Solutions to RSPL/3

1. (*a*), $$BA = \begin{bmatrix} 1 & 2 \\ -1 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 12 \\ 0 & 7 & 12 \\ 6 & -3 & 0 \end{bmatrix}$$

$$b_{32} + b_{11} = -3 + 4 = 1.$$

2. (*c*), $x = 2, y = 7 \quad x^2 + y = 4 + 7 = 11.$

3. (*b*), $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 4 + 9 - 8 = 5$

$|\vec{a} - \vec{b}| = \sqrt{5}$

4. (*c*), $\sqrt{x} + \sqrt{y} = \frac{5}{2} \Rightarrow \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \left.\frac{dy}{dx}\right]_{(4,\frac{1}{4})} = -\frac{1}{4}$

5. (*b*), $\frac{3}{2}\int_2^4 \frac{2x}{x^2+1}dx = \frac{3}{2}\left[\log|x^2+1|\right]_2^4 = \frac{3}{2}\log\frac{17}{5}$

6. (*d*), $\frac{dy}{dx} - \frac{1}{x}y = x^2$, I.F. $= e^{-\int\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

7. (*a*)

8. (*a*), $$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$$

$$1 = 1 + 1 + 2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{2\pi}{3}.$$

9. (*d*), $2\int \frac{3x^2 + 2\tan x\sec^2 x}{x^3 + \tan^2 x}dx = 2\log|x^3 + \tan^2 x| + C$

10. (*c*), $|2AB| = 2^2|A||B| = 4 \times (-6) \times (-1) = 24$

11. (*a*), $0 + 0 \geq 12$, False

12. (*a*), $$2x - 4 + 9x = 29$$

$$\Rightarrow \quad 11x = 33 \Rightarrow x = 3.$$

13. (*b*)

14. (*a*), $$A = \frac{1}{2}\left|(3\hat{i} + \hat{j} + 4\hat{k}) \times (\hat{i} - \hat{j} + \hat{k})\right|$$

$$= \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}\right| = \frac{1}{2}|5\hat{i} + \hat{j} - 4\hat{k}| = \frac{1}{2}\sqrt{25+1+16} = \frac{1}{2}\sqrt{42} \text{ sq units}$$

15. (*c*), $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{7}{10}}{\frac{4}{5}} = \frac{7}{10} \times \frac{5}{4} = \frac{7}{8}.$

16. (*d*), As equation cannot be written as polynomial of derivatives.

17. (*b*), $\frac{dy}{dx} = 2\frac{1}{\sec^2 x}.\sec x \cdot \sec x \tan x = 2 \tan x$

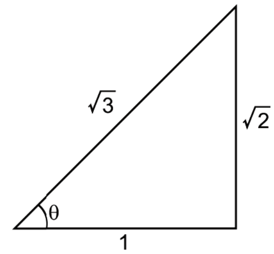
18. (*d*), $\bar{r} = (-\hat{i} + 2\hat{j} + 4\hat{k}) + \lambda(2\hat{j} - \hat{k})$

19. (*d*), A is false but R is true.

20. (*d*), A is false but R is true.

21. $\cos[\cot^{-1}\{\sin(\tan^{-1}1)\}] = \cos[\cot^{-1}\{\sin\frac{\pi}{4}\}]$

$$= \cos\left[\cot^{-1}\frac{1}{\sqrt{2}}\right] = \cos\left[\cos^{-1}\frac{1}{\sqrt{3}}\right] = \frac{1}{\sqrt{3}}.$$



OR

For reflexive: $(a, a) \in R \Rightarrow 1 + a^2 > 0$ always true. Hence reflexive

For symmetric: Let $(a, b) \in R$

$\Rightarrow \quad 1 + ab > 0 \Rightarrow 1 + ba > 0$

$\Rightarrow \quad (b, a) \in R$. Hence, symmetric.

22. Let r be radius, S surface area and V volume at any time t.

$$\frac{dS}{dt} = 6 \text{ cm}^2/\text{s}$$

$$\frac{d}{dt}(4\pi r^2) = 6$$

$$\Rightarrow \quad 8\pi r\frac{dr}{dt} = 6$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{3}{4\pi r} \quad ...(i)$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi r^2 . \frac{3}{4\pi r} = 3r$$

$$\left.\frac{dV}{dt}\right]_{r=4} = 3 \times 4 = 12 \text{ cm}^3/\text{s}.$$

23.

$$|\vec{a} - \vec{b}|^2 = 900$$

$$\Rightarrow \quad (\vec{a} - \vec{b})^2 = 900 \quad \Rightarrow \quad \vec{a}^2 + \vec{b}^2 - 2\vec{a}\cdot\vec{b} = 900$$

$$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 900$$

$$\Rightarrow \quad 121 + 529 - 2\vec{a}\cdot\vec{b} = 900$$

$$\Rightarrow \quad 2\vec{a}\cdot\vec{b} = -250.$$

Consider $|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b})^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a}\cdot\vec{b}$

$$\Rightarrow \quad = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$$

$$= 121 + 529 - 250 = 400$$

$$\therefore \quad |\vec{a} + \vec{b}|^2 = 400$$

$$\Rightarrow \quad |\vec{a} + \vec{b}| = 20.$$

OR

$$|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b})^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a} \cdot \vec{b}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|\,|\vec{b}|\cos\theta$$

$$= 1 + 1 + 2\cos\theta = 2(1 + \cos\theta) = 2 \times 2\cos^2\frac{\theta}{2}$$

$$\Rightarrow \quad |\vec{a} + \vec{b}| = 2\cos\frac{\theta}{2}.$$

24. Consider $e^x + e^y = e^{x+y}$ ...(*i*)

Differentiating w.r.t x, we get

$$e^x + e^y\frac{dy}{dx} = e^{x+y}[1 + \frac{dy}{dx}]$$

$$\Rightarrow \quad [e^y - e^{x+y}]\frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \quad (-e^x)\frac{dy}{dx} = e^y \qquad \text{[from (i)]}$$

$$\Rightarrow \quad \frac{dy}{dx} = -e^{y-x}$$

25. α, β and γ are the angles with x, y and z axis respectively.

$\Rightarrow$ $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are its direction cosines.

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$$

$\Rightarrow 3 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1.$

Note: In question it is given $\cos 2\alpha + \cos 2\beta + \cos 2\alpha = -1$

Whereas it shall be $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$.

26. Consider $\displaystyle\int \frac{x+2}{\sqrt{(x-2)(x-3)}}dx = \int \frac{x+2}{\sqrt{x^2-5x+6}}dx = \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2-5x+6}}dx$

$$= \frac{1}{2}\int \frac{(2x-5)+9}{\sqrt{x^2-5x+6}}dx$$

$$= \frac{1}{2}\left[\int \frac{2x-5}{\sqrt{x^2-5x+6}}\,dx + 9\int \frac{1}{\sqrt{x^2-5x+6}}\,dx\right] \qquad ...(i)$$

Consider $\displaystyle\int \frac{2x-5}{\sqrt{x^2-5x+6}}dx = \int \frac{1}{\sqrt{t}}dt = 2\sqrt{t} + C_1$ | Let $x^2 - 5x + 6 = t$ $\Rightarrow (2x-5)dx = dt$

$$= 2\sqrt{x^2-5x+6} + C_1 \qquad (ii)$$

Consider $\int \frac{1}{\sqrt{x^2-5x+6}}dx = \int \frac{1}{\sqrt{\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+6}}dx = \int \frac{1}{\sqrt{\left(x-\frac{5}{2}\right)^2-\frac{1}{4}}}dx$

$$= \log\left|\left(x-\frac{5}{2}\right)+\sqrt{x^2-5x+6}\right|+C_2 \quad ...(iii)$$

Substituting from (*ii*) and (*iii*) in (*i*), we get

$$\int \frac{x+2}{\sqrt{(x-2)(x-3)}}dx = \frac{1}{2}\left[2\sqrt{x^2-5x+6}+9\log\left|\left(x-\frac{5}{2}\right)+\sqrt{x^2-5x+6}\right|\right]+C$$

27. X: Number of successes(S)

X: Can take values 0, 1, 2

S: getting a total of 9

$$P(S) = \frac{4}{36}=\frac{1}{9},\ P(\bar{S}) = \frac{8}{9}$$

$$P(0) = {}^2C_0 \cdot [P(\bar{S})]^2 = 1 \times \left(\frac{8}{9}\right)^2 = \frac{64}{81}$$

$$P(1) = {}^2C_1 \cdot P(S)\ P(\bar{S}) = 2 \times \frac{1}{9}\times\frac{8}{9}=\frac{16}{81}$$

$$P(2) = {}^2C_2 \cdot [P(S)]^2 = 1 \times \left(\frac{1}{9}\right)^2 = \frac{1}{81}$$

$\therefore$ Probability distribution is

X	***P(X)***
0	$\frac{64}{81}$
1	$\frac{16}{81}$
2	$\frac{1}{81}$
	$\frac{81}{81}$

OR

A : Total of 8 : {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

B : Doublet : {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$A \cap B$: {(4, 4)}

$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{36}}{\frac{5}{36}} = \frac{1}{5}$$

28. Consider $\int \underset{(1)}{\tan^{-1}x}\ \underset{(2)}{x^2}dx = \tan^{-1}x \cdot \frac{x^3}{3} - \int \frac{1}{1+x^2}\cdot\frac{x^3}{3}dx$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int\frac{x^3}{1+x^2}dx \quad ...(i)$$

Consider $\int \frac{x^3}{1+x^2}dx = \int \frac{x^2 \cdot x\,dx}{1+x^2}$ | Let $1 + x^2 = t$ $\Rightarrow$ $2x\,dx = dt$

$$= \frac{1}{2}\int \frac{t-1}{t}dt$$

$$= \frac{1}{2}\int \left(1-\frac{1}{t}\right)dt = \frac{1}{2}(t - \log t) + C_1$$

$$= \frac{1}{2}(1 + x^2 - \log|1 + x^2|) + C_1$$

Substituting in (*i*), we get

$$\int x^2 \tan^{-1}x\,dx = \frac{x^3}{3}\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\log|1 + x^2| + C$$

OR

Let $I = \int_0^{2\pi} \frac{dx}{1+e^{\sin x}}$ ...(*i*)

Using property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{2\pi} \frac{dx}{1+e^{\sin(2\pi - x)}}$$

$$= \int_0^{2\pi} \frac{dx}{1+e^{-\sin x}}$$

$$= \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1}dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{2\pi} 1 \cdot dx = \Big[x\Big]_0^{2\pi} = 2\pi$$

$\Rightarrow$ $I = \pi$

29. $\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2},$

$$\text{I.F.} = e^{\int \frac{1}{x\log x}dx} = e^{\log|\log x|} = \log|x|,$$

Solution is $y\,.\,\log|x| = 2\int \frac{\log|x|}{x^2}dx$

$$= 2\left[\log|x| \cdot \left(\frac{x^{-1}}{-1}\right) - \int \frac{1}{x}\cdot\left(\frac{x^{-1}}{-1}\right)dx\right]$$

$\Rightarrow$ $y\log|x| = 2\left[-\frac{1}{x}\log|x| + \int x^{-2}dx\right]$

$\Rightarrow$ $y\log|x| = 2\left[-\frac{1}{x}\log|x| - \frac{1}{x}\right] + C$ is the required solution.

OR

$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$, homogeneous equation

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \quad v + x\frac{dv}{dx} = v - \tan v$

$\Rightarrow \quad \int \frac{1}{\tan v} dv = -\int \frac{1}{x} dx$

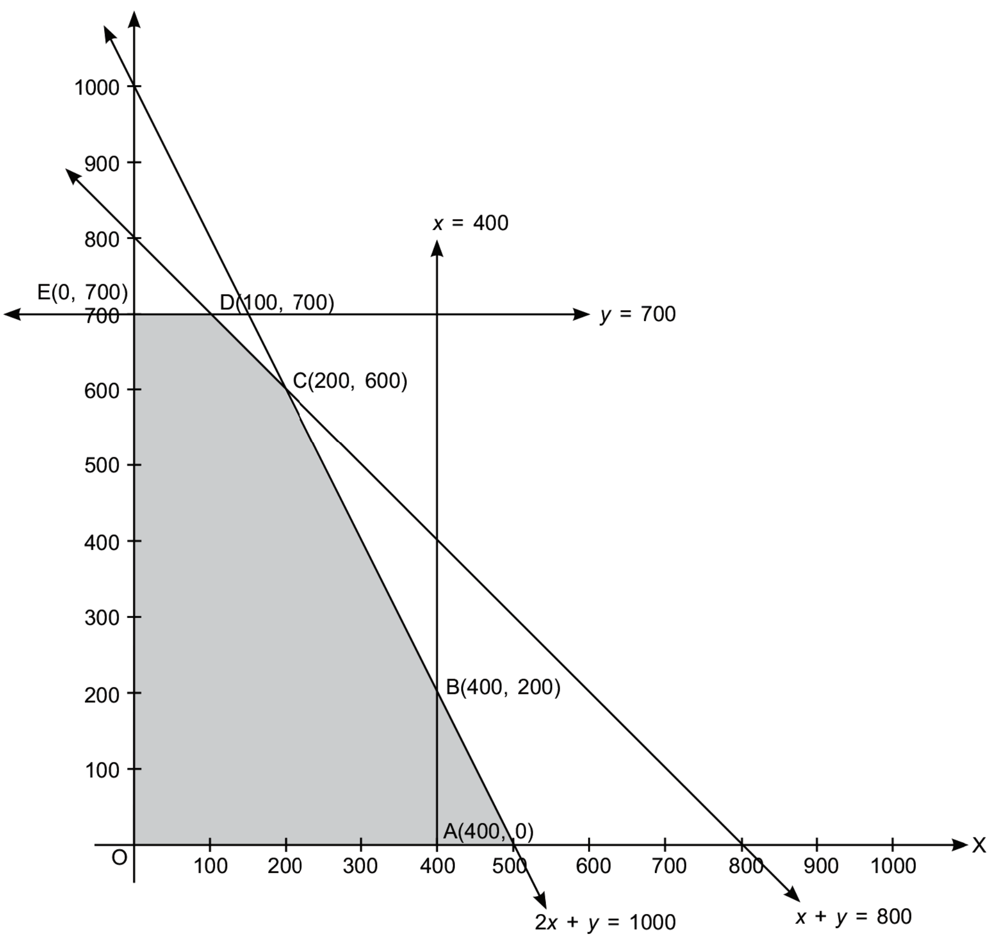
$\Rightarrow \quad \int \cot v \, dv = -\int \frac{1}{x} dx$

$\Rightarrow \quad \log|\sin v| = -\log|x| + \log C$

$\Rightarrow \quad \log|\sin v| = \log\left|\frac{C}{x}\right|$

$\Rightarrow \quad x \sin\frac{y}{x} = C$ is the required solution.

30. On plotting the inequations, we get shaded portion represents feasible solution.



Possible points for maximum Z are $A(400, 0)$, $B(400, 200)$, $C(200, 600)$, $D(100, 700)$, $E(0, 700)$

Points	$Z = 8x + 6y$	Values
$A(400, 0)$	3200 + 0	3200
$B(400, 200)$	3200 + 1200	4400
$C(200, 600)$	1600 + 3600	5200
$D(100, 700)$	800 + 4200	5000
$E(0, 700)$	0 + 4200	4200

← Maximum (for $C(200, 600)$)

Z is maximum for $C(200, 600)$, i.e. $x = 200$, $y = 600$

31. Consider $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

Let $$\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+3)} \qquad ...(i)$$

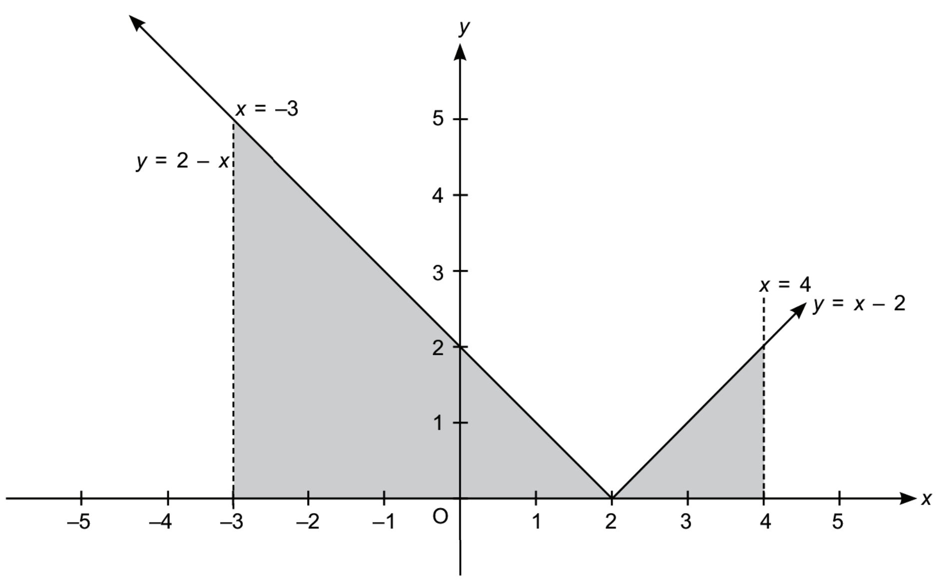
$$\Rightarrow \quad x^2 + 1 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$
$$= A(x^2 + 2x - 3) + B(x + 3) + C(x^2 - 2x + 1)$$
$$= x^2(A + C) + x(2A + B - 2C) + (-3A + 3B + C)$$

Comparing coefficients

$$A + C = 1$$
$$2A + B - 2C = 0 \Rightarrow 2A + B + 2A - 2 = 0 \Rightarrow 4A + B = 2$$
$$-3A + 3B + C = 1 \Rightarrow -3A + 3B + 1 - A = 1 \Rightarrow -4A + 3B = 0$$
$$\Rightarrow B = \frac{1}{2}, A = \frac{3}{8}, C = \frac{5}{8}$$

$\therefore$ From (i), $$\int \frac{x^2+1}{(x-1)^2(x+3)}dx = \frac{3}{8}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{1}{(x-1)^2}dx + \frac{5}{8}\int \frac{1}{x+3}dx$$
$$= \frac{3}{8}\log|x-1| - \frac{1}{2(x-1)} + \frac{5}{8}\log|x+3| + C$$

32. Graph of $y = |x - 2|$



$$\text{Area} = \int_{-3}^{4} y\,dx = \int_{-3}^{2} (2-x)\,dx + \int_{2}^{4} (x-2)\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_{-3}^{2} + \left[\frac{x^2}{2} - 2x\right]_{2}^{4}$$

$$= (4-2) - \left(-6 - \frac{9}{2}\right) + (8-8) - (2-4) = 2 + \frac{21}{2} + 0 + 2 = 4 + \frac{21}{2} = \frac{29}{2} \text{ sq units}$$

33. $R = \{(a, b) : |a - b| \text{ is divisible by 5 and } a, b \in Z\}$

For reflexive: $(a, a) \in R \Rightarrow |a - a| = 0$ is divisible by 5, true for all $a \in Z$.

Hence, relation R is reflexive.

For symmetric: Let $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 5

$\Rightarrow |b - a|$ is divisible by 5

$\Rightarrow (b, a) \in R$ for $a, b \in Z$.

Hence, relation R is symmetric.

For transitive: Let $(a, b) \in R$ and $(b, c) \in R$, for $a, b, c \in Z$

$\Rightarrow |a - b|$ is divisible by 5 and $|b - c|$ is divisible by 5.

$\Rightarrow a - b$ is divisible by 5 and $b - c$ is divisible by 5.

$\Rightarrow (a - b) + (b - c) = a - c$ is divisible by 5

$\Rightarrow |a - c|$ is divisible by 5 [$\because$ if numbers are divisible by 5, then their sum is also divisible by 5]

$\Rightarrow (a, c) \in R$

As $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$

Hence, R is transitive.

As relation R is reflexive, symmetric and transitive. Hence, relation R is an equivalence relation.

OR

We have $R_1 \cap R_2 = \{(a, b) \mid (a, b) \in R_1 \text{ and } (a, b) \in R_2\}$

For reflexive: Let $a \in A$, then $(a, a) \in R_1$ and $(a, a) \in R_2$

$\Rightarrow (a, a) \in R_1 \cap R_2$. Hence, reflexive.

For symmetric: Let $(a, b) \in R_1 \cap R_2$

$\Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2$

$\Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2$ (as R_1 and R_2 are equivalence relations)

$\Rightarrow (b, a) \in R_1 \cap R_2$.

Hence, symmetric.

For transitive: Let $(a, b), (b, c) \in R_1 \cap R_2$

$\Rightarrow (a, b), (b, c) \in R_1$ and $(a, b), (b, c) \in R_2$

$\Rightarrow (a, c) \in R_1$ and $(a, c) \in R_2$ ($\because$ R_1 and R_2 are equivalence relations)

$\Rightarrow (a, c) \in R_1 \cap R_2$.

As $(a, b), (b, c) \in R_1 \cap R_2$

$\Rightarrow (a, c) \in R_1 \cap R_2$. So, transitive

Since relation $R_1 \cap R_2$ is reflexive, symmetric and transitive. Hence, $R_1 \cap R_2$ is an equivalence relation.

34. Lines are $\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

and $\vec{r} = (5\hat{j} - 2\hat{k}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

$$\cos\theta = \frac{1\times 3 + 2\times 2 + 2\times 6}{\sqrt{1+4+4}\,\sqrt{9+4+36}} = \frac{3+4+12}{3\times 7} = \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Lines passing through point (1, –3, 2) is $\vec{r} = (\hat{i} - 3\hat{j} + 2\hat{k}) + t\,\vec{n}$

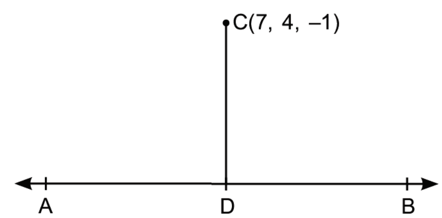
where $$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = 8\hat{i} - 4\hat{k}$$

$\therefore$ $\vec{r} = (\hat{i} - 3\hat{j} + 2\hat{k}) + t(8\hat{i} - 4\hat{k})$

OR

DR's of line passing through $A(0, 6, -9)$ and $B(-3, -6, 3)$ are 0 + 3, 6 + 6, –9 – 3 i.e. 3, 12, –12 or 1, 4, –4

Line is $\frac{x-0}{1} = \frac{y-6}{4} = \frac{z+9}{-4} = \lambda$(say)



General point on line is

D(λ, $4\lambda + 6$, $-4\lambda - 9$) ...(*i*)

DR's of CD are

$\lambda - 7$, $4\lambda + 6 - 4$, $-4\lambda - 9 + 1$

i.e. $\lambda - 7$, $4\lambda + 2$, $-4\lambda - 8$

If CD is perpendicular to *AB*, then

$1(\lambda - 7) + 4(4\lambda + 2) - 4(-4\lambda - 8) = 0$

$\Rightarrow$ $\lambda - 7 + 16\lambda + 8 + 16\lambda + 32 = 0$

$\Rightarrow$ $33\lambda + 33 = 0 \Rightarrow \lambda = -1$

Substitute in (*i*) foot of perpendicular is $D(-1, 2, -5)$

35. $$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

Matrix formed by cofactors in $|A|$

$A_{11} = 0,\quad A_{12} = 2,\quad A_{13} = 1$

$A_{21} = -1,\quad A_{22} = -9,\quad A_{23} = -5$

$A_{31} = 2,\quad A_{32} = 23,\quad A_{33} = 13$

$$\text{Adj } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{ adj } A = -\frac{1}{1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \quad ...(i)$$

Consider equations,

$2x - 3y + 5z = 16$

$3x + 2y - 4z = -4$

$x + y - 2z = -3$

Corresponding matrix equation is

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix}$$

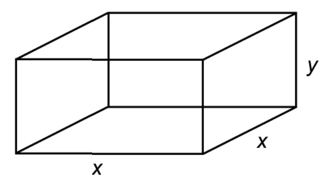
i.e., $AX = B$, Its solution is $X = A^{-1}B$

$$X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix} \qquad \text{[From } (i)\text{]}$$

or
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 4 + 6 \\ -32 - 36 + 69 \\ -16 - 20 + 39 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 2, y = 1, z = 3$

36. (*i*) $x^2 + 4xy = 900$



(*ii*) $V = x^2y = x^2\left(\dfrac{900 - x^2}{4x}\right)$

$V = \dfrac{1}{4}(900x - x^3)$

(*iii*) $\dfrac{dV}{dx} = \dfrac{1}{4}(900 - 3x^2)$

For critical point, $\dfrac{dV}{dx} = 0$

$\Rightarrow$ $900 - 3x^2 = 0 \Rightarrow x^2 = 300 \Rightarrow x = 10\sqrt{3}$

OR

(*iii*) $\dfrac{d^2V}{dx^2} = \dfrac{1}{4}(-6x) < 0$ for $x = 10\sqrt{3}$

From (*i*) $300 + 40\sqrt{3}\,y = 900 \Rightarrow 40\sqrt{3}\,y = 600$

$\Rightarrow$ $y = \dfrac{15}{\sqrt{3}} = 5\sqrt{3}$ or $y = \dfrac{x}{2}$

37. Profit function is $P(x) = -5x^2 + 120x + 37500$

(*i*) $P'(x) = -10x + 120$

(*ii*) For critical point $P'(x) = 0$

$\Rightarrow$ $-10x + 120 = 0 \Rightarrow x = 12$

(*iii*) $P''(x) = -10 < 0$ for $x = 12$

Maximum profit $= P(12) = (-5) \times 144 + 120 \times 12 + 37500$

$= -720 + 1440 + 37500 = 38220$

OR

(*iii*) Marginal profit for $x = 10$

$P'(10) = -10 \times 10 + 120 = -100 + 120 = 20$

38. Let A, B, C be fire extinguisher vans

$$P(A) = P(B) = P(C) = \frac{3}{4}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{1}{4}$$

(*i*) $P(\text{neither of them is available}) = P(\bar{A}\,\bar{B}\,\bar{C}) = \dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4} = \dfrac{1}{64}$

(*ii*) $P(\text{at most two vans are available}) = 1 - P(\text{all three}) = 1 - P(ABC)$

$= 1 - \dfrac{3}{4} \times \dfrac{3}{4} \times \dfrac{3}{4} = 1 - \dfrac{27}{64} = \dfrac{37}{64}$