Solutions to RMM–DS2/Set-1

1. (*b*)

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$3A^3 = 3IA = 3A = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$$

2. (*d*)

$$|A| = 0 - 1(-3) + 2(-9) = -15$$

$$|A^{-1}| = \frac{1}{|A|} = -\frac{1}{15}$$

3. (*a*)

$$2x^2 - 12 = 4$$

$$\Rightarrow \quad x^2 = 8$$

$$\Rightarrow \quad x = \pm 2\sqrt{2}$$

4. (*b*)

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{k}{3}$$

$$\Rightarrow \quad \lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{4 \cdot \frac{x^2}{4}} = \frac{k}{3}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{k}{3}$$

$$\Rightarrow \quad k = \frac{3}{2}$$

5. (*d*) Line through the points (3, 1, –2) and (0, 2, 4) is

$$\frac{x - 0}{-3} = \frac{y - 2}{1} = \frac{z - 4}{6}$$

DR's are –3, 1, 6

If line with DR's a, b, c makes acute angle with y-axis then $b > 0$

$\therefore$ DR's are $\langle -3, 1, 6 \rangle$

6. (*b*),

$$(y - px)^2 = a^2p^2 + b^2$$

$$\Rightarrow \quad (x^2 - a^2)p^2 - 2xyp - b^2 + y^2 = 0$$

$$\text{Degree} = 2, \text{ as } p^2 = \left(\frac{dy}{dx}\right)^2$$

7. (*d*),

$$2Z_{(4, 2)} = Z_{(0, 5)}$$

$$\Rightarrow \quad 2(4p + 2q) = 5q$$

$$\Rightarrow \quad 8p = q$$

8. (*c*) $|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow$ $(\vec{a}+\vec{b})^2 = (\vec{a}-\vec{b})^2$

$\Rightarrow$ $\vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b} = \vec{a}^2+\vec{b}^2-2\vec{a}\cdot\vec{b}$

$\Rightarrow$ $4\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a}\perp\vec{b}$

9. (*a*) $$I = \int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}dx \quad ...(i)$$

Using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$I = \int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_1^3 1\cdot dx = [x]_1^3 = 2$$

$$I = 1$$

10. (*b*) $M_{12} = 1, A_{23} = -(4) = -4$

$3M_{12} - 2A_{23} = 3 + 8 = 11$

11. (*b*) For (3, 5), $x - y \leq 0$ is true

12. (*d*) $$\text{Vectors} = \pm 5\left(\frac{2\hat{i}-\hat{j}+2\hat{k}}{\sqrt{4+1+4}}\right)$$

$$= \pm\left(\frac{10}{3}\hat{i} - \frac{5}{3}\hat{j} + \frac{10}{3}\hat{k}\right)$$

13. (*b*) $A\cdot\text{Adj } A = |A|\cdot I$ and $A^{-1} = \frac{1}{|A|}\text{Adj } A$

$\Rightarrow$ $|A| = -4$

$\Rightarrow$ $K = -4$

$\Rightarrow$ $16K = -64$

14. (*b*)

15. (*c*) $$\int\frac{dx}{\sqrt{1-x^2}} + \int\frac{dy}{\sqrt{1-y^2}} = \int 0dx$$

$\Rightarrow$ $\sin^{-1}x + \sin^{-1}y = C$

16. (*b*) $2\hat{i} - \hat{j} + 3\hat{k} = t(4\hat{i} - 5\lambda\hat{j} + 6\hat{k})$, where t is a scalar.

$\Rightarrow$ $2 = 4t, -1 = -5\lambda t, 3 = 6t$

$\Rightarrow$ $t = \frac{1}{2}$

$\Rightarrow$ $-1 = \frac{-5\lambda}{2}$

$\Rightarrow$ $\lambda = \frac{2}{5}$

17. (*c*) $f'(x) = 1 + \sin x > 0$ for $x \in R$. $\left\{1 + \sin x = \left(\cos\frac{x}{2} + \frac{\sin x}{2}\right)^2 \geq 0\right\}$

As $0 \leq 1 + \sin x \leq 2$

$\therefore$ Always increasing.

18. (*d*) Line is $\frac{x-2}{-2} = \frac{y+3}{3} = \frac{z-5}{0}$

DR's: –2, 3, 0.

19. (*d*) Assertion is false, as function neither increases nor decreases in [0, 5].

Reason is true.

Hence, (A) is false but (R) is true.

20. (*d*) Assertion is false as '*f*' is not a function on *R*.

Reason is true.

Hence, (A) is false but (R) is true.

21. Principal value of $\sin^{-1} x$ lies between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \quad \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1} x = \frac{\pi}{2}, \sin^{-1} y = \frac{\pi}{2}, \sin^{-1} z = \frac{\pi}{2}$$

$$\Rightarrow \quad x = \sin\frac{\pi}{2}, y = \sin\frac{\pi}{2}, z = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad x = 1, y = 1, z = 1$$

$$3x - y + 2z = 3 - 1 + 2 = 4$$

OR

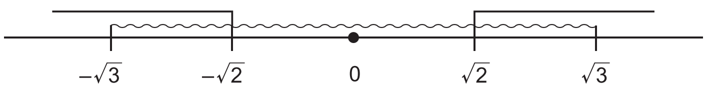
For domain $-1 \leq 2x^2 - 5 \leq 1$

$$\Rightarrow \quad 4 \leq 2x^2 \leq 6$$

$$\Rightarrow \quad 2 \leq x^2 \leq 3$$

$$\Rightarrow \quad x^2 \geq 2 \text{ and } x^2 \leq 3$$

$$x^2 \geq \sqrt{2} \Rightarrow x \leq -\sqrt{2} \text{ or } x \geq 0 \text{ ; } x^2 \leq 3 \Rightarrow -\sqrt{3} \leq x \leq \sqrt{3}$$



$$\Rightarrow \quad -\sqrt{3} \leq x \leq -\sqrt{2} \text{ or } \sqrt{2} \leq x \leq \sqrt{3}.$$

22. $f'(x) = -2\sin\left(2x + \frac{\pi}{4}\right)$ ...(*i*)

Given $\frac{3\pi}{8} \leq x \leq \frac{5\pi}{8}$

$$\Rightarrow \quad \frac{3\pi}{4} \leq 2x \leq \frac{5\pi}{4}$$

$$\Rightarrow \quad \pi \leq 2x + \frac{\pi}{4} \leq \frac{3\pi}{2}$$

$$\Rightarrow \quad 2x + \frac{\pi}{4} \in \text{3rd quadrant}$$

$$\sin\left(2x + \frac{\pi}{4}\right) < 0$$

From (*i*), $f'(x) > 0 \Rightarrow$ function increases.

23. $$f'(x) = x \cdot \frac{1}{2\sqrt{1-x}}(-1) + \sqrt{1-x} = \frac{2-3x}{2\sqrt{1-x}}$$

For a point of local maximum or minimum

$$f'(x) = 0 \Rightarrow x = \frac{2}{3}$$

$$f''(x) = \frac{2\sqrt{1-x}\,(-3) + (2-3x)\left(1-x\right)^{\frac{-1}{2}}}{(2\sqrt{1-x})^2}$$

$f''\left(\frac{2}{3}\right) < 0.$

$\therefore$ Maximum at $x = \frac{2}{3}$ and $f\left(\frac{2}{3}\right) = \frac{2\sqrt{3}}{9}$.

OR

$$3y = ax^3 + 1$$

$$3\frac{dy}{dt} = 3ax^2\frac{dx}{dt}$$

Also $$\frac{dy}{dt} = 2\frac{dx}{dt}$$

$\Rightarrow$ $$6\frac{dx}{dt} = 3ax^2\frac{dx}{dt}$$

$\Rightarrow$ $$2 = ax^2$$

For $x = 1$, $$2 = a(1)^2 \Rightarrow a = 2$$

24. $$\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}dx = 2\int \sec^2 t\, dt$$
$$= 2\tan t + C = 2\tan\sqrt{x} + C$$

Let $\sqrt{x} = t$

$\Rightarrow$ $\frac{1}{2\sqrt{x}}dx = dt$

25. $$f'(x) = 3x^2 + 2x + 1$$

For critical points, $3x^2 + 2x + 1 = 0$

$$D = 4 - 12 < 0$$

No solution, as $f'(x) \neq 0$ for any x, no maximum or minimum value.

26. $$\int \frac{x}{x^3 + x^2 + x + 1}dx = \int \frac{x}{(x+1)(x^2+1)}dx$$

$$\frac{x}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \quad ...(i)$$

$\Rightarrow$ $$x = A(x^2+1) + (Bx+C)(x+1)$$
$$= Ax^2 + A + Bx^2 + Bx + Cx + C = x^2(A+B) + x(B+C) + (A+C)$$

Comparing the coefficients of x^2, x and constant terms, we get

$$A + B = 0, B + C = 1, A + C = 0 \Rightarrow A = -B = -C = -\frac{1}{2}$$

$\therefore$ From (i), we get

$$\int \frac{x}{x^3+x^2+x+1}dx = -\frac{1}{2}\int \frac{1}{x+1}dx + \int \frac{\frac{1}{2}x+\frac{1}{2}}{x^2+1}dx$$

$$= -\frac{1}{2}\int \frac{1}{x+1}dx + \frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

27. Let green signals be on D_1, D_2 and D_3.

$$P(D_1) = P(D_2) = P(D_3) = \frac{30}{100} = \frac{3}{10}$$

$\therefore$ Probability of green signal on two consecutive days $= P(D_1\ D_2\ \overline{D}_3) + \mathrm{P}(\overline{D}_1\ D_2\ D_3)$

$$= \frac{3}{10}\times\frac{3}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{3}{10}\times\frac{3}{10} = \frac{126}{1000} = 0.0126$$

28. $$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}dx = \int \frac{2\cos^2 x - 2\cos^2\alpha}{\cos x - \cos\alpha}dx \qquad \{\text{As } \cos 2x = 2\cos^2 x - 1\}$$

$$= 2\int(\cos x + \cos\alpha)\,dx = 2\sin x + 2x\cdot\cos\alpha + C.$$

OR

$$\int \underset{②}{x}\cdot\underset{①}{\log(1+2x)}\,dx = \log(1+2x)\cdot\frac{x^2}{2} - \int \frac{2}{1+2x}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2}{2}\log(1+2x) - \int \frac{x^2}{2x+1}dx$$

$$= \frac{x^2}{2}\log(2x+1) - \frac{1}{4}\int \frac{(4x^2-1)+1}{2x+1}dx$$

$$= \frac{x^2}{2}\log(2x+1) - \frac{1}{4}\int\left(2x-1+\frac{1}{2x+1}\right)dx$$

$$= \frac{x^2}{2}\log(2x+1) - \frac{1}{4}x^2 + \frac{x}{4} - \frac{1}{8}\log|2x+1|$$

$$\int_0^1 x\log(1+2x)\,dx = \left[\frac{x^2}{2}\log(2x+1) - \frac{x^2}{4} + \frac{x}{4} - \frac{1}{8}\log|2x+1|\right]_0^1$$

$$= \left[\frac{1}{2}\log 3 - \frac{1}{4} + \frac{1}{4} - \frac{1}{8}\log 3\right] - [0-0+0-0]$$

$$= \frac{3}{8}\log 3$$

29. Consider the equation, $(1+x^2)\dfrac{dy}{dx} + y = e^{\tan^{-1}x}$

$$\Rightarrow \qquad \frac{dy}{dx} + \frac{1}{1+x^2}\cdot y = \frac{e^{\tan^{-1}x}}{1+x^2}$$

Here, $P(x) = \dfrac{1}{1+x^2}$ and $Q(x) = \dfrac{e^{\tan^{-1}x}}{1+x^2}$

$$\text{Integrating factor } (I.F.) = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$$

$$\text{Solution is} \quad (I.F.)\, y = \int \{(I.F.)\, Q(x)\}\, dx$$

$$\Rightarrow \quad e^{\tan^{-1}x} \cdot y = \int e^{\tan^{-1}x} \cdot \frac{e^{\tan^{-1}x}}{1+x^2} dx \qquad \left| \text{Let } e^{\tan^{-1}x} = t \Rightarrow \frac{e^{\tan^{-1}x}}{1+x^2} dx = dt \right.$$

$$= \int t\, dt$$

$$= \frac{t^2}{2} + C$$

$$e^{\tan^{-1}x} \cdot y = \frac{1}{2}(e^{\tan^{-1}x})^2 + C$$

$$\Rightarrow \quad y = \frac{1}{2} e^{\tan^{-1}x} + C \cdot e^{-\tan^{-1}x} \text{ is the required solution.}$$

OR

$$x\frac{dy}{dx} + x\cos^2\frac{y}{x} = y$$

$$\Rightarrow \quad \frac{dy}{dx} + \cos^2\frac{y}{x} = \frac{y}{x} \qquad ...(i)$$

Let $y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (i), we get

$$v + x\frac{dv}{dx} + \cos^2 v = v$$

$$\Rightarrow \quad x\frac{dv}{dx} = -\cos^2 v$$

$$\Rightarrow \quad \int \sec^2 v\, dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \tan v = -\log|x| + C$$

$$\Rightarrow \quad \tan\frac{y}{x} = -\log|x| + C \qquad ...(ii)$$

Given when $x = 1$, $y = \frac{\pi}{4}$

$$\Rightarrow \quad \tan\frac{\pi}{4} = -\log 1 + C \quad \Rightarrow \quad C = 1$$

$\therefore$ From (ii)

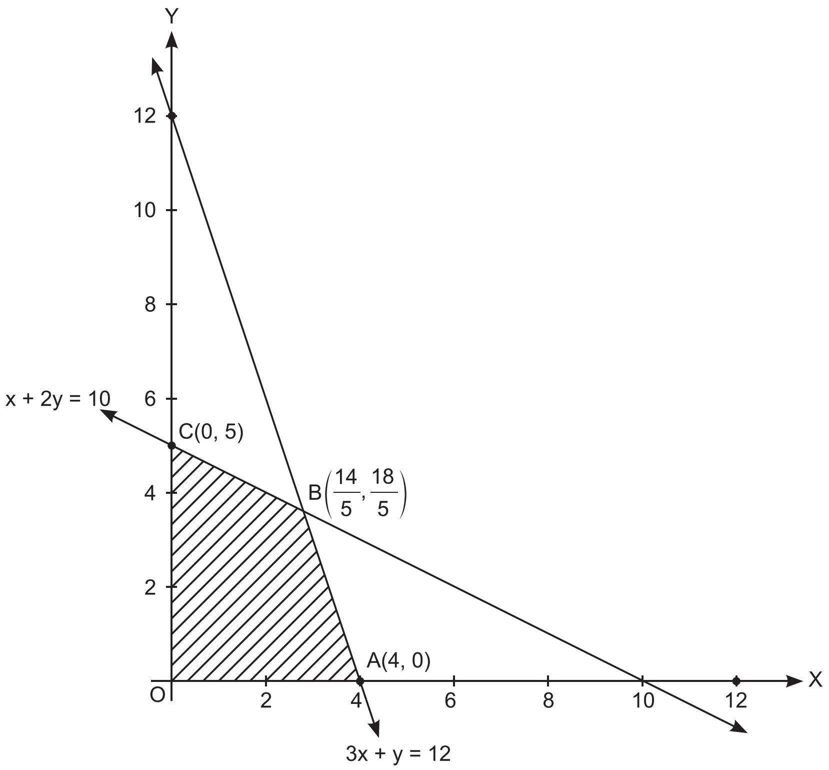
$$\tan\frac{y}{x} = -\log|x| + 1 \text{ is the required solution.}$$

30. To maximise $Z = 15x + 30y$

Subject to the constraints:

$$3x + y \le 12,\ x + 2y \le 10,\ x \ge 0,\ y \ge 0$$

On plotting inequations, we get shaded portion as feasible solution.



Possible points for maximise Z are $A(4, 0)$, $B\left(\frac{14}{5}, \frac{18}{5}\right)$, $C(0, 5)$.

Points	Z = 15x + 30y	Values
$A(4, 0)$	60 + 0	60
$B\left(\frac{14}{5}, \frac{18}{5}\right)$	42 + 108	150
$C(0, 5)$	0 + 150	150

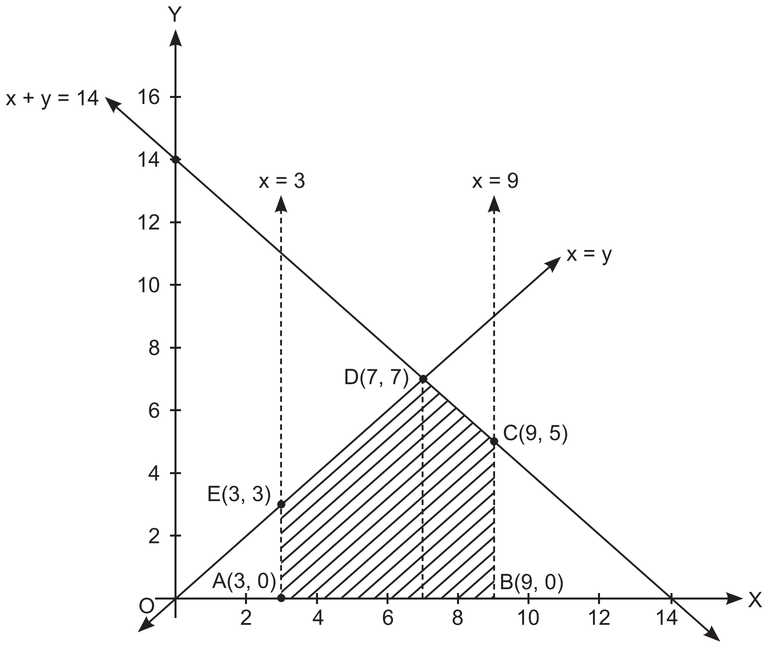
Maximum value of Z is 150 at $C(0, 5)$ or $B\left(\frac{14}{5}, \frac{18}{5}\right)$. Maximum Z is for any point on line segment BC.

OR

To minimise $Z = 2x + y$

Subject to the constraints

$x \ge 3,\ x \le 9,\ y \ge 0,\ x - y \ge 0,\ x + y \le 14$



On plotting inequations we notice shaded portion is feasible solution.

Possible points for minimum Z are $A(3, 0)$, $B(9, 0)$, $C(9, 5)$, $D(7, 7)$, $E(3, 3)$.

Points	Z = 2x + y	Values	
$A(3, 0)$	6 + 0	6	← Minimum
$B(9, 0)$	18 + 0	18	
$C(9, 5)$	18 + 5	23	
$D(7, 7)$	14 + 7	21	
$E(3, 3)$	6 + 3	9	

Z is minimum at $A(3, 0)$ i.e. $x = 3$, $y = 0$.

Minimum value $Z = 6$

31. Given $$y = e^{a\cos^{-1}x} \qquad ...(i)$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{a\cos^{-1}x} \cdot \frac{-a}{\sqrt{1-x^2}} = \frac{-ay}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2} \cdot \frac{dy}{dx} = -ay \qquad \text{[From } (i)\text{]}$$

On squaring both sides, we get

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

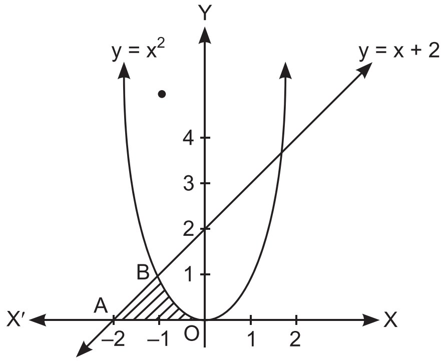
Differentiating both sides w.r.t. x, we get

$$(1-x^2)\left(2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}\right) - 2x\cdot\left(\frac{dy}{dx}\right)^2 = a^2\left(2y\cdot\frac{dy}{dx}\right)$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0 \qquad \left(\text{On dividing by } 2\frac{dy}{dx}\right)$$

32. Given curves are $x^2 = y$ and $y = x + 2$.

Plotting the curves we notice we have to find the shaded area.



Eliminating y, we get

$$x^2 = x + 2$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad (x-2)(x+1) = 0$$

$$\Rightarrow \quad x = 2 \text{ or } x = -1$$

$$\therefore \quad \text{Area} = \int_{-2}^{-1} y_{\text{AB}}\,dx + \int_{-1}^{0} y_{\text{OB}}\,dx$$

$$= \int_{-2}^{-1} (x+2)dx + \int_{-1}^{0} x^2 dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$$

$$= \left(\frac{1}{2} - 2\right) - \left(\frac{4}{2} - 4\right) + \left[0 - \left(\frac{-1}{3}\right)\right]$$

$$= \frac{-3}{2} + 2 + \frac{1}{3} = \frac{-9+12+2}{6} = \frac{5}{6} \text{ sq units}$$

33. Given relation $R = \{(a, b) \in N \times N : a \text{ is divisor of } b\}$

Reflexive: Let $a \in N$, $(a, a) \in R \Rightarrow a$ is divisor of a, which is true.

Hence, reflexive

Symmetric: Let for $a, b \in N$, $(a, b) \in R \Rightarrow a$ is divisor of b.

This may not imply b is divisor of a *e.g.,* 3 is divisor of 15 but 15 is not divisor of 3.

$\therefore$ $(a, b) \in R$ may not imply $(b, a) \in R$.

Hence, not symmetric.

Transitive: Let for $a, b, c \in N$

$$(a, b) \in R \text{ and } (b, c) \in R$$

$\Rightarrow a$ is divisor of $b \quad \Rightarrow \quad b = \lambda a, \lambda \in N$

and b is divisor of $c \quad \Rightarrow \quad c = \mu b, \mu \in N$

$$c = \mu(\lambda a) = (\lambda\mu)a, \lambda, \mu \in N$$

$\Rightarrow a$ is divisor of $c \quad \Rightarrow \quad (a, c) \in R$

as $\quad (a, b) \in R, (b, c) \in R \quad \Rightarrow \quad (a, c) \in R$

Hence, R is transitive relation.

OR

Given $f : R \to R$, defined as $f(x) = \dfrac{x}{x^2+1}$

Let for $x_1, x_2 \in R$,

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow x_1x_2^2 + x_1 = x_2x_1^2 + x_2$$

$$\Rightarrow x_1x_2^2 - x_2x_1^2 + x_1 - x_2 = 0$$

$$\Rightarrow x_1x_2(x_2 - x_1) + 1(x_1 - x_2) = 0$$

$$\Rightarrow (x_1 - x_2)(1 - x_1x_2) = 0$$

$$\Rightarrow x_1 = x_2 \quad \text{or} \quad x_1x_2 = 1$$

So $f(x_1) = f(x_2)$ is possible if $x_1 \cdot x_2 = 1$

e.g., let $x_1 = 2$ and $x_2 = \dfrac{1}{2}$

$$f(x_1) = \frac{2}{5}, f(x_2) = \frac{\frac{1}{2}}{\frac{1}{4}+1} = \frac{2}{5}$$

$\therefore f(x_1) = f(x_2)$ does not imply $x_1 = x_2$

Hence, not one-one.

Let for $y \in R$ (co-domain), there exists $x \in R$ (domain) such that $y = f(x)$.

$$\Rightarrow y = \frac{x}{x^2+1} \Rightarrow x^2y + y = x$$

$$\Rightarrow yx^2 - x + y = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y},$$

Here for $y = 0 \in$ co-domain, there is no value of $x \in R$ (domain) *i.e.,* for 0 from co-domain there is no preimage in domain. Hence, not onto.

34. Given matrix is

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj } A$$

$$|A| = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} = 1(-3) - 2(-2) + 0 = 1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}' = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad ...(i)$$

Given equations are

$$x - 2y = 10$$

$$2x - y - z = 8$$

$$-2y + z = 7$$

Matrix equation is

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$A'X = B$$

$$\text{Solution is } X = (A')^{-1}B$$

$$= (A^{-1})'B$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}' \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 0, y = -5, z = -3$ is the solution.

35. Let the line passing through the point (2, – 1, 3) is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda'(a\hat{i} + b\hat{j} + c\hat{k}) \qquad ...(i)$$

If line (*i*) is perpendicular to the lines

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$$

and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

then $(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k}) = 0$

$\Rightarrow$ $2a - 2b + c = 0$

and $(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) = 0 \Rightarrow a + 2b + 2c = 0$

$\Rightarrow$ $\dfrac{a}{-4-2} = \dfrac{-b}{4-1} = \dfrac{c}{4+2}$, i.e. $\dfrac{a}{-6} = \dfrac{b}{-3} = \dfrac{c}{6}$

$\Rightarrow a : b : c$ is – 6 : – 3 : 6 or 2 : 1 : – 2

From (*i*), line is $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda'(2\hat{i} + \hat{j} - 2\hat{k})$

Point through which line passes (2, –1, 3) and DR's of the line are 2, 1, –2.

$\therefore$ Cartesian equation is $\dfrac{x-2}{2} = \dfrac{y+1}{1} = \dfrac{z-3}{-2}$

OR

Given lines are

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

and $$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu\left(4\hat{i} + 6\hat{j} + 12\hat{k}\right)$$

Here,

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

and $$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k},\ \vec{b}_2 = 4\hat{i} + 6\hat{j} + 12\hat{k}$$

We notice $\vec{b}_1$ and $\vec{b}_2$ are parallel vectors as $\frac{2}{4} = \frac{3}{6} = \frac{6}{12}$ or $\vec{b}_1 \times \vec{b}_2 = \vec{0}$

$\therefore$ lines are parallel.

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$
$$= 2\hat{i} + \hat{j} - \hat{k}$$

$$\text{The shortest distance} = \left|\frac{\left|\vec{b} \times \left(\vec{a}_2 - \vec{a}_1\right)\right|}{|\vec{b}\,|}\right|$$

$$\vec{b} \times \left(\vec{a}_2 - \vec{a}_1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix} = -9\hat{i} + 14\hat{j} - 4\hat{k}$$

$\therefore$ $$\text{The shortest distance} = \left|\frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}\,|}{|2\hat{i} + 3\hat{j} + 6\hat{k}\,|}\right|$$

$$= \frac{\sqrt{81 + 196 + 16}}{\sqrt{4 + 9 + 36}} = \frac{\sqrt{293}}{7} \text{ units}$$

36. $P(A_1) = \frac{2}{5}, P(A_2) = \frac{2}{5}, P(A_3) = \frac{1}{5}$

E : Seed germinates

$P(E/A_1) = \frac{45}{100}, P(E/A_2) = \frac{60}{100}, P(E/A_3) = \frac{35}{100}$

(*i*) $P(\text{chosen } A_1) = P(A_1)\ P(E/A_1) = \frac{2}{5} \times \frac{45}{100} = 0.18$

(*ii*) $P(\text{randomly chosen seed germinate}) = P(A_1)\ P(E/A_1) + P\left(A_2\right) P(E/A_2) + P\left(A_3\right)(E/A_3)$

$$= \frac{2}{5} \times \frac{45}{100} + \frac{2}{5} \times \frac{60}{100} + \frac{1}{5} \times \frac{35}{100}$$

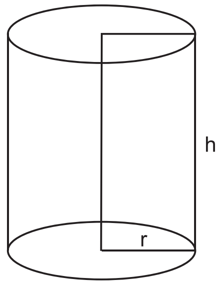
$$= 0.18 + 0.24 + 0.07 = 0.49$$

(*iii*) $P(A_2/E) = \frac{P\left(A_2\right) P\left(E/A_2\right)}{P\left(\text{Seed germinates}\right)} = \frac{0.24}{0.49} = \frac{24}{49}$

OR

(*iii*) $P(A_3/E) = \frac{P\left(A_3\right) P\left(E/A_3\right)}{P\left(\text{Seed germinates}\right)} = \frac{0.07}{0.49} = \frac{1}{7}$

37. (*i*) Volume, $V = \pi r^2 h$



Surface area, $S = 2\pi rh + \pi r^2$

$\Rightarrow$ $75\pi = 2\pi rh + \pi r^2$

$\Rightarrow$ $75 = 2rh + r^2 \Rightarrow h = \dfrac{75 - r^2}{2r}$

$$V = \pi r^2\left[\frac{75 - r^2}{2r}\right] = \frac{\pi}{2}\left[75r - r^3\right]$$

(*ii*) $$\frac{dV}{dr} = \frac{\pi}{2}\left(75 - 3r^2\right)$$

$$\frac{dV}{dr} = 0$$

$\Rightarrow$ $3r^2 = 75$

$\Rightarrow$ $r^2 = 25$

$\Rightarrow$ $r = 5$

(*iii*) $$\frac{d^2V}{dr^2} = \frac{\pi}{2}(-6r) = -3\pi r$$

$$\left.\frac{d^2V}{dr^2}\right]_{r=5} = -15\pi < 0$$

$\therefore$ Volume is maximum for $r = 5$.

OR

(*iii*) $$V_{max} = \frac{\pi}{2}[75 \times 5 - (5)^3] = \frac{\pi}{2}(375 - 125)$$

$$= \frac{\pi}{2} \times 250 = 125\pi \text{ cm}^3$$

Also $$h = \frac{75 - r^2}{2r} = \frac{75 - 25}{10} = \frac{50}{10} = 5 = r$$

$\Rightarrow h = r$ for maximum volume.

38. (*i*) $A(3, 1, 2)$, $B(0, 4, 1)$, $C(3, 2, 1)$, $D(1, 1, 1)$

$$\overrightarrow{AB} = -3\hat{i} + 3\hat{j} - \hat{k}$$

$$\overrightarrow{AC} = \hat{j} - \hat{k}$$

$\therefore$ $$\text{Area }(\Delta ABC) = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & -1 \\ 0 & 1 & -1 \end{vmatrix} = -2\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\therefore \quad \text{Area} = \frac{1}{2}\left|-2\hat{i} - 3\hat{j} - 3\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{4+9+9} = \frac{1}{2}\sqrt{22} \text{ sq units}$$

(*ii*)

$$\overrightarrow{AB} = -3\hat{i} + 3\hat{j} - \hat{k}$$

$$\overrightarrow{BC} = 3\hat{i} - 2\hat{j}$$

$$\overrightarrow{BD} = \hat{i} - 3\hat{j}$$

$$\therefore \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BD} = \hat{i} - 2\hat{j} - \hat{k}$$

$$\left|\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BD}\right| = \left|\hat{i} - 2\hat{j} - \hat{k}\right|$$

$$= \sqrt{1+4+1} = \sqrt{6}$$