} \text{ cm}^2/\text{min}$$

26. $\int_1^4 (|x-1|) + |x-2|)\,dx$

When $x < 1$:

$\therefore$ $|x-1| + |x-2| = -(x-1) - (x-2)$
$= -2x + 3$ (Rejected no interval given for integrals)

When $1 \le x < 2$:

$\therefore$ $|x-1| + |x-2| = x - 1 - x + 2$
$= 1$

When $x \ge 2$:

$\therefore$ $|x-1| + |x-2| = x - 1 + x - 2$
$= 2x - 3$

$\therefore$ $$\int_1^4 (|x-1| + |x-2|)dx = \int_1^2 1\,dx + \int_2^4 (2x-3)\,dx$$
$$= [x]_1^2 + \left[\frac{2x^2}{2} - 3x\right]_2^4$$
$$= 2 - 1 + (16 - 12) - (4 - 6)$$
$$= 1 + 4 + 2$$
$$= 7$$

27. $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}, P(C) = \frac{1}{4}$

Problem is solved by exactly two students.

$\therefore$ P(Problem solved by exactly two students) $= P(AB\overline{C})$ or $P(A\overline{B}C)$ or $P(\overline{A}BC)$

$$= \frac{1}{3}\times\frac{1}{2}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}\times\frac{1}{4}+\frac{2}{3}\times\frac{1}{2}\times\frac{1}{4}$$

$$= \frac{1}{24}(3+1+2) = \frac{1}{4}$$

28. $\int \frac{x+2}{\sqrt{x^2+5x+8}}dx$

Let $x+2 = A\frac{d}{dx}(x^2+5x+8)+B$

$= A(2x+5)+B$

$\Rightarrow$ $x+2 = 2Ax+(5A+B)$

Comparing coefficient of 'x' on both sides, we get

$$1 = 2A \quad \Rightarrow \quad A = \frac{1}{2}$$

Comparing the constant on both sides,

$$5A+B = 2$$

$\Rightarrow$ $B = 2-\frac{5}{2} = -\frac{1}{2}$

$$\therefore \quad \int \frac{x+2}{\sqrt{x^2+5x+8}}dx = \int \frac{\frac{1}{2}(2x+5)-\frac{1}{2}}{\sqrt{x^2+5x+8}}dx$$

$$= \frac{1}{2}\int \underbrace{\frac{2x+5}{\sqrt{x^2+5x+8}}dx}_{I_1} - \frac{1}{2}\int \underbrace{\frac{dx}{\sqrt{x^2+5x+8}}}_{I_2}$$

For $I_1 = \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+8}}dx$

Let $x^2+5x+8 = t$
$\Rightarrow (2x+5)dx = dt$

$$I_1 = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2}\times 2\sqrt{t}+C_1$$

$$= \sqrt{x^2+5x+8}+C_1$$

$$I_2 = \int \frac{dx}{\sqrt{x^2+5x+8}}$$

$$= \int \frac{dx}{\sqrt{x^2+5x+\frac{25}{4}-\frac{25}{4}+8}}$$

$$= \int \frac{dx}{\sqrt{\left(x+\frac{5}{2}\right)^2+\frac{7}{4}}}$$

$$I_2 = \int \frac{dx}{\sqrt{\left(x+\frac{5}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+8}\right|+C_2$$

$\therefore$ $I = \sqrt{x^2+5x+8}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+8}\right|+C$, where C (constant) $= C_1 - C_2$

OR

Let $I = \int \frac{x^2}{(x^2+1)(x^2+4)} dx$

put $x^2 = y$

$$\Rightarrow \quad \frac{x^2}{(x^2+1)(x^2+4)} = \frac{y}{(y+1)(y+4)}$$

$$\text{Let} \quad \frac{y}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{(y+4)}$$

$$\Rightarrow \quad \frac{y}{(y+1)(y+4)} = \frac{(y+4)A+(y+1)B}{(y+1)(y+4)}$$

$$\Rightarrow \quad y = y(A+B) + (4A+B)$$

Comparing equal degree terms, we get

$\Rightarrow \quad A + B = 1 \; ... (i) \qquad$ and $\qquad 4A + B = 0 \qquad ...(ii)$

Solving (*i*) and (*ii*), we get

$A = -\frac{1}{3}, B = \frac{4}{3}$

$$\therefore \quad \int \frac{x^2}{(x^2+1)(x^2+4)} dx = -\frac{1}{3}\int \frac{dx}{x^2+1} + \frac{4}{3}\int \frac{dx}{x^2+4}$$

$$= -\frac{1}{3} \times \tan^{-1}x + \frac{4}{3} \times \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$= -\frac{1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\frac{x}{2} + C$$

29. $\frac{dy}{dx} + y\tan x = 3x^2 + x^3\tan x \qquad ...(i)$

Comparing with $\frac{dy}{dx} + Py = Q$, we get

$$P = \tan x, Q = 3x^2 + x^3 \tan x$$

$$I.F. = e^{\int P dx}$$

$$= e^{\int \tan x \, dx}$$

$$= e^{\log|\sec x|}$$

$$= \sec x$$

The solution of (*i*) is given by,

Solution is $y \times IF = \int Q \times IF \, dx$

$$\Rightarrow \quad y \cdot \sec x = \int (3x^2 + x^3\tan x)\sec x \cdot dx$$

$$= \int \underset{②}{3x^2} \underset{①}{\sec x} dx + \int x^3 \sec x \tan x \, dx$$

$$\Rightarrow \quad y \sec x = \frac{3x^3}{3}\sec x - 3\int \sec x \tan x \cdot \frac{x^3}{3} dx + \int x^3 \sec x \tan x \, dx + C$$

$\Rightarrow \quad y \sec x = x^3 \sec x + C \qquad ...(ii)$

Putting $y = 0, x = \frac{\pi}{3}$ in (*ii*), we get

$$\Rightarrow \quad 0 = \frac{\pi^3}{27} \cdot 2 + C \Rightarrow C = \frac{-2\pi^3}{27}$$

$\therefore$ Particular solution is $y \sec x = x^3 \sec x - \frac{2\pi^3}{27}$

OR

$$\frac{dy}{dx} = 1 - x + y - xy$$
$$= (1 - x) + y(1 - x)$$
$$= (1 - x)(1 + y)$$

$\Rightarrow$ $$\frac{dy}{1+y} = (1 - x)dx$$

Integrating both sides, we get

$$\int \frac{dy}{1+y} = \int (1 - x)\, dx$$
$$\log|1 + y| = x - \frac{x^2}{2} + C$$

30. $Z = 600x + 400y$

Given inequations are

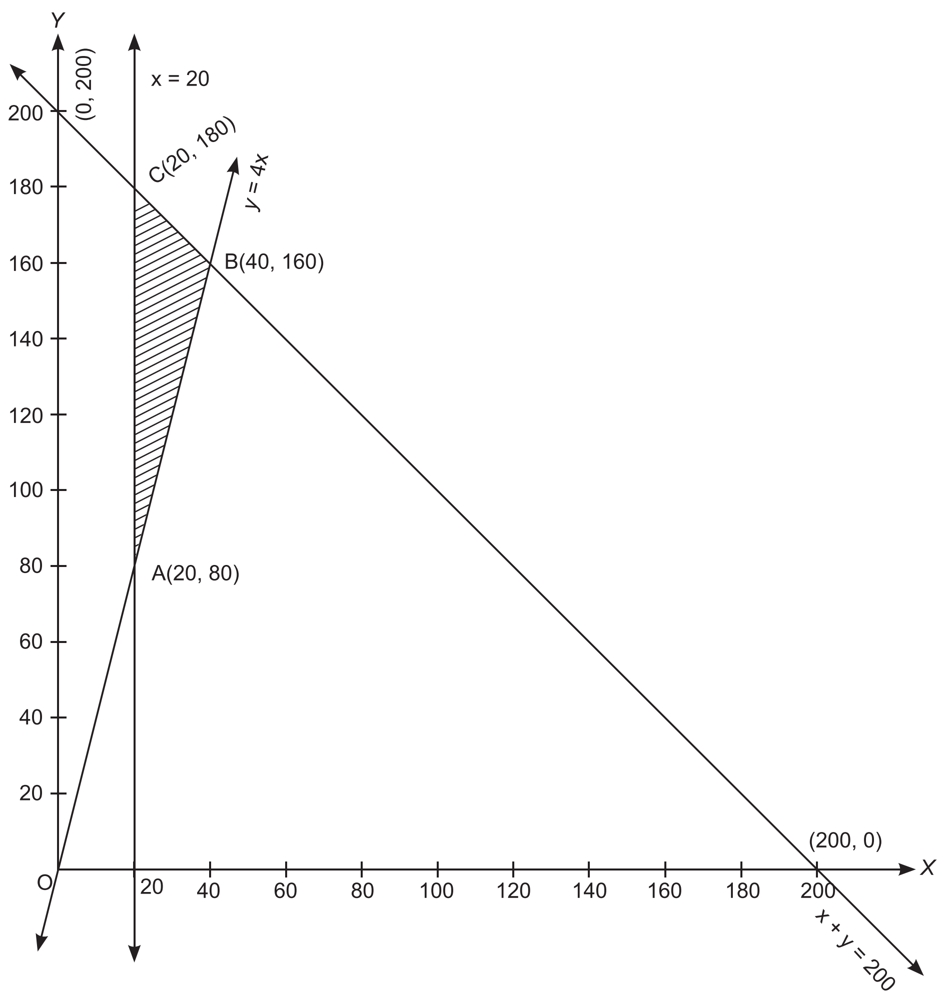
$x + y \le 200$

$y \ge 4x$

$x \ge 20$

$x, y \ge 0$

Plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for maximum Z are $A(20, 80)$, $B(40, 160)$ and $C(20, 180)$.



Points	$Z = 600x + 400y$	Values
A(20, 80)	$600 \times 20 + 400 \times 80$	44000
B(40, 160)	$600 \times 40 + 400 \times 160$	88000 ← Maximum
C(20, 180)	$600 \times 20 + 400 \times 180$	84000

∴ Maximum value of Z = 88000 at $x = 40$, $y = 160$

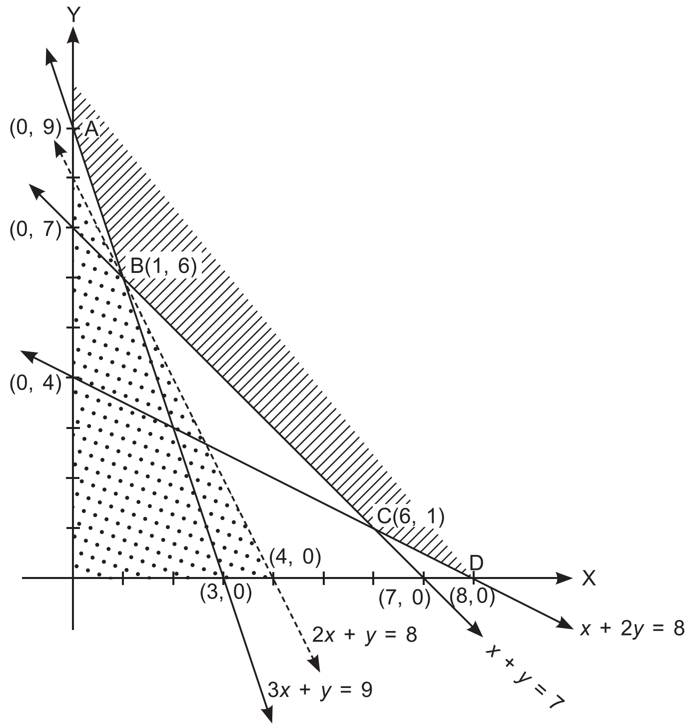
OR

For point B

$$\begin{aligned} x + y &= 7 \\ 3x + y &= 9 \\ - \quad - \quad & \quad - \\ \hline -2x &= -2 \\ x &= 1 \\ y &= 6 \end{aligned} \qquad B(1, 6)$$

For point C

$$\begin{aligned} x + 2y &= 8 \\ x + y &= 7 \\ - \quad - \quad & \quad - \\ \hline y &= 1 \\ x &= 6 \end{aligned} \qquad C(6, 1)$$



Corner points	Values of $Z = 2x + y$
$A(0, 9)$	9
$B(1, 6)$	8 ←Minimum
$C(6, 1)$	13
$D(8, 0)$	16

To check another value if any

$$2x + y < 8$$

If there is any common point of feasible region then there will be no minimum value.

But open half plane represented by $2x + y < 8$ does not have points common with feasible region.

∴ Minimum value of $Z = 8$

31. $x = 3\sin\theta - \sin 3\theta$ ...(*i*) $\qquad y = 3\cos\theta - \cos 3\theta$ ...(*ii*)

Differentiating both sides, w.r.t. θ, we get

$$\frac{dx}{d\theta} = 3\cos\theta - \cos 3\theta \times 3$$
$$= 3(\cos\theta - \cos 3\theta)$$
$$= 3 \times 2\sin 2\theta \cdot \sin\theta$$
$$\frac{dx}{d\theta} = 6.\sin\theta \sin 2\theta \quad ...(iii)$$

Differentiating both sides, w.r.t. θ, we get

$$\frac{dy}{d\theta} = -3\sin\theta + \sin 3\theta \times 3$$
$$= 3(\sin 3\theta - \sin\theta)$$
$$= 3 \cdot 2\cos 2\theta \sin\theta$$
$$= 6\cos 2\theta \cdot \sin\theta \quad ...(iv)$$

From (*iii*) and (*iv*), we get

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$= \frac{6\cos 2\theta \cdot \sin\theta}{6\sin\theta \cdot \sin 2\theta}$$

$$\frac{dy}{dx} = \cot 2\theta \qquad ...(v)$$

Differentiating (*v*) w.r.t. *x*, we get

$$\frac{d^2y}{dx^2} = -\text{cosec}^2 2\theta \times 2 \cdot \frac{d\theta}{dx}$$

$$= -\text{cosec}^2 2\theta \times 2 \times \frac{1}{6\sin\theta\sin 2\theta}$$

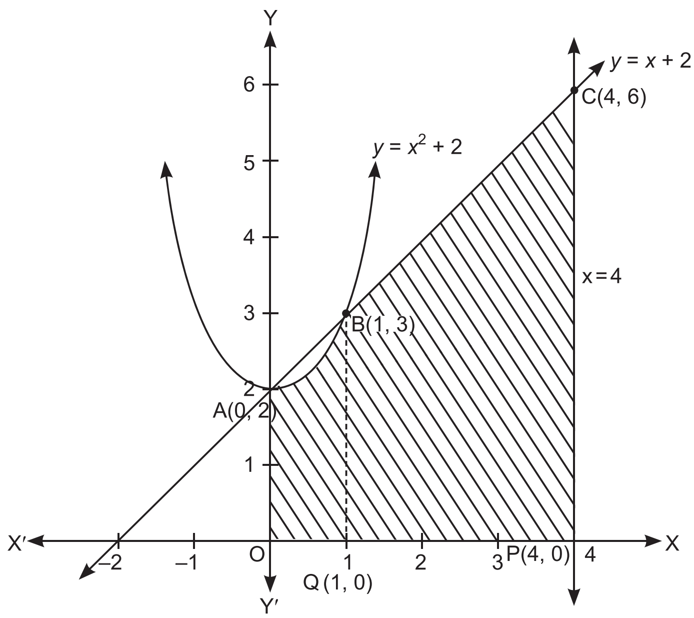
$$= -\frac{1}{3}\text{cosec}^3 2\theta \cdot \text{cosec}\,\theta$$

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{3}} = -\frac{1}{3}\left(\frac{2}{\sqrt{3}}\right)^3 \cdot \frac{2}{\sqrt{3}} = -\frac{1}{3} \times \frac{16}{9} = -\frac{16}{27}$$

32. $y \geq 0, y \leq x^2 + 2$

$y \leq x + 2$

$x \geq 0, x \leq 4$



Since $y = x + 2$ and $y = x^2 + 2$

$\Rightarrow \qquad x^2 + 2 = x + 2$

$\Rightarrow \qquad x(x-1) = 0 \Rightarrow x = 0$ or $x = 1$

Hence point of intersection are (1, 3) and (0, 2).

Area of shaded region $= \text{ar}(OABQO) + \text{ar}(BCPQB)$

$$= \int_0^1 y_1 dx + \int_1^4 y_2 dx$$

$$= \int_0^1 (x^2+2)\,dx + \int_1^4 (x+2)\,dx$$

$$= \left[\frac{x^3}{3}+2x\right]_0^1 + \left[\frac{x^2}{2}+2x\right]_1^4$$

$$= \left[\left(\frac{1}{3}+2\right)-0\right]+\left[(8+8)-\left(\frac{1}{2}+2\right)\right]$$

$$= \frac{7}{3}+\frac{27}{2} = \frac{14+81}{6} = \frac{95}{6} \text{ sq units}$$

33. $(a, b) R (c, d) \Rightarrow \frac{1}{a}+\frac{1}{d} = \frac{1}{c}+\frac{1}{b}$

For reflexive: Let $(a, b)\ R\ (a, b)$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{b} = \frac{1}{b}+\frac{1}{a}$$

Which is true because addition is commutative.

R is reflexive.

For symmetric:

Let for $(a, b)\ R\ (c, d)$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c} \quad ...(i) \qquad \Rightarrow \quad \frac{1}{c}+\frac{1}{b} = \frac{1}{d}+\frac{1}{a} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$(a, b)\ R\ (c, d) \Rightarrow (c, d)\ R\ (a, b)$$

$\therefore$ R is symmetric relation.

For transitive:

Let $(a, b)\ R\ (c, d)$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c} \quad ...(iii)$$

Let $(c, d)\ R\ (e, f)$

$$\Rightarrow \quad \frac{1}{c}+\frac{1}{f} = \frac{1}{d}+\frac{1}{e} \quad ...(iv)$$

From (*iii*) and (*iv*), we get

$$\frac{1}{a}+\frac{1}{c}+\frac{1}{f}+\frac{1}{d} = \frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}$$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{f} = \frac{1}{b}+\frac{1}{e}$$

$$\Rightarrow \quad (a, b)\ R\ (e, f)$$

$\therefore$ R is transitive relation.

Hence, given relation is an equivalence relation.

OR

$$f(x) = \begin{cases} \frac{x+1}{2}, \text{ if } x \text{ is odd} \\ \frac{x}{2}, \text{ if } x \text{ is even} \end{cases}$$

For one-one:

Case 1. Let $x_1, x_2 \in N$, and suppose x_1, x_2 both are odd.

Now, $\quad f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{x_1+1}{2} = \frac{x_2+1}{2}$$

$$\Rightarrow \quad x_1 = x_2$$

Case 2. Let $x_1, x_2 \in N$ and suppose x_1, x_2 both are even.

Now, $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{x_1}{2} = \frac{x_2}{2}$

$\Rightarrow \quad x_1 = x_2$

Case 3. Let $x_1, x_2 \in N$. Suppose x_1 is odd and x_2 is even.

Now, $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{x_1+1}{2} = \frac{x_2}{2}$

$\Rightarrow \quad x_1 - x_2 = -1$

So, $f(x_1) = f(x_2) \Rightarrow x_1 \neq x_2$

$\therefore$ f is not one-one.

For onto:

Let $f(x) = y$, for instance $y \in N$.

When x is odd

$$y = \frac{x+1}{2}$$

$\Rightarrow \quad x = 2y - 1$

So, for each $y \in N$, $x \in N$.

When x is even

$$y = \frac{x}{2}$$

$\Rightarrow \quad x = 2y$

So for each $y \in N$, $x \in N$

So, for every $y \in N$, there exists $x \in N$ for instance $f(x) = y$

Hence f is onto.

$\therefore$ f is not bijective function.

34. Let cost of type A pen be ₹ x, cost of type B pen be ₹ y and cost of type C pen be ₹ z.

According to the question,

$$x + y + z = 37$$
$$4x + 3y + 2z = 106$$
$$6x + 2y + 3z = 129$$

Matrix form is

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37 \\ 106 \\ 129 \end{bmatrix}$$

Which is of the form, $AX = B \quad \Rightarrow \quad X = A^{-1}B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$|A| = 1 \times 5 - 1 \times 0 + 1 \times (-10)$$
$$= -5 \neq 0$$

$\therefore$ A^{-1} exists.

$C_{11} = 5$, $\quad C_{12} = 0$, $\quad C_{13} = -10$

$C_{21} = -1$, $\quad C_{22} = -3$ $\quad C_{23} = 4$

$C_{31} = -1$, $\quad C_{32} = 2$ $\quad C_{33} = -1$

$$\text{adj } A = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix} \qquad \left(\because A^{-1} = \frac{1}{|A|}\text{adj } A\right)$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 37 \\ 106 \\ 129 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} 185 - 106 - 129 \\ 0 - 318 + 258 \\ -370 + 424 - 129 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -50 \\ -60 \\ -75 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 15 \end{bmatrix}$$

$\therefore$ Cost of type A pen = ₹ 10, cost of type B pen = ₹ 12, cost of type C pen = ₹ 15

35. Direction ratios of required line would be same as direction ratios of line AB.

Direction ratios of $\overrightarrow{AB}$ = <4, 5, 3>

Equation of line passing through point (x_1, y_1, z_1) having direction ratios a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Cartesian equation of line passing through (–1, – 6, 3) having direction ratios 4, 5, 3 is

$$\frac{x+1}{4} = \frac{y+6}{5} = \frac{z-3}{3}$$

Vector equation is $\vec{r} = (-\hat{i} - 6\hat{j} + 3\hat{k}) + \lambda(4\hat{i} + 5\hat{j} + 3\hat{k})$

Direction ratios of $\overrightarrow{AP}$ = <–3, –9, –1>

$$|\overrightarrow{AP}| = \sqrt{9 + 81 + 1} = \sqrt{91}$$

$\therefore$ Direction ratios = $<\frac{-3}{\sqrt{91}}, \frac{-9}{\sqrt{91}}, \frac{-1}{\sqrt{91}}>$

OR

Line are $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$

Here, $\vec{a}_1 = 5\hat{i} + 7\hat{j} - 2\hat{k}$, $\vec{b}_1 = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = -3\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{b}_2 = -3\hat{i} + 2\hat{j} + 4\hat{k}$

$$\vec{a}_2 - \vec{a}_1 = -8\hat{i} - 4\hat{j} + 8\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}(-4 - 2) - \hat{j}(12 + 3) + \hat{k}(6 - 3)$$

$$= -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{36 + 225 + 9}$$

$$= \sqrt{270} = 3\sqrt{30}$$

Shortest distance between the lines $= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$$= \left| \frac{(-8\hat{i} - 4\hat{j} + 8\hat{k}) \cdot (-6\hat{i} - 15\hat{j} + 3\hat{k})}{\sqrt{270}} \right|$$

$$= \left| \frac{48 + 60 + 24}{\sqrt{270}} \right| = \frac{132}{\sqrt{270}}$$

$$= \frac{132}{3\sqrt{30}} \text{ units} = \frac{44}{\sqrt{30}} \text{ units}$$

36. Given $x = \dfrac{600 - p}{8}$

$\Rightarrow$ $8x = 600 - p$

$\Rightarrow$ $p = 600 - 8x$

$R(x) = p.x$

$= 600x - 8x^2$

Cost function, $C(x) = x^2 + 78x + 2500$

(*i*) $p = 600 - 8x$

(*ii*) $P(x) = R(x) - C(x)$

$= 600x - 8x^2 - x^2 - 78x - 2500$

$= -9x^2 + 522x - 2500$

(*iii*) $P(x) = -9x^2 + 522x - 2500$

Differentiating both sides w.r.t. x, we get

$P'(x) = -18x + 522$

For maximum or minimum profit

put $P(x) = 0$

$\Rightarrow$ $18x = 522$

$\Rightarrow$ $x = \dfrac{522}{18} = 29$

$P''(x) = -18 < 0$

$\therefore$ $P(x)$ is maximum when $x = 29$.

OR

(*iii*) $P(x) = -9x^2 + 522x - 2500$

Differentiating both sides w.r.t. x, we get

$P'(x) = -18x + 522$

Put $P'(x) = 0$, for critical point

$\therefore$ $-18x = -522 \Rightarrow x = 29$

Interval	sign of $P'(x)$
(0, 29)	+ve
(29, ∞)	–ve

$\therefore$ $P(x)$ is increasing in the interval (0, 29)

37. A: Event that helicopter I hits correctly.

B: Event that helicopter II hits correctly.

$P(A) = 0.3, P(B) = 0.2$

(*i*) $P(\text{Target hit by only one helicopter}) = P(A)P(\overline{B}) + P(\overline{A})\, P(B)$

$= 0.3 \times 0.8 + 0.7 \times 0.2$

$= 0.24 + 0.14$

$= 0.38$

(*ii*) $P(\text{Both hit target}) = P(A) \times P(B)$

$= 0.3 \times 0.2$

$= 0.06$

(*iii*) $P(\overline{A} \cap B) = P(\overline{A}) \times P(B)$

$= 0.7 \times 0.2$

$= 0.14$

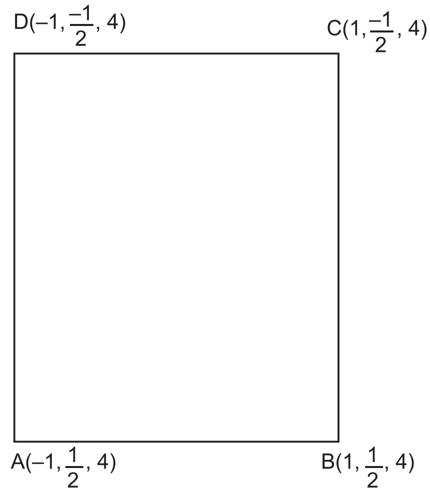
OR

(*iii*) $\text{P(target not hit)} = P(\overline{A}) \times P(\overline{B})$

$= 0.7 \times 0.8$

$= 0.56$

38. (*i*) $\overrightarrow{AB} = 2\hat{i}$



$\overrightarrow{AD} = -\hat{j}$

$\text{Area of rectangle} = |\overrightarrow{AB} \times \overrightarrow{AD}|$

$$= \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{matrix} \right\|$$

$= |\hat{i}(0) - \hat{j}(0) + \hat{k}(-2)|$

$= |-2\hat{k}|$

$= 2$ sq units

(*ii*) $\overrightarrow{AC} = 2\hat{i} - \hat{j}$

$\overrightarrow{BD} = -2\hat{i} - \hat{j}$

$$\cos\theta = \frac{\overrightarrow{AC} \cdot \overrightarrow{BD}}{|\overrightarrow{AC}| \cdot |\overrightarrow{BD}|}$$

$$= \frac{-4+1}{\sqrt{5} \cdot \sqrt{5}}$$

$$= \frac{-3}{5}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{-3}{5}\right)$$