Solutions to RSPL/1

1. (*d*),

2. (*b*), $|A^2| = |A|^2 = (-4)^2 = 16$

3. (*d*),

4. (*c*), If continuous $\lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow \quad \lim_{x \to 0} \frac{kx}{\sin 3x} = 2$$

$$\Rightarrow \quad \frac{k}{3} \lim_{x \to 0} \frac{3x}{\sin 3x} = 2$$

$$\Rightarrow \quad \frac{k}{3} \times 1 = 2 \Rightarrow k = 6$$

5. (*c*),

6. (*a*), Differential equation is $\left(\frac{d^3y}{dx^3}\right)^2 = x^3$

So, $m = 3, n = 2$

$\therefore$ $3n - m = 6 - 3 = 3$

7. (*c*), As for (1, –2), $3x - y < 4$ is false.

8. (*a*), Diagonals of a rhombus are perpendicular.

9. (*b*),

$$\int_0^2 \frac{1}{x^2+4} dx = \frac{1}{2}\left[\tan^{-1}\frac{x}{2}\right]_0^2$$

$$= \frac{1}{2}[\tan^{-1}1 - \tan^{-1}0] = \frac{\pi}{8}$$

10. (*d*), For non singular matrix $|A| \neq 0$

$$\Rightarrow \quad \begin{vmatrix} 2 & k \\ 3 & -3 \end{vmatrix} \neq 0 \Rightarrow -6 - 3k \neq 0 \Rightarrow k \neq -2$$

11. (*d*), $3p + 2q = p + 6q$

$\Rightarrow$ $p = 2q$

12. (*c*),

$$\begin{vmatrix} 2x & 4 \\ 3 & x \end{vmatrix} = 12$$

$$\Rightarrow \quad 2x^2 - 12 = 12$$

$$\Rightarrow \quad x^2 = 12 \Rightarrow x = \pm 2\sqrt{3}$$

13. (*b*), $|A.\text{Adj}\,A| = |A|^3 = 64 \Rightarrow |A| = 4$

14. (*d*), For independent events

$$P(\overline{A} \cap B) = P(\overline{A}) \cdot P(B)$$

$$= 0.6 \times 0.3 = 0.18$$

15. (*d*), $y = \int \log x \, dx$

$\Rightarrow$ $y = x \log x - x + C$

16. (*a*), $y = \sin^{-1}x$

$\Rightarrow \quad y_1 = \dfrac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\cdot y_1 = 1$

17. (*b*), $\vec{a}+\vec{b}+\vec{c} = \vec{0}$

$\Rightarrow \quad \vec{a}\times\vec{b}+\vec{b}\times\vec{b}+\vec{c}\times\vec{b} = \vec{0}\times\vec{b}$

$\Rightarrow \quad \vec{a}\times\vec{b}+\vec{0}-\vec{b}\times\vec{c} = \vec{0}$

$\Rightarrow \quad \vec{a}\times\vec{b} = \vec{b}\times\vec{c}$

18. (*b*), DR'S are 2–0, 3–1, 4–6, i.e. 2, 2, – 2 or 1, 1, – 1

19. (*d*), A is false, but R is true.

20. (*d*), A is false, but R is true.

21. $\tan^{-1}\left[\tan\dfrac{3\pi}{4}\right] = \tan^{-1}[-1]$

$= -\tan^{-1}1 = -\dfrac{\pi}{4}$

OR

Let for $x_1, x_2 \in R - \{1\}$

$f(x_1) = f(x_2)$

$\Rightarrow \quad \dfrac{x_1}{x_1-1} = \dfrac{x_2}{x_2-1}$

$\Rightarrow \quad x_1x_2 - x_1 = x_1x_2 - x_2 \Rightarrow x_1 = x_2$

As $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence injective.

22. Let radius of circular wave at any time t be r.

$\therefore \quad \dfrac{dr}{dt} = 0.7$ cm/s

$A = \pi r^2$

$\Rightarrow \quad \dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$

$= 2\pi r \times 0.7$

$\Rightarrow \quad \dfrac{dA}{dt} = 1.4\pi r$

$\therefore \quad \left.\dfrac{dA}{dt}\right]_{r=7} = 1.4\pi \times 7$

$= 9.8\pi \text{ cm}^2/\text{s}$

$= 9.8 \times \dfrac{22}{7} \text{cm}^2/\text{s} = 30.8 \text{ cm}^2/\text{s}$

23. $\vec{a}+2\vec{c}$ is perpendicular to $\vec{b}$

$\Rightarrow \quad (\vec{a}+2\vec{c})\cdot\vec{b} = 0$

$\Rightarrow \quad (\vec{a}\cdot\vec{b})+2(\vec{c}\cdot\vec{b}) = 0$

$\Rightarrow \quad (p-4-4)+2(5p+4) = 0$

$\Rightarrow \quad p-8+10p+8 = 0$

$\Rightarrow \quad 11p = 0 \Rightarrow p = 0$

OR

(*i*) Components of vector $\vec{a} = 6\hat{i} - 8\hat{j} + 12\hat{k}$ or 6, – 8, 12.

(*ii*) DR's are 6, – 8, 12 or 3, – 4, 6

$\therefore$ Dividing by $\sqrt{9+16+36} = \sqrt{61}$

$\therefore$ DC'S are $\frac{3}{\sqrt{61}}, \frac{-4}{\sqrt{61}}, \frac{6}{\sqrt{61}}$

24.

$$y = Ae^{7x} + Be^{-7x}$$

$$y_1 = A \cdot e^{7x} \cdot 7 + Be^{-7x}(-7)$$

$\Rightarrow$ $$y_1 = 7Ae^{7x} - 7Be^{-7x}$$

Again,

$$y_2 = 7A \cdot e^{7x} \cdot 7 - 7Be^{-7x}(-7)$$

$$= 49Ae^{7x} + 49Be^{-7x}$$

$$= 49(Ae^{7x} + Be^{-7x})$$

$\Rightarrow$ $$y_2 = 49y \Rightarrow y_2 - 49y = 0$$

25. If orthogonal, then

$$\left(|\vec{b}|\vec{a} + |\vec{a}|\vec{b}\right) \cdot \left(|\vec{b}|\vec{a} - |\vec{a}|\vec{b}\right) = 0$$

$\Rightarrow$ $$|b|^2(\vec{a} \cdot \vec{a}) - |\vec{a}|^2(\vec{b} \cdot \vec{b}) = 0$$

$\Rightarrow$ $$|\vec{b}|^2|\vec{a}|^2 - |\vec{a}|^2|\vec{b}|^2 = 0, \text{ true}$$

Hence, $|\vec{b}|\vec{a} + |\vec{a}|\vec{b}$ and $|\vec{b}|\vec{a} - |\vec{a}|\vec{b}$ are orthogonal vectors.

26. Consider $$\int \frac{1}{\sqrt{x^2+x+1}}dx = \int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}}dx$$

$$= \log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + C$$

27. S: getting a six

$$P(S) = \frac{1}{6}, P(\bar{S}) = \frac{5}{6}$$

$$P(A) = P(1) + P(3) + P(5) + \ldots$$

$$= P(S) + [P(\bar{S})]^2P(S) + [P(\bar{S})]^4.P(S) + \ldots$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \ldots$$

$$= \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{\frac{1}{6}}{1-\frac{25}{36}} = \frac{6}{11}$$

$$P(B) = P(2) + P(4) + P(6) + \ldots$$

$$= P(\bar{S})\,P(S) + [P(\bar{S})]^3P(S) + [P(\bar{S})]^5P(S) + \ldots$$

$$= \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6}$$

$$= \frac{\frac{5}{36}}{1-\left(\frac{5}{6}\right)^2} = \frac{\frac{5}{36}}{1-\frac{25}{36}} = \frac{5}{11}$$

OR

X : Number of defective bulbs

X can take values 0, 1, 2

$$\therefore \quad P(0) = \frac{{}^2C_0 \times {}^6C_3}{{}^8C_3} = \frac{1 \times 6 \times 5 \times 4}{8 \times 7 \times 6} = \frac{5}{14} = \frac{10}{28}$$

$$P(1) = \frac{{}^2C_1 \times {}^6C_2}{{}^8C_3} = \frac{2 \times 15 \times 6}{8 \times 7 \times 6} = \frac{15}{28}$$

$$P(2) = \frac{{}^2C_2 \times {}^6C_1}{{}^8C_3} = \frac{1 \times 6 \times 6}{8 \times 7 \times 6} = \frac{3}{28}$$

$\therefore$ Probability distribution table and table for mean:

X	$P(X)$	$X \cdot P(X)$
0	$\frac{10}{28}$	0
1	$\frac{15}{28}$	$\frac{15}{28}$
2	$\frac{3}{28}$	$\frac{6}{28}$
	$\frac{28}{28} = 1$	$\frac{21}{28} = \frac{3}{4}$

Mean = $\frac{3}{4}$

28. Let $I = \int_3^8 \frac{\sqrt{11-x}}{\sqrt{x} + \sqrt{11-x}} dx$ ...(*i*)

Using property $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

$$I = \int_3^8 \frac{\sqrt{11-(11-x)}}{\sqrt{11-x} + \sqrt{11-(11-x)}} dx$$

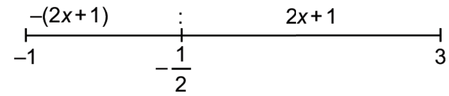
$$\Rightarrow \quad I = \int_3^8 \frac{\sqrt{x}}{\sqrt{11-x} + \sqrt{x}} dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_3^8 1.dx = [x]_3^8 = 8 - 3$$

$$\Rightarrow \quad 2I = 5 \Rightarrow I = \frac{5}{2}$$

OR



Consider $\int_{-1}^{3} |2x+1|\, dx$

$$= -\int_{-1}^{-\frac{1}{2}} (2x+1)\,dx + \int_{-\frac{1}{2}}^{3} (2x+1)\,dx$$

$$= -\left(x^2+x\right)_{-1}^{-\frac{1}{2}} + \left(x^2+x\right)_{-\frac{1}{2}}^{3}$$

$$= -\left(\frac{1}{4}-\frac{1}{2}\right) + (1-1) + (9+3) - \left(\frac{1}{4}-\frac{1}{2}\right)$$

$$= \frac{1}{4} + 0 + 12 + \frac{1}{4} = \frac{25}{2}$$

29. Consider equation, $\tan y.\sec^2 x\, dx + \tan x.\sec^2 y\, dy = 0$

$$\Rightarrow \quad \frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

$$\Rightarrow \quad \int \frac{\sec^2 x}{\tan x}dx + \int \frac{\sec^2 y}{\tan y}dy = C$$

$$\Rightarrow \quad \log|\tan x| + \log|\tan y| = C$$

$$\Rightarrow \quad \log|\tan x \cdot \tan y| = C, \text{ } C \text{ is constant of integration.}$$

$$\Rightarrow \quad \tan x \cdot \tan y = e^C = A \text{ (constant)}$$

OR

Consider equation $\quad x\frac{dy}{dx} = y(\log y - \log x + 1)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$$

Let $\quad y = vx \Rightarrow \frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$

$$\therefore \quad v + x\frac{dv}{dx} = v(\log v + 1)$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = v\log v + v$$

$$\Rightarrow \quad x\frac{dv}{dx} = v\log v$$

$$\Rightarrow \quad \int \frac{dv}{v\log v} = \int \frac{dx}{x}$$

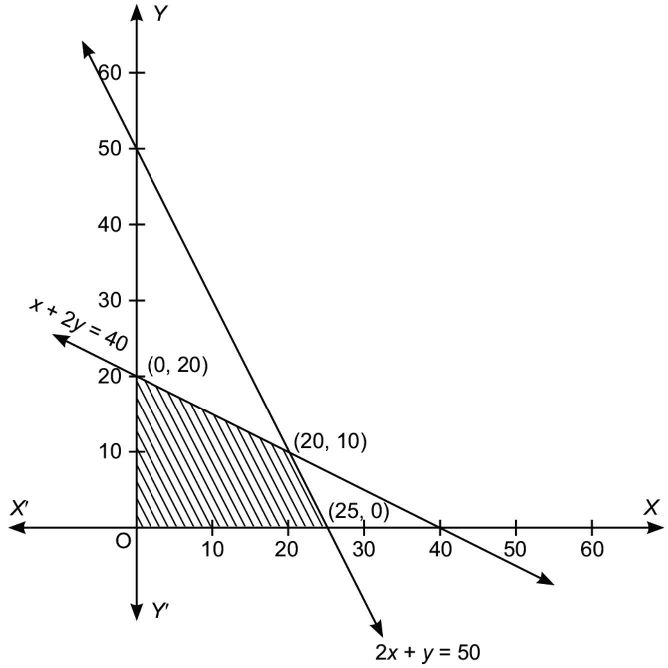
$$\Rightarrow \quad \log|\log v| = \log|x| + \log C$$

$$\Rightarrow \quad \log|\log v| = \log|Cx|$$

$$\Rightarrow \quad \log v = Cx$$

$$\Rightarrow \quad \log\frac{y}{x} = Cx, \text{ } C \text{ is constant of integration.}$$

30. Plotting the constraints $x \geq 0$, $y \geq 0$, $2x + y \leq 50$, $x + 2y \leq 40$,



We notice shaded portion is feasible solution, points for maximum Z are (25, 0), (20, 10), (0, 20)

Points	$Z = x + y$	**Values**	
(25, 0)	25 + 0	25	
(20, 10)	20 + 10	30	⟵ Maximum
(0, 20)	0 + 20	20	

Z is maximum for (20, 10), i.e. $x = 20$, $y = 10$

31. Consider $\int \frac{1}{x^3 - x^2 - x + 1} dx = \int \frac{1}{(x-1)^2(x+1)} dx$

$x^3 - x^2 - x + 1$
$= x^2(x-1) - 1(x-1)$
$= (x^2 - 1)(x - 1) = (x-1)^2(x+1)$

$$\frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} \quad ...(i)$$

$$\Rightarrow \quad 1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$\Rightarrow \quad 1 = A(x^2 - 1) + B(x+1) + C(x^2 - 2x + 1)$$

$$\Rightarrow \quad 1 = x^2(A + C) + x(B - 2C) + (-A + B + C)$$

Comparing coefficients, we get

$$A + C = 0 \Rightarrow A = -C$$

$$B - 2C = 0 \Rightarrow B = 2C$$

$$-A + B + C = 1 \Rightarrow C + 2C + C = 1 \Rightarrow C = \frac{1}{4}$$

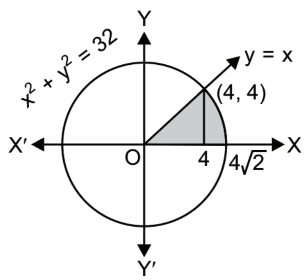
$$A = -\frac{1}{4}, B = \frac{1}{2}, C = \frac{1}{4}$$

From (i)

$$\int \frac{1}{(x-1)^2(x+1)} dx = -\frac{1}{4}\int \frac{1}{x-1} dx + \frac{1}{2}\int \frac{1}{(x-1)^2} dx + \frac{1}{4}\int \frac{1}{x+1} dx$$

$$= -\frac{1}{4}\log|x-1| - \frac{1}{2(x-1)} + \frac{1}{4}\log|x+1| + C$$

32. Eliminating y from $x^2 + y^2 = 32$ and $x = y$, we get $x = 4$



$\therefore$ Area $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2}\sin^{-1}\frac{x}{\sqrt{32}}\right]_4^{4\sqrt{2}}$$

$$= \left(\frac{16}{2} - 0\right) + \left(\frac{4\sqrt{2}}{2} \times 0 + 16\sin^{-1}1\right) - \left(\frac{4}{2}\sqrt{32 - 16} + 16\sin^{-1}\frac{1}{\sqrt{2}}\right)$$

$$= 8 + 16 \times \frac{\pi}{2} - 2 \times 4 - 16 \times \frac{\pi}{4}$$

$$= 8\pi - 4\pi = 4\pi \text{ sq units}$$

33. **For reflexive:** Let for $(a, b) \in A \times A$

$(a, b)\ R\ (a, b) \Rightarrow a + b = b + a$, true. Hence, reflexive.

For symmetric: Let for $(a, b), (c, d) \in A \times A$

$(a, b)\ R\ (c, d) \Rightarrow a + d = b + c$

$\Rightarrow c + b = d + a \Rightarrow (c, d)\ R\ (a, b)$. Hence, symmetric.

For transitive: Let for $(a, b), (c, d), (e, f) \in A \times A$

$\Rightarrow (a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow a + d = b + c$ and $c + f = d + e$

$\Rightarrow a + d + c + f = b + c + d + e$

$\Rightarrow a + f = b + e \Rightarrow (a, b)\ R\ (e, f)$. Hence, R is transitive

As R is reflexive, symmetric and transitive. Hence, R is an equivalence relation.

If $(a, b) \in$ equivalent class $[(1, 3)]$ then $(a, b)\ R(1, 3)$

$\Rightarrow a + 3 = b + 1$

$\therefore [(1, 3)] = \{(1, 3), (2, 4), (3, 5), (4, 6)\}$

OR

Given $f : R - \{0\} \rightarrow R - \{0\}$, defined by $f(x) = \frac{1}{x}$.

For one-one: Let for $x, y \in R - \{0\}$, (domain),

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y}$

$\Rightarrow x = y$. Hence, one-one.

For onto: Let $y \in R - \{0\}$ (co-domain), then there must exists $x \in R - \{0\}$ (domain), such that $f(x) = y$

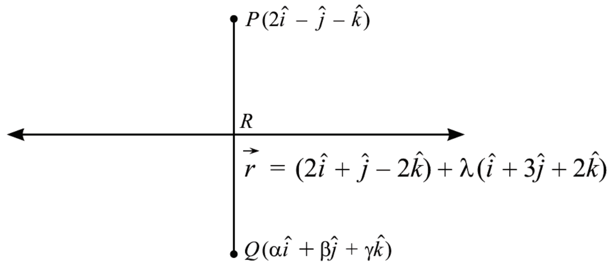
$\Rightarrow \dfrac{1}{x} = y \Rightarrow x = \dfrac{1}{y}$, hence for $y \in R$, there exists unique $\dfrac{1}{y} \in R - \{0\}$.

Hence, onto.

If domain is replaced by N then for $y\ (< 0) \in R - \{0\}$ we must have $x \in N$ such that $y = f(x) \Rightarrow y = \dfrac{1}{x}$

$\Rightarrow x = \dfrac{1}{y} < 0 \notin N$. So, result is not true if domain is replaced by N.

34. Let $Q(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ be image of $P(2\hat{i} - \hat{j} - \hat{k})$ in the line.



General point on line is

$$\vec{r} = (2+\lambda)\hat{i} + (1+3\lambda)\hat{j} + (-2+2\lambda)\hat{k} \quad ...(i)$$

$$\overrightarrow{PR} = (2+\lambda-2)\hat{i} + (1+3\lambda+1)\hat{j} + (-2+2\lambda+1)\hat{k}$$

$$= \lambda\hat{i} + (3\lambda+2)\hat{j} + (2\lambda-1)\hat{k}$$

If PR is perpendicular to the line then

$$1(\lambda) + 3(3\lambda+2) + 2(2\lambda-1) = 0$$

$$\Rightarrow \quad \lambda + 9\lambda + 6 + 4\lambda - 2 = 0$$

$$\Rightarrow \quad 14\lambda + 4 = 0 \Rightarrow 7\lambda + 2 = 0 \Rightarrow \lambda = -\frac{2}{7}$$

From (i) foot of perpendicular R is

$$\vec{r} = \left(2 - \frac{2}{7}\right)\hat{i} + \left(1 - \frac{6}{7}\right)\hat{j} + \left(-2 - \frac{4}{7}\right)\hat{k}$$

$$= \frac{12}{7}\hat{i} + \frac{1}{7}\hat{j} - \frac{18}{7}\hat{k} \quad ...(ii)$$

Also R is mid point of PQ

$$\vec{r} = \left(\frac{2+\alpha}{2}\right)\hat{i} + \left(\frac{-1+\beta}{2}\right)\hat{j} + \left(\frac{-1+\gamma}{2}\right)\hat{k} \quad ...(iii)$$

From (ii) and (iii), we get

$$\frac{2+\alpha}{2} = \frac{12}{7},\ \frac{-1+\beta}{2} = \frac{1}{7},\ \frac{-1+\gamma}{2} = -\frac{18}{7}$$

$$\Rightarrow \quad \alpha = \frac{24}{7} - 2,\ \beta = \frac{2}{7} + 1,\ \gamma = \frac{-36}{7} + 1$$

$$\Rightarrow \quad \alpha = \frac{10}{7},\ \beta = \frac{9}{7},\ \gamma = \frac{-29}{7}$$

$\therefore$ Image is $\dfrac{10}{7}\hat{i} + \dfrac{9}{7}\hat{j} - \dfrac{29}{7}\hat{k}$

OR

Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-0}{1}$ and $\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

and $$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})$$

Here, $$\vec{a}_1 = \hat{i} - \hat{j},\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k}$$

and $$\vec{a}_2 = -\hat{i} + 2\hat{j} + 2\hat{k},\ \vec{b}_2 = 5\hat{i} + \hat{j}$$

$$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j} = -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$\text{Shortest distance} = \left| \frac{(-2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 5\hat{j} - 13\hat{k})}{\sqrt{1 + 25 + 169}} \right|$$

$$= \left| \frac{2 + 15 - 26}{\sqrt{195}} \right| = \frac{9}{\sqrt{195}} \text{ units}$$

Lines are not intersecting as shortest distance is not zero.

35. Matrix equation is $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow$ $$AX = B$$

Solution is $X = A^{-1}B$ ...(*i*)

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$

$$= 150 + 330 + 720 = 1200 \neq 0. \text{ Hence } A^{-1} \text{ exists.}$$

Cofactors of elements of $|A|$ are

$$A_{11} = 75, \quad A_{12} = 110, \quad A_{13} = 72$$
$$A_{21} = 150, \quad A_{22} = -100, \quad A_{23} = 0$$
$$A_{31} = 75, \quad A_{32} = 30, \quad A_{33} = -24$$

$$\text{adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}'$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$\therefore$ $$A^{-1} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

From (i),

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

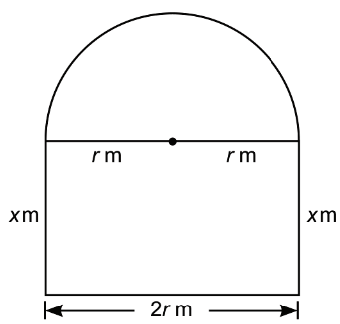
$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0 \quad -48 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$

$\Rightarrow \quad x = 2, y = 3, z = 5$

36. (i)



$2x + 2r + \pi r = 10$

$\Rightarrow \quad 2x + (2 + \pi)r = 10$ ...(i)

(ii) Area $A = x(2r) + \frac{1}{2}\pi r^2$

$= r[10 - (2 + \pi)r] + \frac{1}{2}\pi r^2$

$A = 10r - (2 + \pi)r^2 + \frac{1}{2}\pi r^2$

(iii) $\frac{dA}{dr} = 10 - 2(2 + \pi)r + \pi r$

For critical point, $\frac{dA}{dr} = 0$

$\Rightarrow \quad 10 - 2(2 + \pi)r + \pi r = 0$

$\Rightarrow \quad 10 - 4r - 2\pi r + \pi r = 0$

$\Rightarrow \quad 10 - (4 + \pi)r = 0 \Rightarrow r = \frac{10}{4 + \pi}$

OR

(*iii*) $$\frac{d^2A}{dr^2} = -2(2+\pi)+\pi < 0 \text{ for } r = \frac{10}{4+\pi}$$

Hence for $r = \frac{10}{4+\pi}$ m area is maximum

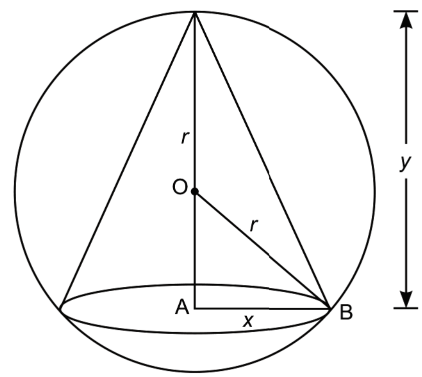
From (*i*), $$2x + (2+\pi)\cdot\frac{10}{4+\pi} = 10$$

$$\Rightarrow \quad 2x = 10 - \frac{10(2+\pi)}{4+\pi} = \frac{40+10\pi-20-10\pi}{4+\pi}$$

$$\Rightarrow \quad 2x = \frac{20}{4+\pi}$$

$$\Rightarrow \quad x = \frac{10}{4+\pi} \text{ m}$$

37. (*i*) OA = $y - r$, AB = x, OB = r



$$\Rightarrow \quad (y-r)^2 + x^2 = r^2$$

$$\Rightarrow \quad y^2 - 2yr + r^2 + x^2 = r^2$$

$$\Rightarrow \quad x^2 = 2yr - y^2$$

(*ii*) $$V = \frac{1}{3}\pi x^2 y$$

$$= \frac{1}{3}\pi y(2yr - y^2)$$

$$V = \frac{1}{3}\pi(2ry^2 - y^3)$$

(*iii*) $$\frac{dV}{dy} = \frac{1}{3}\pi(4ry - 3y^2)$$

For critical value, $$\frac{dV}{dy} = 0$$

$$\Rightarrow \quad 4ry - 3y^2 = 0$$

$$\Rightarrow \quad y = \frac{4r}{3}, (y \neq 0)$$

OR

(*iii*) $$\frac{d^2V}{dy^2} = \frac{1}{3}\pi(4r - 6y) < 0 \text{ for } y = \frac{4r}{3}$$

Hence for $y = \frac{4r}{3}$, volume of cone is maximum.

38. $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{5}$

$P(\bar{A}) = \frac{1}{2}, P(\bar{B}) = \frac{2}{3}, P(\bar{C}) = \frac{4}{5}$

(*i*) $P(\text{exactly two will solve}) = P(AB\bar{C} \text{ or } A\bar{B}C \text{ or } \bar{A}BC)$

$$= \frac{1}{2}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{5} + \frac{1}{2}\times\frac{1}{3}\times\frac{1}{5} = \frac{4+2+1}{30} = \frac{7}{30}$$

(*ii*) $P(\text{problem is solved}) = 1 - P(\text{none solves})$

$$= 1 - P(\bar{A}\bar{B}\bar{C})$$

$$= 1 - \frac{1}{2}\times\frac{2}{3}\times\frac{4}{5} = 1 - \frac{4}{15} = \frac{11}{15}$$