Solutions to RSPL/2

1. (*d*), Order can be $1 \times n$ or $n \times 1$, n is prime

2. (*c*), $|3A| = 3^3|A| = 27 \times (-5) = -135$.

3. (*d*), $\overrightarrow{AB} = \vec{r} \Rightarrow \vec{b} - \vec{a} = \vec{r} \Rightarrow \vec{b} = \vec{a} + \vec{r}$

4. (*c*), If continuous $\lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x \to 0} \frac{\sin 2x}{\tan 5x} = 3k \Rightarrow \frac{2}{5} = 3k$$

5. (*d*)

6. (*c*), $n = 4$, $m = 3$, $4m - 3n = 0$

7. (*c*)

8. (*d*)

9. (*d*), Odd function

10. (*a*), $6x + 10 = 12 + 10 \Rightarrow x = 2$

11. (*d*)

12. (*b*), If collinear $\begin{vmatrix} 2 & 3 & 1 \\ 3 & k & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0$

$$\Rightarrow 2(k - 0) - 3(3 - 1) + 1(0 - k) = 0 \Rightarrow 2k - 6 - k = 0 \Rightarrow k = 6$$

13. (*b*)

14. (*a*), $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot P(B)}{P(B)} = P(A)$

15. (*d*), $\operatorname{cosec} x \dfrac{dy}{dx} = 1 \Rightarrow \int dy = \int \sin x \cdot dx \Rightarrow y = -\cos x + C$.

16. (*d*), $y = \log x \Rightarrow y_1 = \dfrac{1}{x} \Rightarrow y_2 = -\dfrac{1}{x^2}$

17. (*b*)

18. (*c*)

19. (*d*), A is false but R is true.

20. (*a*), Both A and R are true and R is the correct explanation of A.

21. Consider $2\tan^{-1}\sqrt{3} - \operatorname{cosec}^{-1}\left(\dfrac{2}{\sqrt{3}}\right) = 2 \cdot \dfrac{\pi}{3} - \dfrac{\pi}{3} = \dfrac{\pi}{3}$

OR

Given $y = \dfrac{x+2}{3} \Rightarrow 3y = x + 2$

$\Rightarrow \quad x = 3y - 2 \in$ Domains

Hence, surjective.

22. $$R(x) = 3x^2 - 5x + 9$$

$$R'(x) = 6x - 5$$

Marginal revenue, when $x = 5$

$$R'(5) = 30 - 5 = 25$$

23. $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 0 & 1 & 3 \end{vmatrix} = \hat{i}(-6-0) - \hat{j}(3-0) + \hat{k}(1-0) = -6\hat{i} - 3\hat{j} + \hat{k}$$

$\therefore$ $$\sin\theta = \frac{\sqrt{36+9+1}}{\sqrt{1+4}\ \sqrt{1+9}} = \frac{\sqrt{46}}{\sqrt{5}\cdot\sqrt{10}} = \frac{\sqrt{46}}{5\sqrt{2}}$$

$\Rightarrow$ $$\theta = \sin^{-1}\left(\frac{\sqrt{46}}{5\sqrt{2}}\right)$$

OR

Unit vector along $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ is

$$\hat{a} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$\therefore$ vector of magnitude 5 units along $\vec{a}$

$$5\vec{a} = \frac{10}{3}\hat{i} - \frac{5}{3}\hat{j} + \frac{10}{3}\hat{k}$$

24. Given equation is

$$(1 + y^2)dx - (\tan^{-1}y - x)\, dy = 0$$

$\Rightarrow$ $$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

Here, $P(y) = \dfrac{1}{1+y^2}$

$\Rightarrow$ Integrating factor $= e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1}y}$

25. Consider $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq \vec{0}$

$\Rightarrow$ $$\vec{a} \times \vec{b} - \vec{b} \times \vec{c} = \vec{0}$$

$\Rightarrow$ $$\vec{a} \times \vec{b} + \vec{c} \times \vec{b} = \vec{0}$$

$\Rightarrow$ $$(\vec{a} + \vec{c}) \times \vec{b} = \vec{0}$$

$\Rightarrow$ $$(\vec{a} + \vec{c}) \parallel \vec{b}$$

$\Rightarrow$ $\vec{a} + \vec{c} = t\,\vec{b}$, where t is a scalar.

26. $\int \sqrt{2x^2 - 4x + 3}\, dx = \sqrt{2}\int \sqrt{x^2 - 2x + \frac{3}{2}}\, dx = \sqrt{2}\int \sqrt{(x-1)^2 + \frac{1}{2}}\, dx$

$$= \sqrt{2}\left[\frac{x-1}{2}\sqrt{x^2 - 2x + \frac{3}{2}} + \frac{1}{4}\log\left|(x-1) + \sqrt{x^2 - 2x + \frac{3}{2}}\right|\right] + C$$

27. A: Youngest child is girl $\{BG, GG\}$

B: Both girl $\{GG\}$

$A \cap B : \{GG\}$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

OR

Bag I : $3R + 4B$ → (1 ball) → Bag II : $4R + 5B$

Case I : When ball transferred is red $P\left(\frac{R}{I}\right) = \frac{3}{7}$,

Number of balls in bag II : $5R + 5B$

$\therefore \quad P\left(\frac{R}{II}\right) = \frac{5}{10} = \frac{1}{2}$

$\therefore$ Probability in this case $= \frac{3}{7} \times \frac{1}{2} = \frac{3}{14}$

Case II : When ball transferred is black $P\left(\frac{B}{I}\right) = \frac{4}{7}$,

Number of balls in bag II : $4R + 6B$

$\therefore \quad P\left(\frac{R}{II}\right) = \frac{4}{10} = \frac{2}{5}$

$\therefore$ Probability in this case $= \frac{4}{7} \times \frac{2}{5} = \frac{8}{35}$

Probability of drawing red ball from bag II, when one ball is transferred from bag I to bag II is

$$= \frac{3}{14} + \frac{8}{35} = \frac{15+16}{70} = \frac{31}{70}$$

28. Let $I = \int_0^{\pi} \frac{x}{1+\sin x}\, dx$ ...(*i*)

Using properties $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$$I = \int_0^{\pi} \frac{\pi - x}{1 + \sin(\pi - x)}\, dx = \int_0^{\pi} \frac{\pi - x}{1 + \sin x} dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\pi} \frac{\pi}{1+\sin x}\, dx = \pi\int_0^{\pi} \frac{1-\sin x}{\cos^2 x}\, dx$$

$$= \pi \int_0^{\pi} \left(\sec^2 x - \sec x \cdot \tan x\right) dx$$

$$= \pi[\tan x - \sec x]_0^{\pi} = \pi\left[(\tan \pi - \sec \pi) - (\tan 0 - \sec 0)\right]$$

$$2I = \pi(1 + 1) = 2\pi$$

$$I = \pi$$

OR

Consider $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x|\, dx$

$|\sin x|$ is even function $= 2\int_0^{\frac{\pi}{4}} \sin x\, dx$

$$= 2[-\cos x]_0^{\frac{\pi}{4}} = 2\left[-\cos\frac{\pi}{4} + \cos 0\right]$$

$$= 2\left[-\frac{1}{\sqrt{2}} + 1\right] = \left(2 - \sqrt{2}\right)$$

29. From differential equation $\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx$ ...(*i*)

For $\int \frac{e^x}{1+e^{2x}} dx = \int \frac{1}{1+t^2} dt$ | Let $e^x = t$ $\Rightarrow e^x dx = dt$

$$= \tan^{-1} t + C_1 = \tan^{-1} e^x + C_1$$

From (*i*), we get

$$\tan^{-1} y = -\tan^{-1} e^x + C \quad ...(ii)$$

When $x = 0$, $y = 1$

$\Rightarrow$ $\tan^{-1} 1 = -\tan^{-1} 1 + C$

$\Rightarrow$ $C = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ $(e^{\circ} = 1)$

Substituting in (*ii*), we get

$$\tan^{-1} y = -\tan^{-1} e^x + \frac{\pi}{2}$$

$\Rightarrow$ $\tan^{-1} y + \tan^{-1} e^x = \frac{\pi}{2}$ is the required solution.

OR

Consider equation $(x - y)\frac{dy}{dx} = x + 2y$

$\Rightarrow$ $\frac{dy}{dx} = \frac{x+2y}{x-y}$

Let $y = vx$ $\Rightarrow$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow$ $v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx} = \frac{1+2v}{1-v}$

$\Rightarrow$ $x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+2v-v+v^2}{1-v}$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v+v^2}{1-v}$$

$$\Rightarrow \quad \int \frac{1-v}{1+v+v^2}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{2-2v}{1+v+v^2}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{3-(1+2v)}{1+v+v^2}\,dv = \int \frac{dx}{x}$$

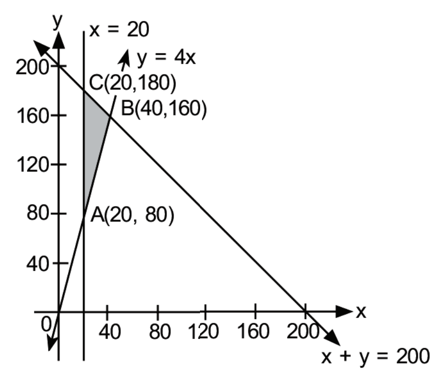
$$\Rightarrow \quad \frac{3}{2}\int \frac{1}{v^2+v+1}\,dv - \frac{1}{2}\int \frac{2v+1}{v^2+v+1}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{3}{2}\int \frac{1}{\left(v+\frac{1}{2}\right)^2+\frac{3}{4}}\,dv - \frac{1}{2}\int \frac{2v+1}{v^2+v+1}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{3}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\frac{\left(v+\frac{1}{2}\right).2}{\sqrt{3}} - \frac{1}{2}\log|v^2+v+1| = \int \frac{dx}{x}$$

$$\Rightarrow \quad \sqrt{3}\tan^{-1}\frac{\left(2\frac{y}{x}+1\right)}{\sqrt{3}} - \frac{1}{2}\log\left|\frac{y^2}{x^2}+\frac{y}{x}+1\right| = \log|x| + C$$

30. Plotting the inequations, feasible solution is the shaded portion. Possible points for maximum Z are $A(20, 80)$, $B(40, 160)$, $C(20, 180)$



Points	$Z = 400x + 300y$	Values
$A(20, 80)$	8000 + 24000	32000
$B(40, 160)$	16000 + 48000	64000 → Maximum
$C(20, 180)$	8000 + 54000	62000

Maximum is at $B(40, 160)$ i.e. $x = 40$, $y = 160$.

31. $$\frac{x}{x^3+x^2+x+1} = \frac{x}{(x^2+1)(x+1)}$$

$$= \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \quad \text{...}(i)$$

$$\Rightarrow \quad x = A(x^2+1) + (Bx+C)(x+1)$$

$$= Ax^2 + A + Bx^2 + Bx + Cx + C = x^2(A+B) + x(B+C) + (A+C)$$

Comparing the coefficients, we get

$$A + B = 0,\ B + C = 1,\ A + C = 0$$

$$\Rightarrow \quad A = -B = -C = -\frac{1}{2}$$

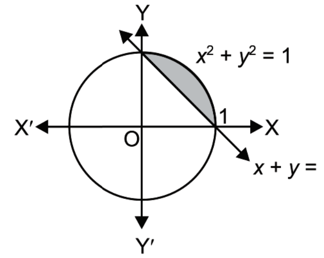
$\therefore$ From (i), we get

$$\int \frac{x}{x^3+x^2+x+1}dx = -\frac{1}{2}\int \frac{1}{x+1}dx + \int \frac{\frac{1}{2}x+\frac{1}{2}}{x^2+1}$$

$$= -\frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

32. Curves are $x^2 + y^2 = 1$ and $x + y = 1$



$$\text{Area bounded} = \int_0^1 \{\sqrt{1-x^2} - (1-x)\}\,dx$$

$$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$$

$$= \left(0 + \frac{1}{2}\sin^{-1}1 - 1 + \frac{1}{2}\right) - 0$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2} = \frac{1}{4}(\pi - 2) \text{ sq units}$$

33. Given $f(x) = 5x^2 + 6x - 9$

For one-one: Let for $x_1, x_2 \in R_+$

$$f(x_1) = f(x_2)$$

$\Rightarrow$ $5x_1^2 + 6x_1 - 9 = 5x_2^2 + 6x_2 - 9 \Rightarrow 5(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow$ $(x_1 - x_2)(5x_1 + 5x_2 + 6) = 0$

$\Rightarrow$ $x_1 - x_2 = 0$ as $5x_1 + 5x_2 + 6 \neq 0$

$\therefore$ $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, for $x_1, x_2 \in R_+$

Hence one-one

For onto: Let for $y \in [-9, \infty)$ there exists $x \in R_+$ such that

$$y = 5x^2 + 6x - 9$$

$\Rightarrow$ $5x^2 + 6x - (9 + y) = 0$

$\Rightarrow$ $x = \dfrac{-6 \pm \sqrt{36 + 20(9+y)}}{10} = \dfrac{-6 \pm \sqrt{216 + 20y}}{10}$

$\Rightarrow$ $x = \dfrac{-6 + \sqrt{216 + 20y}}{10} \in R$. Hence, onto.

OR

Given relation $R = \{(a, b) : a \le b^2\}$ in real numbers.

For reflexive: Let $\left(\frac{1}{2}, \frac{1}{2}\right) \in R \Rightarrow \frac{1}{2} \le \frac{1}{4}$, false

$\therefore$ $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$. Hence, not reflexive.

For symmetric: Let $(-2, 5) \in R \Rightarrow -2 \le 25$, true

consider $(5, -2) \Rightarrow 5 \le 4$, false

$\therefore$ $(-2, 5) \in R \not\Rightarrow (5, -2) \in R$, Hence, not symmetric.

For transitive: Let $(45, 7) \in R$, $(7, 3) \in R$

$\Rightarrow$ $45 \le 49$, true $7 \le 9$, true

Consider $(45, 3) \in R \Rightarrow 45 \le 9$, false.

$\therefore$ $(a, b) \in R, (b, c) \in R \not\Rightarrow (a, c), \in R$, Hence, not transitive.

34. Line is $6x - 2 = 3y + 1 = 2z - 2$

$$\Rightarrow \quad 6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right) = 2(z - 1)$$

$$\Rightarrow \quad \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z - 1}{3}$$

$\therefore$ DR's are 1, 2, 3

Dividing by $\sqrt{1 + 4 + 9} = \sqrt{14}$

$\therefore$ DC's are $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

Cartesian equation of a line passing through point (2, –1, 5) which is parallel to given line is $\frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z - 5}{3}$ and vector equation is $\vec{r} = (2\hat{i} - \hat{j} + 5\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$.

OR

Consider line

$$r = (\hat{i} + \hat{j} - \hat{k}) + \lambda (3\hat{i} - \hat{j})$$

General point on line is $\vec{r} = (1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k}$ ...(*i*)

Consider line $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

General point on line is $\vec{r} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$ ...(*ii*)

If lines intersect then for some values of λ, μ, (*i*) and (*ii*) represent the same point.

$\Rightarrow$ $1 + 3\lambda = 4 + 2\mu$ ...(*iii*)

$1 - \lambda = 0 \Rightarrow \lambda = 1$ ...(*iv*)

and $-1 = -1 + 3\mu \Rightarrow \mu = 0$ ...(*v*)

Substituting $\lambda = 1$, $\mu = 0$ in (iii) we notice it satisfies the equation.

Hence, lines intersect.

Substituting $\lambda = 1$ or $\mu = 0$ in general point, position vector of point of intersection is

$$\vec{r} = 4\hat{i} - \hat{k}$$

$\therefore$ Point of intersection is (4, 0, –1).

35. Consider
$$BA = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 2+4-0 & -2+6-4 & 0+8-8 \\ -4+4-0 & 4+6-4 & 0+8-8 \\ 2-2+0 & -2-3+5 & 0-4+10 \end{bmatrix}$$
$$BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I \qquad ...(i)$$

Given equations are
$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7$$

Matrix equation is $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\Rightarrow$ $$AX = C \quad \Rightarrow \quad X = A^{-1} C.$$

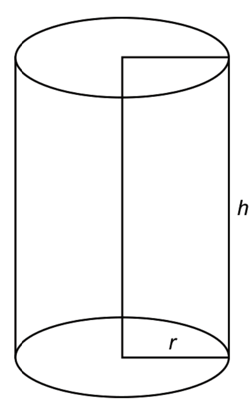
From (*i*), we have
$$BA = 6I \quad \Rightarrow \quad B = 6IA^{-1}$$

$\Rightarrow$ $$A^{-1} = \frac{1}{6}B$$

$\therefore$ $$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$
$$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\Rightarrow$ $$x = 2, y = -1, z = 4.$$

36. (*i*) $$S = 2\pi rh + 2\pi r^2$$



(*ii*) $$V = \pi r^2 h = \pi r^2\left[\frac{S - 2\pi r^2}{2\pi r}\right]$$
$$V = \frac{1}{2}[Sr - 2\pi r^3]$$

(*iii*) $$\frac{dV}{dr} = \frac{1}{2}[S - 6\pi r^2]$$

For critical value, $$\frac{dV}{dr} = 0 \Rightarrow S - 6\pi r^2 = 0 \Rightarrow r = \sqrt{\frac{S}{6\pi}}.$$

OR

(*iii*) $\dfrac{d^2V}{dr^2} = \dfrac{1}{2}[-12\pi r] < 0$, for $r = \sqrt{\dfrac{S}{6\pi}}$. Volume of maximum

$\therefore$ $6\pi r^2 = S \Rightarrow 6\pi r^2 = 2\pi rh + 2\pi r^2$

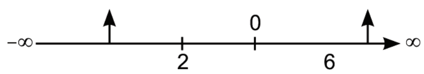
$\Rightarrow$ $2\pi rh = 4\pi r^2 \Rightarrow h = 2r$

37. $f(x) = x^3 - 12x^2 + 36x + 17$

(*i*) $f'(x) = 3x^2 - 24x + 36$

(*ii*) For critical point, $f'(x) = 0$

$\Rightarrow$ $3x^2 - 24x + 36 = 0 \Rightarrow x^2 - 8x + 12 = 0 \Rightarrow (x-6)(x-2) = 0 \Rightarrow x = 6, 2$

(*iii*) 

In (2, 6) he is able to concentrate, as for (2, 6), $f'(x) < 0$.

OR

(*iii*) In $(-\infty, 2) \cup (6, \infty)$ he is not able to concentrate as for $(-\infty, 2)$ or $(6, \infty)$, $f'(x) > 0$.

38. $P(H) = \dfrac{1}{2}, P(\overline{H}) = \dfrac{1}{2}$

(*i*) P(*C* gets head in sixth throw), $P(6) = [P(\overline{H})]^5 P(H)$

$$= \left(\frac{1}{2}\right)^5 \times \frac{1}{2} = \frac{1}{64}$$

(*ii*) $P(B) = P(2) + P(5) + P(8) + ...$

$$= P(\overline{H}) \cdot P(H) + P(\overline{H})^4 \cdot P(H) + P(\overline{H})^7 \cdot P(H) + ...$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^4 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^7 \cdot \frac{1}{2} + ...$$

$$= \frac{\frac{1}{4}}{1 - \frac{1}{8}} = \frac{\frac{1}{4}}{\frac{7}{8}} = \frac{2}{7}$$