

# Solutions to RMT–DS2/Set-1

1. (c) 108, as HCF is a factor of LCM.

2. (a) 
$$y = (x - 1)(x - 2)$$

$$= x^2 - 3x + 2$$

3. (d) If the lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  are parallel, then  $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ . Also, if lines are parallel, then system of equations has no solution.

4. (a) We have, 
$$a = 3, b = -5, c = 7$$

Now, 
$$D = 25 - 4 \times 3 \times 7 = -59 < 0 \quad (\because D = b^2 - 4ac)$$

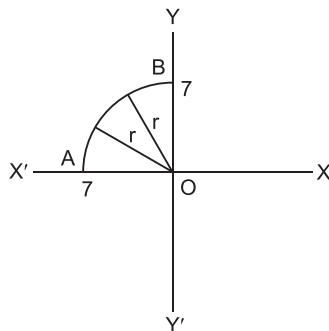
$\therefore$  No real roots

5. (c) 
$$a_7 + a'_7 = (5 + 12) + (5 - 12) = 10 \quad [\because n^{\text{th}} \text{ term} = a + (n - 1)d]$$

6. (b) 
$$\text{Distance} = \sqrt{(3 - 0)^2 + (-4 - 0)^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}$$

7. (d) Infinitely many, as any point on arc AB of a circle of radius 7 units will be the required point.



8. (a) As, 
$$\triangle ABO \sim \triangle CDO \Rightarrow \frac{AB}{CD} = \frac{BO}{DO}$$

$$\Rightarrow \frac{9}{3} = \frac{4 + x}{x} \Rightarrow 3x = 4 + x$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

9. (d) As, tangents drawn from an external point to a circle are always equally inclined to line segment joining external point to the centre.

So, 
$$\angle OPA = \frac{1}{2} \angle APB = \frac{1}{2} \times 70^\circ = 35^\circ$$

Also, tangent to a circle is  $\perp$  to its radius at the point of contact, then  $\angle OAP = 90^\circ$ .

Now, 
$$\angle AOP = 90^\circ - 35^\circ = 55^\circ$$

10. (a) Tangents drawn from an external point to a circle are equal. Then,

$$CR = CQ, AR = AP, BQ = BP \quad \dots(i)$$

Perimeter of  $\triangle ABC = 40$  cm

$$\Rightarrow CA + CB + AB = 40$$

$$\Rightarrow CR - AR + CQ - BQ + AP + BP = 40$$

$$\Rightarrow CR - AR + CR - BP + AR + BP = 40$$

$$\Rightarrow CR = 20 \text{ cm}$$

11. (a) As,

$$\sin \theta = \frac{4}{a}$$

Now,

$$\begin{aligned} \tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} \\ &= \frac{\frac{4}{a}}{\sqrt{1 - \frac{16}{a^2}}} = \frac{4}{\sqrt{a^2 - 16}} \end{aligned}$$

12. (d)

$$\begin{aligned} x^2 - 9y^2 &= 9\sec^2 A - 9\tan^2 A \\ &= 9(\sec^2 A - \tan^2 A) = 9 \end{aligned}$$

13. (b) As,

$$\frac{AB}{BC} = \cot 60^\circ \Rightarrow \frac{AB}{15} = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

$\Rightarrow$

$$AB = \frac{15}{\sqrt{3}} = 5\sqrt{3} \text{ m}$$

14. (c)

$$\pi r^2 = 4 \times 2\pi r \Rightarrow r = 8$$

15. (c) As, 18 sectors will be formed by 9 diameters of a circle.

Now, central angle of each sector,  $\theta = \frac{360^\circ}{18} = 20^\circ$

$$\begin{aligned} \therefore \text{Area of each sector} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{20^\circ}{360^\circ} \times \pi \times (12)^2 \\ &= 8\pi \text{ cm}^2 \end{aligned}$$

16. (a)

$$\begin{aligned} \text{Probability} &= \frac{\frac{1}{2}\pi(5)^2}{16 \times 10} = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{160} \\ &= \frac{55}{224} \end{aligned}$$

17. (d) After reshuffling, 49 cards are left in the deck that includes 2 red queens.

$$\therefore P(\text{a red queen}) = \frac{2}{49}$$

18. (c)

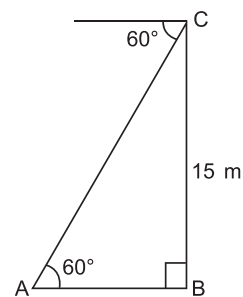
C.I.	<i>f</i>	<i>c.f.</i>
0 – 10	3	3
10 – 20	8	11
20 – 30	11	22
30 – 40	9	31
40 – 50	7	38

← Median class and Modal class

Upper limit of modal class – Lower limit of median class = 30 – 20 = 10

19. (d) Assertion (A) is false, but Reason (R) is true.

20. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).



21. Let, if possible,  $2 + \sqrt{2}$  is a rational number. Then,

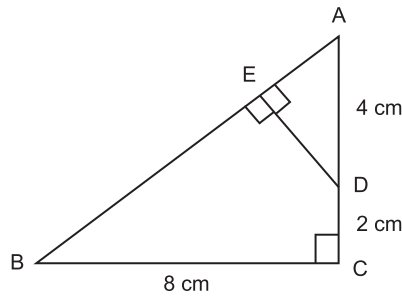
$$2 + \sqrt{2} = \frac{a}{b} \quad (\text{where } a \text{ and } b \text{ are coprime integers and } b \neq 0)$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 2$$

$$\Rightarrow \sqrt{2} = \frac{a-2b}{b} \quad \dots(i)$$

Since  $a$  and  $b$  are integers so, RHS of (i) is a rational number but LHS (i.e.  $\sqrt{2}$ ) is not a rational number. Hence, this is a contradiction. So our supposition is wrong. Hence  $2 + \sqrt{2}$  is an irrational number.

22.



Consider triangles ACB and AED,

$$\angle C = \angle AED \quad [90^\circ \text{ each}]$$

$$\angle A = \angle A \quad [\text{common}]$$

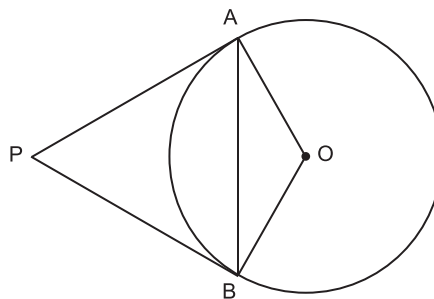
$$\therefore \Delta ACB \sim \Delta AED \quad (\text{AA similarity})$$

$$\Rightarrow \frac{AC}{AE} = \frac{BC}{DE} = \frac{AB}{AD} \Rightarrow \frac{AE}{AC} = \frac{DE}{BC} = \frac{AD}{AB}$$

$$\Rightarrow \frac{AE}{6} = \frac{DE}{8} = \frac{4}{10} \quad [\because AB = \sqrt{64+36} = 10 \text{ cm}]$$

$$\Rightarrow AE = 2.4 \text{ cm, } DE = 3.2 \text{ cm}$$

23.



We have,  $\angle APB = 60^\circ$

Tangents drawn from an external point to a circle are equal.

$$\text{So, } PB = PA \Rightarrow \angle PAB = \angle PBA = 60^\circ$$

$$\text{Also, } \angle PAO = 90^\circ \quad (\text{tangent to circle is } \perp \text{ to its radius at point of contact})$$

$$\therefore \angle OAB = 90^\circ - 60^\circ = 30^\circ$$

$$\text{Now, } \angle OBA = \angle OAB \quad (\because OA = OB)$$

$$\Rightarrow \angle OBA = 30^\circ$$

$$24. \quad \sin(A + B - C) = \frac{1}{2}$$

$$\Rightarrow A + B - C = 30^\circ \quad \dots(i)$$

$$\text{and } \cos(B + C - A) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad B + C - A = 45^\circ \quad \dots(ii)$$

$$\text{Also,} \quad A + B + C = 180^\circ \quad \dots(iii)$$

$$\text{From (i),} \quad A + B = 30^\circ + C$$

$\therefore$  from (iii), we get

$$30^\circ + 2C = 180^\circ \quad [\because A + B = 30^\circ + C]$$

$$\Rightarrow \quad C = 75^\circ$$

Substituting  $C = 75^\circ$  in (ii) and (iii), we get

$$-A + B = -30^\circ \quad \dots(iv)$$

$$A + B = 105^\circ \quad \dots(v)$$

Solving (iv) and (v), we get

$$B = 37.5^\circ \text{ and } A = 67.5^\circ$$

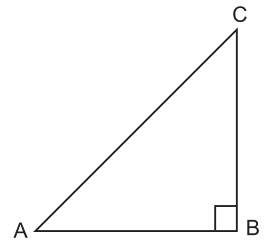
**OR**

$$AB : AC = 1 : \sqrt{2}$$

$$\frac{AB}{AC} = \frac{1}{\sqrt{2}} = \cos A$$

$$\Rightarrow \quad A = 45^\circ$$

$$\therefore \quad \frac{2 \tan A}{1 + \tan^2 A} = \frac{2 \tan 45^\circ}{1 + \tan^2 45^\circ} = \frac{2}{2} = 1. \quad (\because \tan 45^\circ = 1)$$



25. Perimeter of shaded region = circumference of semicircle on diameter DC + circumference of semicircle on diameter AB + AD + BC

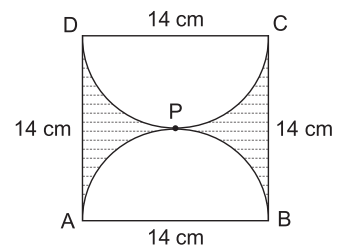
$$= \pi r + \pi r + AD + BC$$

$$= \pi(7) + \pi(7) + 14 + 14$$

$$= 14\pi + 28$$

$$= 14 \times \frac{22}{7} + 28$$

$$= 44 + 28 = 72 \text{ cm.}$$



**OR**

As, QR is diameter, then

$$\angle QPR = 90^\circ$$

[Angle in a semicircle is right angle]

Now,

$$QR^2 = PR^2 + PQ^2 \quad (\text{Pythagoras Theorem})$$

$\Rightarrow$

$$QR^2 = (24)^2 + (7)^2 = 576 + 49 = 625$$

$\Rightarrow$

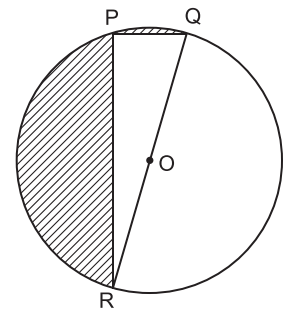
$$QR = 25 \text{ cm}$$

$$\text{Shaded area} = \text{ar}(\text{semicircle}) - \text{ar}(\Delta PQR) = \frac{1}{2} \pi r^2 - \frac{1}{2} \times PQ \times PR$$

$$= \frac{1}{2} \pi \left( \frac{25}{2} \right)^2 - \frac{1}{2} \times 24 \times 7$$

$$= \frac{625}{8} \pi - 84 = \frac{625}{8} \times \frac{22}{7} - 84$$

$$= 245.54 - 84 = 161.54 \text{ cm}^2$$



26. English	Social Science	Hindi
192	240	168

We find HCF of 192, 240 and 168.

$$192 = 2^6 \times 3$$

$$240 = 2^4 \times 3 \times 5$$

$$168 = 2^3 \times 3 \times 7$$

$$\text{HCF} = 2^3 \times 3 = 24$$



In  $\Delta PBO$ ,  $OP^2 = OB^2 + PB^2$   
 $\Rightarrow 676 = (4)^2 + PB^2 \Rightarrow PB^2 = 676 - 16$   
 $\Rightarrow PB^2 = 660 \Rightarrow PB = 2\sqrt{165}$  cm

**OR**

**Given:** PA and PB are tangents drawn to a circle with centre O, from an external point P, with points of contact A and B respectively.

**To Prove:** PA = PB

**Construction:** Join OA, OB and OP.

**Proof:** As tangent to a circle is  $\perp$  to its radius at the point of contact.

So,  $\angle OAP = \angle OBP = 90^\circ$

Now, in  $\Delta OAP$  and  $\Delta OBP$ ,

$\angle OAP = \angle OBP$  (90° each)

$OP = OP$

(common)

$OA = OB$

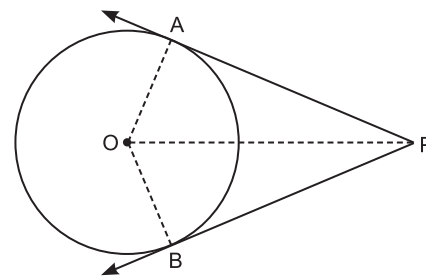
(radii)

$\Rightarrow \Delta OAP \cong \Delta OBP$

(RHS)

$\Rightarrow PA = PB$

(CPCT)



30. As,  $15\cot^2 \theta + 4\operatorname{cosec}^2 \theta = 23$   
 $\Rightarrow 15(\operatorname{cosec}^2 \theta - 1) + 4\operatorname{cosec}^2 \theta = 23$   
 $\Rightarrow 15\operatorname{cosec}^2 \theta - 15 + 4\operatorname{cosec}^2 \theta = 23$   
 $\Rightarrow 19\operatorname{cosec}^2 \theta = 38$   
 $\Rightarrow \operatorname{cosec}^2 \theta = 2$   
 $\Rightarrow 1 + \cot^2 \theta = 2$   
 $\Rightarrow \cot^2 \theta = 1$   
 $\Rightarrow \tan^2 \theta = \frac{1}{\cot^2 \theta} = 1$

Now,  $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$

31.

% of female Teachers (C.I)	$x_i$	No. of States/UT's ( $f_i$ )	$u_i = \frac{x_i - A}{h} = \frac{x_i - 50}{10}$	$f_i u_i$
15 - 25	20	6	-3	-18
25 - 35	30	11	-2	-22
35 - 45	40	7	-1	-7
45 - 55	<span style="border: 1px solid black; padding: 2px;">50</span> = A	4	0	0
55 - 65	60	4	1	4
65 - 75	70	2	2	4
75 - 85	80	1	3	3
$\Sigma f_i = 35$				$\Sigma f_i u_i = -36$

We have, A = 50 and h = 10

$$\begin{aligned} \text{Mean} &= A + \frac{\sum f_i u_i}{\sum f_i} \times h = 50 + \frac{(-36)}{35} \times 10 \\ &= 50 - \frac{72}{7} = \frac{278}{7} \approx 39.71 \end{aligned}$$

Mean percentage of female teachers is 39.71% (approx.).

32. Let the first part =  $x$

Then, the second part =  $15 - x$

ATQ,

$$x^2 + (15 - x)^2 = 197$$

$$\Rightarrow x^2 + 225 + x^2 - 30x = 197$$

$$\Rightarrow 2x^2 - 30x + 28 = 0$$

$$\Rightarrow 2[x^2 - 15x + 14] = 0$$

$$\Rightarrow x^2 - 15x + 14 = 0$$

$$\Rightarrow x^2 - 14x - x + 14 = 0$$

$$\Rightarrow x(x - 14) - 1(x - 14) = 0$$

$$\Rightarrow (x - 1)(x - 14) = 0$$

$$\Rightarrow x - 1 = 0 \quad \text{or} \quad x - 14 = 0$$

$$\Rightarrow x = 1 \quad \text{or} \quad x = 14$$

when  $x = 1$ ,  $15 - x = 14$

when  $x = 14$ ,  $15 - x = 1$

So, the required parts are 14 and 1.

**OR**

Let the width =  $x$  cm

Then, the length =  $(x + 10)$  cm

Now, area of rectangle =  $39 \text{ cm}^2$

$$\Rightarrow \text{length} \times \text{breadth} = 39$$

$$\Rightarrow (x + 10) \times x = 39$$

$$\Rightarrow x^2 + 10x - 39 = 0$$

$$\Rightarrow x^2 + 13x - 3x - 39 = 0$$

$$\Rightarrow x(x + 13) - 3(x + 13) = 0$$

$$\Rightarrow (x - 3)(x + 13) = 0$$

$$\Rightarrow x - 3 = 0 \quad \text{or} \quad x + 13 = 0$$

$$\Rightarrow x = 3 \quad \text{or} \quad x = -13 \quad (\text{Rejected})$$

$\therefore$  Width =  $x$  cm = 3 cm and length =  $(x + 10)$  cm = 13 cm

$$\begin{aligned} \text{Perimeter} &= 2(l + b) \\ &= 2(13 + 3) \\ &= 2 \times 16 = 32 \text{ cm} \end{aligned}$$

33. **Given:** In  $\triangle ABC$ ,  $DE \parallel AB$  and  $BD \parallel EF$ .

**To Prove:**  $DC^2 = CF \times AC$

**Proof:** In  $\triangle ABC$ ,  $DE \parallel AB$ , then

$$\frac{DC}{DA} = \frac{EC}{EB} \quad [\text{BPT}]$$

$$\Rightarrow \frac{DA}{DC} = \frac{EB}{EC}$$

$$\Rightarrow \frac{DA}{DC} + 1 = \frac{EB}{EC} + 1 \quad [\text{Adding '1' on both sides}]$$

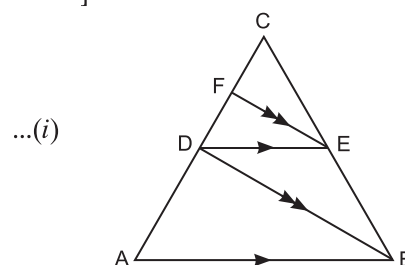
$$\Rightarrow \frac{DA + DC}{DC} = \frac{EB + EC}{EC}$$

$$\Rightarrow \frac{AC}{DC} = \frac{BC}{EC}$$

In  $\triangle CDB$ ,  $EF \parallel BD$ , then

$$\frac{CF}{FD} = \frac{CE}{EB}$$

$$\Rightarrow \frac{FD}{CF} = \frac{EB}{CE}$$



...(i)

$$\begin{aligned} \Rightarrow \quad & \frac{FD}{CF} + 1 = \frac{EB}{CE} + 1 && \text{[Adding '1' on both sides]} \\ \Rightarrow \quad & \frac{FD + CF}{CF} = \frac{EB + CE}{CE} \\ \Rightarrow \quad & \frac{DC}{CF} = \frac{BC}{CE} && \dots(ii) \end{aligned}$$

∴ From (i) and (ii), we get

$$\frac{AC}{DC} = \frac{DC}{CF} \Rightarrow DC^2 = CF \times AC$$

**34. For cylinder:**

$$\begin{aligned} \text{diameter of base} &= 4.3 \text{ m} \Rightarrow \text{radius, } r = \frac{4.3}{2} \text{ m} \\ \text{height, } h &= 3.8 \text{ m} \end{aligned}$$

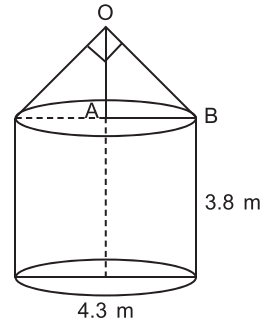
**For cone:**

$$\text{diameter of base} = 4.3 \text{ m} \Rightarrow r = \frac{4.3}{2} \text{ m}$$

$$\text{In } \triangle AOB, \angle AOB = \frac{1}{2} \text{ of vertical angle of cone} = \frac{1}{2} \times 90^\circ = 45^\circ$$

$$\text{Now, } \frac{AB}{OA} = \tan 45^\circ \Rightarrow OA = \frac{4.3}{2} \text{ m} = h_1$$

$$\text{Now, } \frac{OB}{AB} = \operatorname{cosec} 45^\circ \Rightarrow OB = \sqrt{2} \times \frac{4.3}{2} \text{ m} = l$$



$$\begin{aligned} \text{Surface area of building} &= 2\pi rh + \pi rl \\ &= 2\pi \left(\frac{4.3}{2}\right) \times 3.8 + \pi \times \left(\frac{4.3}{2}\right) \times \sqrt{2} \times \frac{4.3}{2} \\ &= 4.3\pi \left[3.8 + \frac{4.3\sqrt{2}}{4}\right] \\ &= 4.3 \times 3.14 \left[3.8 + \frac{4.3 \times 1.41}{4}\right] \\ &= 13.502 \times \frac{21.263}{4} \\ &= 71.77 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume of building} &= \pi r^2 h + \frac{1}{3} \pi r^2 h_1 \\ &= \pi \left(\frac{4.3}{2}\right)^2 \times 3.8 + \frac{1}{3} \pi \left(\frac{4.3}{2}\right)^2 \times \frac{4.3}{2} \\ &= \pi \left(\frac{4.3}{2}\right)^2 \left[3.8 + \frac{4.3}{6}\right] \\ &= 3.14(2.15)^2 \left[3.8 + \frac{4.3}{6}\right] = 65.56 \text{ m}^3 \text{ (approx.)} \end{aligned}$$

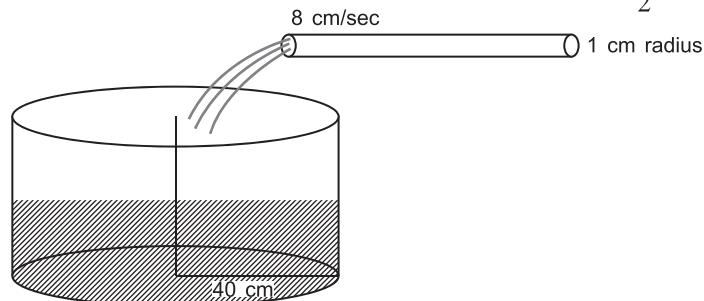
**OR**

**For Pipe:**

$$\text{radius} = 1 \text{ cm}$$

$$\text{In 1 sec water flows} = 8 \text{ cm}$$

$$\text{In 1800 sec water flows} = 8 \times 1800 \text{ cm} \quad \left[ \because \frac{1}{2} \text{ hr} = 1800 \text{ sec} \right]$$





**For cylinder:**

Radius of base = 40 cm

Let

height raised =  $h$  cm

ATQ, Volume of water that flows out in  $\frac{1}{2}$  hr from pipe = volume of water raised in cylindrical tank in  $\frac{1}{2}$  hr

$$\pi(1)^2 \times 8 \times 1800 = \pi(40)^2 h$$

$$\Rightarrow h = \frac{8 \times 1800}{1600} = 9 \text{ cm}$$

$\therefore$  Height raised = 9 cm

35.

C.I.	$f$
0 – 10	7
10 – 20	14
20 – 30	13
30 – 40	12
40 – 50	$p$
50 – 60	18
60 – 70	15
70 – 80	8

← Modal class

Mode = 48

Modal class: 40 – 50

$$l = 40, f_1 = p, f_0 = 12, f_2 = 18, h = 10$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\Rightarrow 48 = 40 + \frac{p - 12}{2p - 12 - 18} \times 10$$

$$\Rightarrow 8(2p - 30) = 10(p - 12)$$

$$\Rightarrow 16p - 240 = 10p - 120$$

$$\Rightarrow 6p = 120 \Rightarrow p = 20$$

36. We have  $a = 50, d = 5$

$$(i) a_8 = 50 + (8 - 1)5 = 50 + 35 = 85$$

$$(ii) a_n = 80 \Rightarrow a + (n - 1)d = 80$$

$$\Rightarrow 50 + (n - 1)5 = 80$$

$$\Rightarrow (n - 1)5 = 30 \Rightarrow n - 1 = 6 \Rightarrow n = 7$$

So, she will take 80 push-ups on 7<sup>th</sup> day.

$$(iii) S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore S_{10} = \frac{10}{2}[2 \times 50 + (10 - 1)5]$$

$$= 5[100 + 45] = 5 \times 145 = 725$$

**OR**

$$a_{15} = 50 + (15 - 1)5 = 50 + 70 = 120$$

37. (i) Mid point is  $\left(\frac{20+100}{2}, \frac{50+40}{2}\right)$  i.e., (60, 45).

(ii) Distance between locations of seating guests G and seating parents Q =  $\sqrt{(100-50)^2 + (10-10)^2}$   
 $= \sqrt{50^2 + 0} = \sqrt{50^2} = 50$  units

**OR**

$$BG = \sqrt{(50-60)^2 + (10-30)^2} = \sqrt{100 + 400} = 10\sqrt{5} \text{ units}$$

(iii) X(40, 60), Y(70, 60), Z(90, 60) are the given points

$$\text{Distance between X and Y} = \sqrt{(70-40)^2 + (60-60)^2} = 30 \text{ units}$$

$$\text{Distance between Y and Z} = \sqrt{(90-70)^2 + (60-60)^2} = 20 \text{ units}$$

$$\text{Distance between X and Z} = \sqrt{(90-40)^2 + (60-60)^2} = 50 \text{ units}$$

As,  $XY + YZ = XZ$

Hence seating place of X, Y, Z are in same straight line.

38. (i) In  $\triangle ABD$ ,  $\frac{BD}{AB} = \cot 30^\circ$

$\Rightarrow BD = 80\sqrt{3}$  m

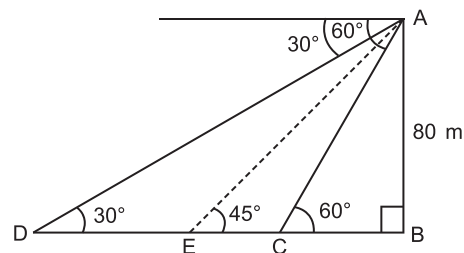
Distance =  $80\sqrt{3}$  m

(ii) We have,  $\angle ACB = 60^\circ$

In  $\triangle ABC$ ,  $\frac{BC}{AB} = \cot 60^\circ$

$\Rightarrow BC = 80 \times \frac{1}{\sqrt{3}} = \frac{80}{\sqrt{3}} = \frac{80\sqrt{3}}{3}$  m

$\therefore$  Distance =  $\frac{80\sqrt{3}}{3}$  m



**OR**

$$DC = DB - BC = 80\sqrt{3} - \frac{80}{\sqrt{3}}$$

$$= 80\left(\frac{3-1}{\sqrt{3}}\right) = \frac{160}{\sqrt{3}} = \frac{160\sqrt{3}}{3} \text{ m}$$

So,  $\frac{160\sqrt{3}}{3}$  m distance is covered in 5 minutes.

Now,  $\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{160\sqrt{3}}{3 \times 5} = \frac{32\sqrt{3}}{3}$  m/min

(iii) When angle of depression is  $45^\circ$ , ship is at E and so  $\angle AEB = 45^\circ$

In  $\triangle ABE$ ,  $\frac{BE}{AB} = \cot 45^\circ \Rightarrow BE = AB = 80$  m [ $\because \cot 45^\circ = 1$ ]

So, distance = 80 m