Solutions to RMM–DS2/Set-2

1. (*d*) 3×1

2. (*b*) $\begin{vmatrix} 1 & 1 & 1+\cos\theta \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1(\sin\theta) - 1(0) + (1+\cos\theta)(-\sin\theta)$

$$= \sin\theta - \sin\theta - \sin\theta\cos\theta = -\sin\theta\cos\theta$$

$$= -\frac{1}{2}\sin 2\theta$$

Now, $-1 \le \sin 2\theta \le 1$

$\Rightarrow \frac{-1}{2} \le \frac{-1}{2}\sin 2\theta \le \frac{1}{2}$

So, maximum value of given determinant is $\frac{1}{2}$.

3. (*a*) As, $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$

$\Rightarrow 2x^2 + 6x - 6x = 8$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

As, $x \in N$, so we take $x = 2$.

4. (*a*) Since f is continuous at $x = 1$,

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{x \to 1^-} (4) = \lim_{x \to 1^+} (kx^2) = k(1)^2$

$\Rightarrow 4 = k = k \Rightarrow k = 4$

5. (*b*) Let (l, m, n) be the dc's and (a, b, c) be the dr's of the line passing through points (3, 2, 5) and (1, 3, 9)

As, line makes an acute angle with x-axis, so $l > 0$.

Now, dr's of a line are always proportional to dc's.

Now, $l > 0 \Rightarrow a > 0$

$\therefore$ dr's of the given line are $(3 - 1, 2 - 3, 5 - 9)$ i.e. $(2, -1, -4)$.

6. (*b*) order = 2, degree = 1

Product = $2 \times 1 = 2$

7. (*b*) As (1, 2) does not satisfy inequation $3x \ge 5$.

8. (*d*) As $|2\vec{c} + \vec{a} + \vec{b}|^2 = (2\vec{c} + \vec{a} + \vec{b})^2$

$$= (2\vec{c} + \vec{a} + \vec{b}) \cdot (2\vec{c} + \vec{a} + \vec{b})$$

$$= 4|\vec{c}|^2 + |\vec{a}|^2 + |\vec{b}|^2 + 4\vec{c}\cdot\vec{a} + 2\vec{a}\cdot\vec{b} + 4\vec{b}\cdot\vec{c}$$

$$= 4 + 1 + 1 + 0 + 0 + 0$$

$$= 6$$

$\Rightarrow |2\vec{c} + \vec{a} + \vec{b}| = \sqrt{6}$

9. (*c*) $f(x) = \int \left(x + \frac{1}{x}\right)dx = \frac{x^2}{2} + \log|x| + C$

10. (*d*) As for skew-symmetric matrix, $a_{ij} = -a_{ji} \ \forall\ i, j \rightarrow a = 0$ and $a = -7$

$\therefore$ No value of 'a'.

11. (*a*)
$$Z_B = Z_A + Z_C + 2$$
$$\Rightarrow \quad 3a + 5b = 2a + b + 7b + 2$$
$$\Rightarrow \quad a = 3b + 2$$

12. (*c*) Vector component of $\vec{r}$ along z-axis $= -3\vec{k}$

13. (*d*) If A is a skew symmetric matrix of odd order, then $|A| = 0$.

So, $|A| = x = 0$. Then $(2024)^x = (2024)^0 = 1$

14. (*c*)
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.4 + 0.3 - 0.5 = 0.2$$
$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$
$$= 0.3 - 0.2 = 0.1$$

15. (*c*) The differential equation is $\frac{dy}{dx} + \frac{2}{x}y = x$

Comparing with $\frac{dy}{dx} + Py = Q$,

we get $P = \frac{2}{x}$ and $Q = x$

$$\text{Integrating factor} = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log |x|}$$
$$= e^{\log |x|^2} = x^2$$

16. (*a*) On evaluating the determinant,
$$\Delta = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$
$$= |\vec{a}|^2 |\vec{b}|^2 - \{|\vec{a}|\,|\vec{b}| \cos\theta\}^2$$
$$= |\vec{a}|^2 |\vec{b}|^2 - |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta$$
$$= |\vec{a}|^2\ |\vec{b}|^2 \sin^2\theta$$
$$= \{|\vec{a}|^2\ |\vec{b}|^2 \sin\theta\}^2$$
$$= |\vec{a} \times \vec{b}|^2$$
$$= (\vec{a} \times \vec{b})^2$$

17. (*c*) Let 'x' be the side and 'A' be the area of the equilateral Δ at any time 't'.
$$\frac{dx}{dt} = 2 \text{ cm/s}$$
Now,
$$A = \frac{\sqrt{3}}{4}x^2 \Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}x \cdot \frac{dx}{dt}$$
$$\left.\frac{dA}{dt}\right]_{x=10} = \frac{\sqrt{3}}{2} \times 10 \times 2 = 10\sqrt{3} \text{ cm}^2\text{/s}$$

18. (*d*) As, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$$\Rightarrow \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$$
$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

19. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

20. (*a*) Both (A) and (R) are true and (R) is the correct explanation of (A).

21. $$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 2\tan^{-1}(1) = -\frac{\pi}{4} + 2\cdot\frac{\pi}{4} = \frac{\pi}{4}$$

22. $$y = \log_2(\sqrt{x-a} + \sqrt{x-b}) = \frac{\log(\sqrt{x-a} + \sqrt{x-b})}{\log 2}$$

$$\frac{dy}{dx} = \frac{1}{\log 2}\cdot\frac{1}{(\sqrt{x-a} + \sqrt{x-b})}\cdot\left\{\frac{1}{2\sqrt{x-a}}\cdot 1 + \frac{1}{2\sqrt{x-b}}\cdot 1\right\}$$

$$= \frac{1}{\log 2}\cdot\frac{1}{(\sqrt{x-a} + \sqrt{x-b})}\cdot\frac{1}{2}\left\{\frac{\sqrt{x-b} + \sqrt{x-a}}{\sqrt{x-a}\,\sqrt{x-b}}\right\}$$

$$= \frac{1}{2\log 2\cdot\left(\sqrt{x-a}\cdot\sqrt{x-b}\right)}$$

OR

$$y = \log(\log x^2)$$

Differentiating w.r.t. x both sides,

$$y_1 = \frac{1}{\log x^2}\cdot\frac{1}{x^2}\cdot 2x = \frac{2}{x\log x^2}$$

$\Rightarrow$ $$y_1 = \frac{2}{x\cdot 2\log x} = \frac{1}{x\log x}$$

Differentiating again w.r.t. x both sides,

$$y_2 = \frac{x\log x\times 0 - 1\left\{x\times\frac{1}{x} + \log x\right\}}{(x\log x)^2} = \frac{-(1+\log x)}{(x\log x)^2}$$

23. Consider function $f(x) = x^2$

$\Rightarrow$ $f'(x) = 2x$

For increasing function, $f'(x) \geq 0$

$\Rightarrow$ $2x \geq 0 \Rightarrow x \geq 0$

For decreasing function, $f'(x) \leq 0$

$\Rightarrow$ $2x \leq 0 \Rightarrow x \leq 0$

So, f increases on $(-\infty, 0]$ and decreases $[0, \infty)$. Hence function is neither increasing nor decreasing on set of real numbers.

OR

We have, $f(x) = \sin 3x$

$\Rightarrow$ $f'(x) = 3\cos 3x$ ...(*i*)

For critical points, $f'(x) = 0$

$\Rightarrow$ $\cos 3x = 0$

$\Rightarrow$ $3x = \frac{\pi}{2}, \frac{3\pi}{2}$

So, $x = \frac{\pi}{6}$ and $\frac{\pi}{2}$

Case I: When $0 < x < \frac{\pi}{6}$ i.e. $0 < 3x < \frac{\pi}{2}$

In this case, we have

$\cos 3x > 0$

$\Rightarrow$ $3\cos 3x > 0$

$\Rightarrow$ $f'(x) > 0$

$\therefore$ $f(x)$ is increasing on $\left(0, \frac{\pi}{6}\right)$.

Case II: When $\frac{\pi}{6} < x < \frac{\pi}{2}$ i.e. $\frac{\pi}{2} < 3x < \frac{3\pi}{2}$

In this case, we have

$$\cos 3x < 0$$

$$\Rightarrow \quad 3\cos 3x < 0$$

$$\Rightarrow \quad f'(x) < 0$$

$\therefore$ $f(x)$ is decreasing on $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$.

24.
$$\int \sin x \cdot \sin 3x\, dx = \frac{1}{2}\int (2\sin 3x \sin x)\, dx$$
$$= \frac{1}{2}\int (\cos 2x - \cos 4x)\, dx$$
$$= \frac{1}{2}\left[\frac{\sin 2x}{2} - \frac{\sin 4x}{4}\right] + C$$
$$= \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$$

25. Let r be the radius, V be the volume and S be the surface area of the sphere at any instant t.

$$\frac{dV}{dt} = 3 \text{ cm}^3\text{/s}$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 3 \Rightarrow 4\pi r^2 \frac{dr}{dt} = 3$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{3}{4\pi r^2} \quad ...(i)$$

Now, $$S = 4\pi r^2$$

$$\Rightarrow \quad \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} = 8\pi r \cdot \frac{3}{4\pi r^2} = \frac{6}{r} \quad \text{[using } (i)\text{]}$$

$$\left.\frac{dS}{dt}\right]_{r=2} = \frac{6}{2} = 3 \text{ cm}^2\text{/s}$$

26. $$\int \frac{1}{\cos(x+a)\sin(x+b)}\, dx$$

$$= \frac{1}{\cos(a-b)} \int \frac{\cos\{(x+a)-(x+b)\}}{\cos(x+a)\sin(x+b)}\, dx$$

$$= \frac{1}{\cos(a-b)} \int \frac{\cos(x+a)\cos(x+b) + \sin(x+a)\sin(x+b)}{\cos(x+a)\sin(x+b)}\, dx$$

$$= \frac{1}{\cos(a-b)} \int \{\cot(x+b) + \tan(x+a)\}\, dx$$

$$= \frac{1}{\cos(a-b)} [\log|\sin(x+b)| + \log|\sec(x+a)|] + C$$

27. Let S be the sample space.

$\therefore$ $$S = \{1, 2, 3, 4, 5, 6\}$$

So, $$n(S) = 6$$

A : number obtained is even.

$$A = \{2, 4, 6\}$$

So, $$n(A) = 3$$

So, $P(A) = \frac{n(A)}{n(S)} \Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B : number obtained is red.

Now, $B = \{1, 2, 3\}$

So, $n(B) = 3$

$\therefore$ $P(B) = \frac{3}{6} = \frac{1}{2}$

Now, $A \cap B = \{2\} \Rightarrow n(A \cap B) = 1$

$\therefore$ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6} = \frac{1}{6}$

As, $P(A \cap B) \neq P(A) \cdot P(B)$

So, events A and B are not independent.

28. $$I = \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx \quad ...(i)$$

Using property $\int_0^a f(x)dx = \int_0^a f(a-x)\, dx$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \sin(\pi - x)}\, dx$$

$$= \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1 + \sin x} dx \quad ...(ii)$$

$\therefore$ $$2I = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x}\, dx \quad \text{[On adding } (i) \text{ and } (ii)\text{]}$$

$$= \pi \int_0^{\pi} \frac{\sin x (1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$$

$$= \pi \int_0^{\pi} (\sec x \tan x - \tan^2 x) dx$$

$$= \pi \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1)\, dx$$

$$= \pi \Big[\sec x - \tan x + x\Big]_0^{\pi}$$

$$= \pi[(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0)]$$

$$2I = \pi(-2 + \pi)$$

$\Rightarrow$ $$I = \frac{\pi}{2}\, (\pi - 2) = \pi\left(\frac{\pi}{2} - 1\right)$$

OR

Let $$I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx \quad ...(i)$$

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right) dx\right] \quad \left[\text{using property : } \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$$

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right] dx$$

$\Rightarrow \qquad I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right)dx \qquad ...(ii)$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{4}} \left[\log(1+\tan x) + \log\left(\frac{2}{1+\tan x}\right)\right]dx$$

$\Rightarrow \qquad 2I = \int_0^{\frac{\pi}{4}} [\log(1+\tan x) + \log 2 - \log(1+\tan x)]dx$

$\Rightarrow \qquad 2I = \log 2 \int_0^{\frac{\pi}{4}} dx$

$\Rightarrow \qquad 2I = \log 2 \times \left[x\right]_0^{\frac{\pi}{4}}$

$\Rightarrow \qquad 2I = \frac{\pi}{4}\log 2$

$\Rightarrow \qquad I = \frac{\pi}{8}\log 2$

29. We have, $\qquad \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right) \qquad ...(i)$

Let $\qquad y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ From (i), we get

$$v + x\frac{dv}{dx} = v(\log v + 1) = v\log v + v$$

$\Rightarrow \qquad x\frac{dv}{dx} = v\log v$

$\Rightarrow \qquad \frac{dv}{v\log v} = \frac{dx}{x}$

On integrating both sides,

$$\int \frac{1}{v\log v}dv = \int \frac{dx}{x}$$

$\Rightarrow \qquad \log|\log v| = \log|x| + \log|C| \Rightarrow \log|\log v| = \log|Cx| \Rightarrow \log v = xC$

$\Rightarrow \qquad \log\left(\frac{y}{x}\right) = xC$ is the required solution.

OR

Consider equation $\qquad xe^{y/x} - y + x\frac{dy}{dx} = 0$

$\Rightarrow \qquad x\frac{dy}{dx} = y - xe^{y/x}$

$\Rightarrow \qquad \frac{dy}{dx} = \frac{y}{x} - e^{y/x} \qquad ...(i)$

Let $\qquad y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (i), we get

$$v + x\frac{dv}{dx} = v - e^v$$

$$\Rightarrow \qquad x\frac{dv}{dx} = -e^{v}$$

$$\Rightarrow \qquad e^{-v}dv = -\frac{dx}{x}$$

On integrating both sides,

$$\int e^{-v}dv = -\int \frac{dx}{x}$$

$$\Rightarrow \qquad \frac{e^{-v}}{-1} = -\log|x| + C_1$$

$$\Rightarrow \qquad \frac{1}{e^{v}} = \log|x| - C_1$$

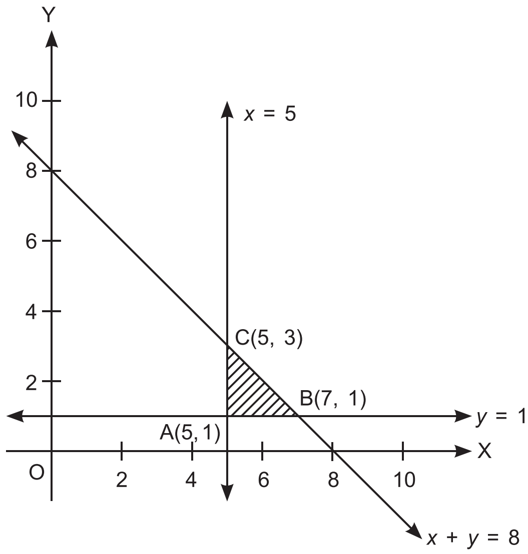
$$\Rightarrow \qquad \frac{1}{e^{y/x}} = \log|x| + C \qquad \{\text{where } C = -C_1\}$$

$$\Rightarrow \qquad e^{y/x}[\log|x| + C] = 1$$

30. To maximize, $Z = 3x + y$

subject to constraints,

$$x \geq 5,\ y \geq 1,\ x + y - 8 \leq 0,\ x \geq 0,\ y \geq 0$$



On plotting the inequations on graph, we notice shaded portion is feasible solution.

Possible points for maximum Z are $A(5, 1)$, $B(7, 1)$, $C(5, 3)$

Corner Points	$Z = 3x + y$	Values	
$A(5, 1)$	15 + 1	16	
$B(7, 1)$	21 + 1	22	← Maximum
$C(5, 3)$	15 + 3	18	

$\therefore$ Z is maximum at $B(7, 1)$ i.e. $x = 7$, $y = 1$.

Maximum value of $Z = 22$.

OR

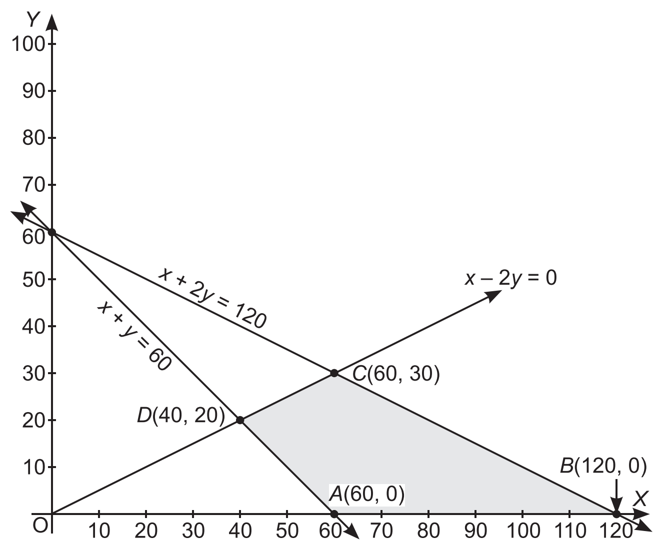
To minimise

$$Z = 5x + 10y$$

subject to the constraints

$$x \geq 0,\ y \geq 0,\ x - 2y \geq 0,\ x + y \geq 60,\ x + 2y \leq 120$$

Plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for minimum Z are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$



Corner Points	$Z = 5x + 10y$	Values	
$A(60, 0)$	300 + 0	300	← Minimum
$B(120, 0)$	600 + 0	600	
$C(60, 30)$	300 + 300	600	
$D(40, 20)$	200 + 200	400	

$\therefore$ Z is minimum at $A(60, 0)$. Hence, for $x = 60$ and $y = 0$, Z is minimum.

Minimum value of $Z = 300$.

31. We have, $y = e^x(\sin x + \cos x)$ ...(*i*)

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^x(\cos x - \sin x) + e^x(\sin x + \cos x)$$

$$\Rightarrow \quad \frac{dy}{dx} = e^x(\cos x - \sin x) + y \quad ...(ii)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x) + \frac{dy}{dx}$$

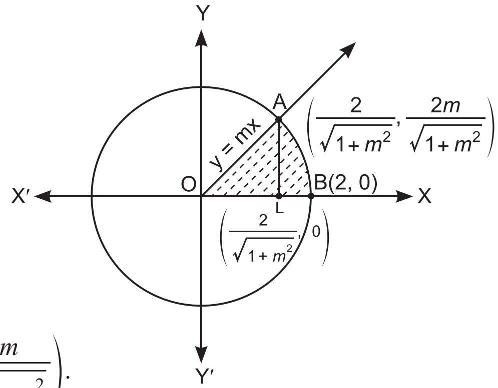
$$\Rightarrow \quad \frac{d^2y}{dx^2} = \left(\frac{dy}{dx} - y\right) - y + \frac{dy}{dx} \quad \text{[from } (i) \text{ and } (ii)]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

32. Given curves are : $y = mx (m > 0)$ ...(*i*)

and $x^2 + y^2 = 4$ ...(*ii*)

Put $y = mx$ from (*i*) in (*ii*), we get



$$x^2 + m^2x^2 = 4$$

$$\Rightarrow \quad x^2(1 + m^2) = 4$$

$$\Rightarrow \quad x = \frac{2}{\sqrt{1+m^2}}$$

From (*i*), $$y = \frac{2m}{\sqrt{1+m^2}}$$

So, point of intersection of (*i*) and (*ii*) in 1st quadrant is $A\left(\frac{2}{\sqrt{1+m^2}}, \frac{2m}{\sqrt{1+m^2}}\right)$.

On plotting the given curves on graph, we notice that area of shaded region is to be found.

$\therefore$ Area(OAB) = ar(OAL) + ar(LAB)

$$\Rightarrow \quad \int_0^{\frac{2}{\sqrt{1+m^2}}} mx\,dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} \sqrt{4-x^2}\,dx = \frac{\pi}{2}$$

$$\Rightarrow \left[\frac{mx^2}{2}\right]_0^{\frac{2}{\sqrt{1+m^2}}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{2}{\sqrt{1+m^2}}}^{2}$$

$$\Rightarrow \frac{m}{2}\cdot\frac{4}{1+m^2} + \{0 + 2\sin^{-1}(1)\} - \left\{\frac{1}{\sqrt{1+m^2}}\sqrt{4 - \frac{4}{1+m^2}} + 2\cdot\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right)\right\} = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2m}{1+m^2} + 2\times\frac{\pi}{2} - \frac{2m}{1+m^2} - 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad 2\sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{2} \quad \Rightarrow \quad \sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1+m^2}} = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sqrt{1+m^2} = \sqrt{2} \quad \Rightarrow \quad 1 + m^2 = 2 \quad \Rightarrow \quad m^2 = 1 \quad \Rightarrow \quad m = \pm 1$$

As, $m > 0$, so we take $m = 1$.

33. $$f(x) = 5x^2 + 6x - 9$$

For one-one: Let $x_1, x_2 \in R^+$.

Then, $f(x_1) = f(x_2)$

$\Rightarrow \quad 5x_1^2 + 6x_1 - 9 = 5x_2^2 + 6x_2 - 9$

$\Rightarrow \quad 5(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow \quad (x_1 - x_2)(5x_1 + 5x_2 + 6) = 0$

Now, $x_1, x_2 \in R^+ \quad \therefore \quad 5x_1 + 5x_2 + 6 \neq 0$

Hence, $x_1 - x_2 = 0$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one function.

For onto: Let $5x^2 + 6x - 9 = y$

$\Rightarrow \quad 5x^2 + 6x - (9 + y) = 0$

$\Rightarrow \quad x = \dfrac{-6 \pm \sqrt{36 + 20(9 + y)}}{10}$ (Using quadratic formula)

$\Rightarrow \quad x = \dfrac{-6 \pm \sqrt{36 + 180 + 20y}}{10}$

$= \dfrac{-6 \pm 2\sqrt{54 + 5y}}{10}$

Here $\dfrac{\sqrt{54 + 5y} - 3}{5} \in R^+$ but $\dfrac{-\sqrt{54 + 5y} - 3}{5} \notin R^+$ (Rejected)

$$\therefore \quad f(x) = 5\left(\frac{\sqrt{54 + 5y} - 3}{5}\right)^2 + 6\left(\frac{\sqrt{54 + 5y} - 3}{5}\right) - 9$$

$$= \left(\frac{54 + 5y + 9 - 6\sqrt{54 + 5y}}{5}\right) + \frac{6(\sqrt{54 + 5y} - 3)}{5} - 9$$

$$= \frac{63 + 5y - 6\sqrt{54 + 5y} + 6\sqrt{54 + 5y} - 18 - 45}{5}$$

$$= y$$

$\therefore$ f is onto

Hence f is a bijective function.

OR

$R = \{(a, b) : a - b + \sqrt{5} \in \text{S and } a, b \in \text{R}\}$

For reflexive: Let $a \in R$

Now, $(a, a) \in R$

$\Rightarrow \quad a - a + \sqrt{5} = \sqrt{5} \in S$

$\therefore$ R is reflexive relation.

For symmetric: Let $a, b \in R$

Take $a = \sqrt{5}$, $b = 1$.

Now, $a - b + \sqrt{5} = \sqrt{5} - 1 + \sqrt{5}$

$= 2\sqrt{5} - 1 \in S$

But $b - a + \sqrt{5} = 1 - \sqrt{5} + \sqrt{5} = 1 \notin S$

$\therefore \quad (b, a) \notin R$

$\therefore \quad R$ is not symmetric relation.

For transitive:

Take $a = 1, b = \sqrt{2}, c = \sqrt{5}$

Now, $(a, b) \in R \quad \Rightarrow \quad 1 - \sqrt{2} + \sqrt{5} \in S$

And $(b, c) \in R \quad \Rightarrow \quad \sqrt{2} - \sqrt{5} + \sqrt{5} = \sqrt{2} \in S$

But $(a, c) \notin R$, as $1 - \sqrt{5} + \sqrt{5} = 1 \notin S$

$\therefore$ Given relation is reflexive but neither symmetric nor transitive.

34. Consider

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 1(0) - 2(7) - 3(-7)$$

$$= -14 + 21 = 7 \neq 0$$

So, A^{-1} exists.

Let A_{ij} be the cofactors of a_{ij} in $|A|$. Then,

$A_{11} = (-1)^2 (0) = 0, A_{12} = (-1)^3 (7) = -7, A_{13} = (-1)^4 (-7) = -7$

$A_{21} = (-1)^3 (-1) = 1, A_{22} = (-1)^4 (7) = 7, A_{23} = (-1)^5 (-5) = 5$

$A_{31} = (-1)^4 (2) = 2, A_{32} = (-1)^5 (7) = -7, A_{33} = (-1)^6 (-4) = -4$

Now,

$$\text{Adj } A = \begin{bmatrix} 0 & -7 & -7 \\ 1 & 7 & 5 \\ 2 & -7 & -4 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$\therefore$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \qquad ...(i)$$

Consider equations

$$x + 2y - 3z = 6$$

$$3x + 2y - 2z = 3$$

$$2x - y + z = 2$$

Corresponding matrix equation is,

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \qquad AX = B$$

Its solution is $X = A^{-1}B$.

$$\Rightarrow \qquad X = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} \qquad \text{[from } (i)\text{]}$$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 + 3 + 4 \\ -42 + 21 - 14 \\ -42 + 15 - 8 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\Rightarrow x = 1, y = -5, z = -5$ is solution of the given system of equations.

35. The given equations of lines are :

$$\frac{x+1}{3} = \frac{y+2}{5} = \frac{z+5}{7} = \lambda \text{ (say)} \qquad ...(i)$$

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-4}{5} = \mu \text{ (say)} \qquad ...(ii)$$

The coordinates of any general point on line (*i*) are given by (3λ – 1, 5λ – 2, 7λ – 5).

Also, the coordinates of any general point on line (*ii*) are (μ + 2, 3μ + 4, 5μ + 4).

If lines intersect, then for some values of λ, μ they represent the same point.

i.e.,

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda = \mu + 3 \qquad ...(iii)$$

$$5\lambda - 2 = 3\mu + 4 \Rightarrow 5\lambda = 3\mu + 6 \qquad ...(iv)$$

$$7\lambda - 5 = 5\mu + 4 \Rightarrow 7\lambda = 5\mu + 9 \qquad ...(v)$$

Solving (*iii*) and (*iv*) we get

$$\lambda = \frac{3}{4} \text{ and } \mu = -\frac{3}{4}$$

Substituting the values of λ and μ in (*v*), we get

$$\frac{21}{4} = -\frac{15}{4} + 9 \Rightarrow \frac{21}{4} = \frac{21}{4}, \text{ true.}$$

Hence, for $\lambda = \frac{3}{4}$, $\mu = \frac{-3}{4}$ lines intersect.

Substituting $\lambda = \frac{3}{4}$ in (*i*) or $\mu = \frac{-3}{4}$ in (*ii*),

Coordinates of point of intersection are $\left(\frac{9}{4} - 1, \frac{15}{4} - 2, \frac{21}{4} - 5\right)$ *i.e.,* $\left(\frac{5}{4}, \frac{7}{4}, \frac{1}{4}\right)$.

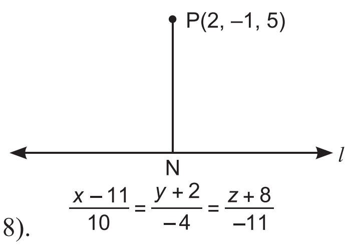
OR

The given equation of line '*l*' is,

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}.$$

Let $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$

⇒ $x = 10\lambda + 11, y = -4\lambda - 2, z = -11\lambda - 8$



Suppose the coordinates of any general point on the line '*l*' is (10λ + 11, –4λ – 2, –11λ – 8).

Let *N* be the foot of the perpendicular drawn from the point *P*(2, –1, 5) on the given line '*l*'.

Suppose the coordinates of point *N* be *N*(10λ + 11, –4λ – 2, –11λ –8).

Now, dr's of line '*l*' are <10, – 4, – 11>

$$\text{Also, dr's of } PN = \langle 10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5\rangle$$
$$= \langle 10\lambda + 9, -4\lambda - 1, -11\lambda - 13\rangle$$

As, *PN* is perpendicular to line '*l*', then

$$10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$$
$$\Rightarrow 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$$
$$\Rightarrow 237\lambda = -237 \Rightarrow \lambda = -1$$

Now, we will put λ = – 1 in the coordinates of *N*. So, coordinates of the foot of the perpendicular drawn from the point *P* on the given line '*l*' is

N(–10 + 11, 4 – 2, 11 – 8) *i.e.* *N*(1, 2, 3).

$$\text{Length of perpendicular, } PN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2}$$
$$= \sqrt{1+9+4}$$
$$= \sqrt{14} \text{ units}$$

36. (*i*) As, $\Sigma P(X) = 1$

$\Rightarrow \quad p + 2p + 2p + p + 2p + p^2 + 2p^2 + 7p^2 + p = 1$

$\Rightarrow \quad 10p^2 + 9p - 1 = 0 \quad \Rightarrow \quad (10p - 1)(p + 1) = 0$

$\Rightarrow \quad 10p - 1 = 0$ or $p + 1 = 0$

$\Rightarrow \quad p = \dfrac{1}{10}$ or $p = -1$ (rejected)

(*ii*) $P(X > 6) = P(7) + P(8) = 2p^2 + 7p^2 + p$

$$= 9p^2 + p = \frac{9}{100} + \frac{1}{10} = \frac{19}{100} = 0.19$$

(*iii*) Required probability $= P(3) + P(6)$

$$= 2p + p^2 = \frac{2}{10} + \frac{1}{100}$$
$$= \frac{21}{100} = 0.21$$

OR

(*iii*) Required probability $= P(2) + P(4) + P(6) + P(8)$

$$= 2p + p + p^2 + 7p^2 + p = 8p^2 + 4p$$
$$= \frac{8}{100} + \frac{4}{10} = \frac{48}{100} = 0.48$$

37. (*i*) Length = $(18 - 2x)$ cm, breadth = $(18 - 2x)$ cm, height = x cm

(*ii*) Volume V = length × breadth × height = $x(18 - 2x)^2$ cm^3

(*iii*) As, $V = x(18 - 2x)^2$

$\Rightarrow \quad \dfrac{dV}{dx} = x \cdot 2(18 - 2x)(-2) + (18 - 2x)^2$

$= (18 - 2x)(18 - 6x)$

For maximum or minimum volume, $\dfrac{dV}{dx} = 0$

$\Rightarrow \quad (18 - 6x)(18 - 2x) = 0$

$\Rightarrow \quad x = 3$ or $x = 9$

Now, $x = 9$ is rejected as length and breadth becomes 0 for $x = 9$.

Now, $\dfrac{d^2V}{dx^2} = (18 - 2x)(-6) + (18 - 6x)(-2)$

$$\left.\frac{d^2V}{dx^2}\right]_{x=3} = (18 - 2 \times 3)(-6) + (-2)(18 - 6 \times 3) = -72 < 0$$

$\therefore$ Volume is maximum at $x = 3$.

OR

(*iii*) Maximum Volume $= 3(18 - 6)^2 = 432$ cm^3

38. (*i*)

$$\overrightarrow{BC} = \text{Position vector of } C - \text{Position vector of } B$$

$$\Rightarrow \quad \overrightarrow{BC} = \left(\hat{i} + 5\hat{j} - 2\hat{k}\right) - \left(-3\hat{i} + 6\hat{j} + 8\hat{k}\right)$$

$$= 4\hat{i} - \hat{j} - 10\hat{k}$$

$$\text{unit vector along } \overrightarrow{BC} = \frac{\overrightarrow{BC}}{|\overrightarrow{BC}|}$$

$$= \frac{4\hat{i} - \hat{j} - 10\hat{k}}{\sqrt{16+1+100}} = \frac{4}{\sqrt{117}}\hat{i} - \frac{1}{\sqrt{117}}\hat{j} - \frac{10}{\sqrt{117}}\hat{k}$$

(*ii*)

$$\overrightarrow{BP} = \text{p.v. of } P - \text{p.v. of } B = \left(-2\hat{i} + 3\hat{j} + 7\hat{k}\right) - \left(-3\hat{i} + 6\hat{j} + 8\hat{k}\right)$$

$$= \hat{i} - 3\hat{j} - \hat{k}$$

$$\overrightarrow{CP} = \text{p.v. of } P - \text{p.v. of } C = \left(-2\hat{i} + 3\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 5\hat{j} - 2\hat{k}\right)$$

$$= -3\hat{i} - 2\hat{j} + 9\hat{k}$$

$$\cos\theta = \frac{\overrightarrow{BP} \cdot \overrightarrow{CP}}{|\overrightarrow{BP}||\overrightarrow{CP}|} = \frac{-3+6-9}{\sqrt{1+9+1}\sqrt{9+4+81}}$$

$$\Rightarrow \quad \cos\theta = \frac{-6}{\sqrt{11}\sqrt{94}} = \frac{-6}{\sqrt{1034}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{-6}{\sqrt{1034}}\right)$$