Solutions to RMM–DS2/Set-3

1. (*b*) As, $$(A + A')' = A' + (A')' = A' + A = A + A' \quad [\because \text{ Matrix addition is commutative}]$$

$\therefore$ $A + A'$ is symmetric matrix.

2. (*b*) $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A = I$$

$\therefore$ $$A^2 = I \text{ and } 2A = 2I$$

$\therefore$ $$A^2 + 2A = I + 2I = 3I = 3A \quad [\because A = I]$$

3. (*d*) $$A + B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$$

Let $$D = A + B, \text{ then } |D| = \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} = -1 - 2 = -3 \neq 0$$

So, D^{-1} exists as $|D| \neq 0$.

Let D_{ij} be the cofactors of d_{ij} in $|D|$.

Now, $$D_{11} = (-1)^2 (1) = 1, D_{12} = (-1)^3 (2) = -2$$

$$D_{21} = (-1)^3 (1) = -1, D_{22} = (-1)^4 (-1) = -1$$

$\therefore$ $$\text{Adj}(D) = \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$\therefore$ $$D^{-1} = \frac{\text{Adj}(D)}{|D|} = \frac{-1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$\therefore$ $$(A + B)^{-1} = \frac{-1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

4. (*c*) Since '*f*' is continuous at $x = \frac{\pi}{2}$, then

$$\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$\Rightarrow$ $$\lim_{x \to \frac{\pi}{2}} \frac{6\cos x}{\pi - 2x} = f\left(\frac{\pi}{2}\right)$$

Put $x - \frac{\pi}{2} = h$. As $x \to \frac{\pi}{2}$, then $h \to 0$

Now, $$\lim_{h \to 0} \frac{6\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)} = f\left(\frac{\pi}{2}\right)$$

$\Rightarrow$ $$\lim_{h \to 0} \frac{-6 \sin h}{-2h} = f\left(\frac{\pi}{2}\right)$$

$\Rightarrow$ $$3\lim_{h \to 0} \frac{\sin h}{h} = f\left(\frac{\pi}{2}\right) \Rightarrow f\left(\frac{\pi}{2}\right) = 3 \times 1 = 3$$

5. (*d*) We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow$ $k^2 + k^2 + k^2 = 1$

$\Rightarrow$ $3k^2 = 1$

$\Rightarrow$ $k = \pm \dfrac{1}{\sqrt{3}}$

6. (*c*) $x\dfrac{dy}{dx} - y = x^4 - 3x$

$\Rightarrow$ $\dfrac{dy}{dx} - \dfrac{y}{x} = x^3 - 3$

Comparing with standard form of linear differential equation, i.e. $\dfrac{dy}{dx} + Py = Q$, we get $P = \dfrac{-1}{x}$; $Q = x^3 - 3$

Now, $IF = e^{\int P dx} = e^{\int -\frac{1}{x}dx}$

$= e^{-\log|x|} = \dfrac{1}{x}$

7. (*b*) We have, $Z = 3x - 4y$

Corner point	Value of $Z = 3x - 4y$	
(0, 0)	$3 \times 0 - 4 \times 0 = 0$	
(5, 0)	$3 \times 5 - 4 \times 0 = 15$	
(6, 5)	$3 \times 6 - 4 \times 5 = -2$	
(6, 8)	$3 \times 6 - 4 \times 8 = -14$	
(4, 10)	$3 \times 4 - 4 \times 10 = -28$	
(0, 8)	$3 \times 0 - 4 \times 8 = -32$	← Minimum

$\therefore$ Minimum of Z occurs at (0, 8).

8. (*a*) Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{j}$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$ [where 'θ' is angle between $\vec{a}$ and $\vec{b}$]

$\Rightarrow$ $\cos\theta = \dfrac{1-1}{\sqrt{2}\sqrt{2}} = 0$

$\Rightarrow$ $\theta = \dfrac{\pi}{2}$

9. (*b*) Let $$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2023}x}{\cos^{2023}x + \sin^{2023}x}dx \quad ...(i)$$

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2023}\left(\frac{\pi}{2} - x\right)}{\cos^{2023}\left(\frac{\pi}{2} - x\right) + \sin^{2023}\left(\frac{\pi}{2} - x\right)}dx$$

[Using property: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$]

$\Rightarrow$ $$I = \int_0^{\frac{\pi}{2}} \frac{\cos^{2023}x}{\sin^{2023}x + \cos^{2023}x}dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2023}x + \cos^{2023}x}{\cos^{2023}x + \sin^{2023}x}dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} dx$$

$$\Rightarrow \quad 2I = \left[x\right]_0^{\frac{\pi}{2}} \quad \Rightarrow \quad 2I = \frac{\pi}{2} - 0 \quad \Rightarrow \quad I = \frac{\pi}{4}$$

10. (*d*) We have, $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$$\Rightarrow \quad 1(-10x^2 - 10x) - 4(5x^2 - 5) + 20(2x + 2) = 0$$

$$\Rightarrow \quad -10x^2 - 10x - 20x^2 + 20 + 40x + 40 = 0$$

$$\Rightarrow \quad -30x^2 + 30x + 60 = 0$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad (x - 2)(x + 1) = 0$$

$$x = 2, \ x = -1$$

$\therefore$ solution set = {2, –1}

11. (*c*)

12. (*a*) Projection of $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$= \frac{(2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}}$$

$$= \frac{2 - 2 + 2}{3} = \frac{2}{3}$$

13. (*c*), Let A be a square matrix of order 'n', then $|kA| = k^n|A|$.

Suppose A is the corresponding matrix such that $\Delta = |A|$. Now, order of matrix A, $n = 3$.

Let $\quad \Delta' = |4A|$

$\Rightarrow \quad \Delta' = 4^3|A| = 64\Delta$

14. (*b*) Total number of ways of selecting 2 numbers from the given set = $2 \times {}^5C_2$

$$= 2 \times \frac{5!}{2!\,3!} = 20$$

Outcomes such that $\frac{a}{b}$ is an integer = $\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{4}{2}$

Number of favourable outcomes = 5

Required probability $= \frac{5}{20} = \frac{1}{4}$

15. (*a*) As, order = 2 and degree = 2

So, required product = 2 × 2 = 4

16. (*a*) Vector along $\overrightarrow{AD}$ $= \frac{3\hat{i} + 0\hat{j} + 5\hat{k}}{2}$

$$|\overrightarrow{AD}| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{34}{4}}$$

$$= \frac{\sqrt{34}}{2} \text{ units}$$

17. (*c*) We have, $y = \sin x°$

$\Rightarrow \quad y = \sin\left(\frac{\pi x}{180}\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\pi}{180} \times \cos\left(\frac{\pi x}{180}\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\pi}{180}\cos x°$

18. (*b*) $\cos a$, $\cos b$, $\cos c$ are the dc's of the line.

19. (*a*) Both (*A*) and (*R*) are true and (*R*) is the correct explanation of (*A*).

20. (*a*) Both (*A*) and (*R*) are true and (*R*) is the correct explanation of (*A*).

21.
$$\tan^{-1}\left[2\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left[2\sin\left(\frac{\pi}{6}\right)\right]$$
$$= \tan^{-1}\left[2 \times \frac{1}{2}\right]$$
$$= \tan^{-1}(1)$$
$$= \frac{\pi}{4}$$

OR

We have, $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = 3\pi$ ...(*i*)

Now, range of $\cos^{-1} x$ is $[0, \pi]$.

$\therefore$ Equation (*i*) is satisfied when $\cos^{-1} p = \pi, \quad \cos^{-1} q = \pi, \quad \cos^{-1} r = \pi$

$\Rightarrow \quad p = \cos \pi, \quad q = \cos \pi, \quad r = \cos \pi$

$\Rightarrow \quad p = -1, \quad q = -1 \quad r = -1$

$\therefore \quad pq + qr + rp = (-1) \times (-1) + (-1) \times (-1) + (-1) \times (-1)$

$= 1 + 1 + 1 = 3$

22. Let '*r*' be the radius and '*V*' be the volume of balloon at any instant '*t*'.

Given $\frac{dV}{dt} = 900 \text{ cm}^3\text{/s},$

We have: $V = \frac{4}{3}\pi r^3$

Differentiating w.r.t. *t*, we get

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$\Rightarrow \quad 900 = \frac{4}{3}\pi \times 3 \times r^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2}$

$\Rightarrow \quad \left.\frac{dr}{dt}\right]_{r=15} = \frac{900}{4\pi \times 15 \times 15}$

$$= \frac{1}{\pi} \text{ cm/s}$$

23. Let the required point on curve be (h, k). Now, point (h, k) must satisfy $x^2 = 2y$ as it lies on curve.

$\therefore \qquad h^2 = 2k \qquad \text{...}(i)$

Distance between (h, k) and $(0, 5)$, $D = \sqrt{(h-0)^2 + (k-5)^2}$

$\Rightarrow \qquad D^2 = h^2 + (k-5)^2$

$\Rightarrow \qquad D^2 = 2k + (k-5)^2 \qquad$ [using (i)]

Let $D^2 = Z$, then $Z = 2k + (k-5)^2$

Differentiating both sides w.r.t. k, we get

$$\frac{dZ}{dk} = 2 + 2(k-5) = 2k - 8$$

For maxima or minima, $\frac{dZ}{dk} = 0 \Rightarrow 2k - 8 = 0 \Rightarrow k = 4$

$$\frac{d^2Z}{dk^2} = 2 > 0$$

So, Z is minimum at $k = 4$ or D^2 is minimum at $k = 4$ i.e. D is minimum at $k = 4$.

If $k = 4$, then $h = \pm 2\sqrt{2} \qquad$ [using (i)]

$\therefore$ Coordinates of required points are $(\pm 2\sqrt{2}, 4)$.

OR

We have, $\qquad f(x) = 2x^2 - 3x$

Differentiating w.r.t. x, we get

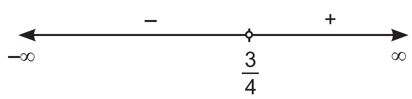
$$f'(x) = 4x - 3$$

For critical points, $f'(x) = 0$

$\Rightarrow \qquad 4x = 3$

$\Rightarrow \qquad x = \frac{3}{4}$

Plotting on number line, we get



Sign of $f'(x)$

Intervals	Sign of $f'(x)$	Nature of 'f'
$\left(-\infty, \frac{3}{4}\right)$	– ve	Strictly decreasing
$\left(\frac{3}{4}, \infty\right)$	+ ve	Strictly increasing

24. $\int_{-1}^{5} |x-3|\, dx$

Let $f(x) = |x-3|$

Now, $f(x) = \begin{cases} -(x-3), \text{ when } x < 3 \\ (x-3), \text{ when } x \geq 3 \end{cases}$

So, $$\int_{-1}^{5} |x-3|\, dx = -\int_{-1}^{3} (x-3)\, dx + \int_{3}^{5} (x-3)\, dx$$

$$= -\left[\frac{x^2}{2} - 3x\right]_{-1}^{3} + \left[\frac{x^2}{2} - 3x\right]_{3}^{5}$$

$$= -\left[\left(\frac{9}{2} - 9\right) - \left(\frac{1}{2} + 3\right)\right] + \left[\left(\frac{25}{2} - 15\right) - \left(\frac{9}{2} - 9\right)\right]$$

$$= \left(\frac{9}{2} + \frac{7}{2}\right) + \left(-\frac{5}{2} + \frac{9}{2}\right) = 8 + 2 = 10$$

25. $f(x) = \tan^{-1}(\sin x + \cos x)$

Differentiating both sides, w.r.t. x, we get

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

So, when $0 < x < \frac{\pi}{4}$, then $\cos x - \sin x > 0$ and $\frac{1}{1 + (\sin x + \cos x)^2} > 0$

$\Rightarrow$ $$\frac{(\cos x - \sin x)}{1 + (\sin x + \cos x)^2} > 0$$

$\therefore$ $f'(x) > 0$

Hence, $f(x)$ is increasing on $\left(0, \frac{\pi}{4}\right)$.

26. $$I = \int \frac{1}{x(x^4 - 1)} dx$$

$$= \int \frac{1}{x^5\left(1 - \frac{1}{x^4}\right)} dx$$

$$= \frac{1}{4}\int \frac{dt}{t} = \frac{1}{4} \log|t| + C$$

$$= \frac{1}{4} \log\left|1 - \frac{1}{x^4}\right| + C$$

Put $1 - \frac{1}{x^4} = t$

$\Rightarrow$ $\frac{4}{x^5} dx = dt$

$\Rightarrow$ $\frac{dx}{x^5} = \frac{dt}{4}$

27. For biased coin :

$$P(H) = \frac{3}{4}, \quad P(T) = \frac{1}{4}$$

X = A random variable that denotes number of tails when coin is tossed twice

So, $X = 0, 1$ or 2

$$P(X = 0) = P(H)\, P(H) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$P(X = 1) = P(H)\ P(T) + P(T)\ P(H) = \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} = \frac{6}{16}$$

$$P(X = 2) = P(T)\ P(T) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

The probability distribution table is given below:

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

28. $$I = \int \underset{\text{I}}{e^{2x}} \underset{\text{II}}{\cos x}\, dy$$

Integrating by parts, we get

$$I = e^{2x} \cdot \int \cos x\, dx - \int \left(\frac{d}{dx} e^{2x} \cdot \int \cos x\right) dx$$

$$= e^{2x} \cdot \sin x - \int e^{2x} \cdot 2 \cdot \sin x\, dx$$

$$= e^{2x} \sin x - 2 \int \underset{\text{I}}{e^{2x}} \underset{\text{II}}{\sin} x\, dx$$

$$= e^{2x} \sin x - 2\left[- e^{2x} \cdot \cos x + \int e^{2x} \cdot 2 \cos x\, dx\right]$$

$$= e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x\, dx$$

$$= e^{2x} \sin x + 2e^{2x} \cos x - 4I$$

$\Rightarrow$ $$5I = e^{2x} (\sin x + 2 \cos x)$$

$\Rightarrow$ $$I = \frac{1}{5} e^{2x} (\sin x + 2 \cos x) + C$$

OR

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx \qquad \text{...}(i)$$

$$= \int_0^{\pi} \frac{(\pi - x) \tan (\pi - x)}{\sec (\pi - x) + \tan (\pi - x)} dx \qquad \left[\because \int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx\right]$$

$$I = \int_0^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x}\, dx \qquad \text{...}(ii)$$

Adding (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x}\, dx$$

$$= \pi \int_0^{\pi} \frac{\tan x (\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} dx$$

$$= \pi \int_0^{\pi} (\tan x \sec x - \tan^2 x)\, dx$$

$$= \pi \int_0^{\pi} (\tan x \sec x - \sec^2 x + 1)\, dx$$

$$= \pi \left[\sec x - \tan x + x\right]_0^{\pi}$$

$$= \pi\,[(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0)]$$
$$= \pi\,[(-1 - 0 + \pi) - (1 - 0 + 0)]$$
$$= \pi\,(-1 + \pi - 1) = \pi\,(\pi - 2)$$
$$\therefore \quad I = \frac{\pi}{2}(\pi - 2)$$

29. $$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2x}{x^2 - 1}y = \frac{2}{(x^2 - 1)^2} \qquad ...(i)$$

Here, $P = \frac{2x}{x^2 - 1}$, $Q = \frac{2}{(x^2 - 1)^2}$, $\left(\text{Comparing } (i) \text{ with } \frac{dy}{dx} + Py = Q\right)$

$$IF = e^{\int P\,dx} = e^{\int \frac{2x}{x^2-1}dx}$$
$$= e^{\int \frac{dt}{t}}$$
$$= e^{\log|t|} = t$$
$$IF = x^2 - 1$$

Let $x^2 - 1 = t$
$\Rightarrow \quad 2x\,dx = dt$

$\therefore$ Solution of (i) is given by,

$$y \times IF = \int Q \times IF\,dx$$
$$\Rightarrow \quad y \cdot (x^2 - 1) = \int \frac{2}{(x^2-1)^2} \times (x^2 - 1)\,dx$$
$$\Rightarrow \quad y \cdot (x^2 - 1) = 2\int \frac{dx}{x^2 - 1}$$
$$\Rightarrow \quad y \cdot (x^2 - 1) = 2 \times \frac{1}{2 \times 1}\log\left|\frac{x-1}{x+1}\right| + C$$
$$\Rightarrow \quad y(x^2 - 1) = \log\left|\frac{x-1}{x+1}\right| + C$$

OR

We have, $$\log\left(\frac{dy}{dx}\right) = 3x + 4y$$
$$\Rightarrow \quad \frac{dy}{dx} = e^{3x+4y} = e^{3x} \cdot e^{4y}$$
$$\Rightarrow \quad \frac{dy}{e^{4y}} = e^{3x}\,dx$$

Integrating both sides, we get

$$\int e^{-4y}dy = \int e^{3x}dx$$
$$\Rightarrow \quad -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C$$

30. The given constraints are :

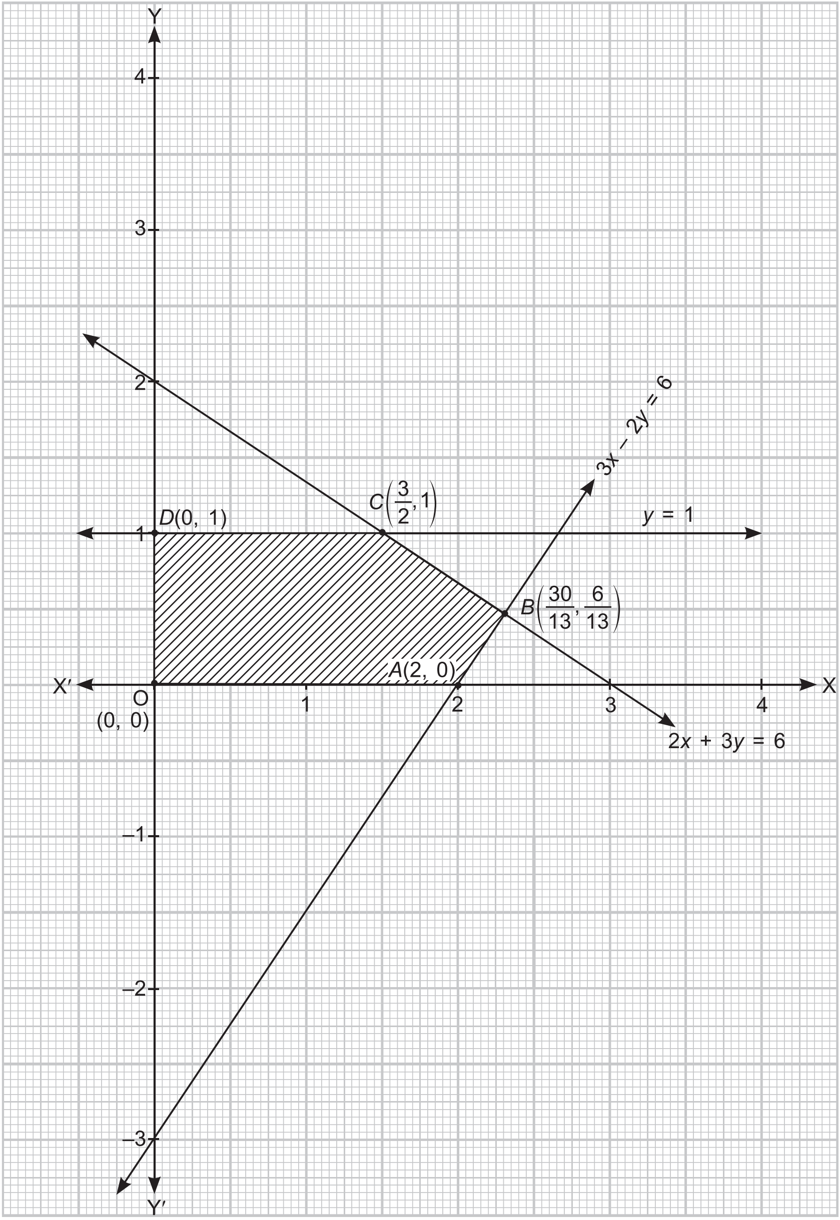
$$2x + 3y \le 6$$
$$3x - 2y \le 6$$
$$y \le 1$$
$$x, y \ge 0$$

Converting given inequations to equations, we get

$2x + 3y = 6$, $3x - 2y = 6$, $y = 1$, $x = 0$ and $y = 0$

These lines are drawn on the graph and shaded region OABCD represents the feasible region for inequations of given LPP.

The coordinates of corner points of the feasible region are : $O(0, 0)$, $A(2, 0)$, $B\left(\frac{30}{13}, \frac{6}{13}\right)$, $C\left(\frac{3}{2}, 1\right)$ and $D(0, 1)$.



The value of objective function Z at the corner points are given in the following table:

Corner point	**Value of $Z = 8x + 9y$**	
$O(0, 0)$	$0 + 0 = 0$	
$A(2, 0)$	$8 \times 2 + 9 \times 0 = 16$	
$B\left(\frac{30}{13}, \frac{6}{13}\right)$	$8 \times \frac{30}{13} + 9 \times \frac{6}{13} = \frac{294}{13}$	← Maximum
$C\left(\frac{3}{2}, 1\right)$	$8 \times \frac{3}{2} + 9 \times 1 = 21$	
$D(0, 1)$	$8 \times 0 + 9 \times 1 = 9$	

So, maximum value of Z occurs at $B\left(\frac{30}{13}, \frac{6}{13}\right)$ and maximum value is $\frac{294}{13}$.

OR

The given constraints are :

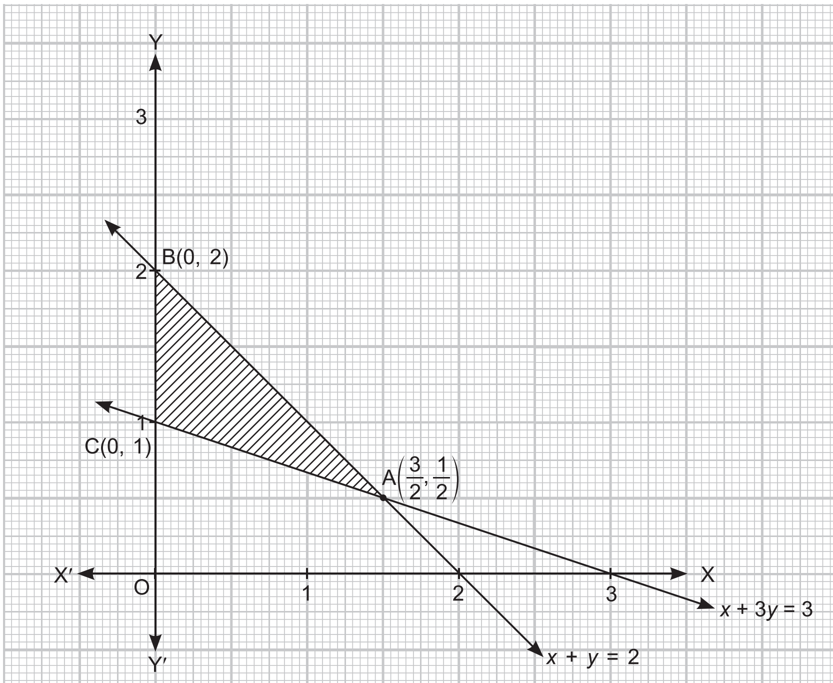
$$x + 3y \geq 3$$
$$x + y \leq 2$$
$$x, y \geq 0$$

Converting given inequations to equations, we get

$x + 3y = 3$, $x + y = 2$, $x = 0$ and $y = 0$.

These lines are drawn on the graph and shaded region represent the feasible region for inequations of given LPP.

The coordinates of corner points of the feasible region are : $A\left(\frac{3}{2}, \frac{1}{2}\right)$, $B(0, 2)$ and $C(0, 1)$



Corner point	Value of $Z = 3x + 5y$	
$A\left(\frac{3}{2}, \frac{1}{2}\right)$	$3 \times \frac{3}{2} + 5 \times \frac{1}{2} = 7$	
$B(0, 2)$	$3 \times 0 + 5 \times 2 = 10$	
$C(0, 1)$	$3 \times 0 + 5 \times 1 = 5$	← Minimum

So, minimum value of Z occurs at $C(0, 1)$.

Minimum value of Z is 5.

31. $$x = \sin t, \quad y = \sin pt$$

Differentiating w.r.t. 't' we get

$$\frac{dx}{dt} = \cos t, \qquad \frac{dy}{dt} = p \cos pt$$

Now, $$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{p \cos pt}{\cos t}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{p \,.\, \sqrt{1 - \sin^2 pt}}{\sqrt{1 - \sin^2 t}}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{p \sqrt{(1 - y^2)}}{\sqrt{1 - x^2}}$$

On squaring both sides, we get

$$\Rightarrow \qquad \left(\frac{dy}{dx}\right)^2 = \frac{p^2 (1 - y^2)}{(1 - x^2)}$$

$$\Rightarrow \qquad (1 - x^2) \left(\frac{dy}{dx}\right)^2 = p^2 (1 - y^2)$$

Differentiating again w.r.t. x, we get

$$\Rightarrow \quad (1 - x^2) \times 2\left(\frac{dy}{dx}\right)\cdot\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 \times (-2x) = p^2 (-2y) \left(\frac{dy}{dx}\right)$$

Dividing both sides by $\left(2 \cdot \frac{dy}{dx}\right)$, we get

$$\Rightarrow \qquad (1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = -p^2 y$$

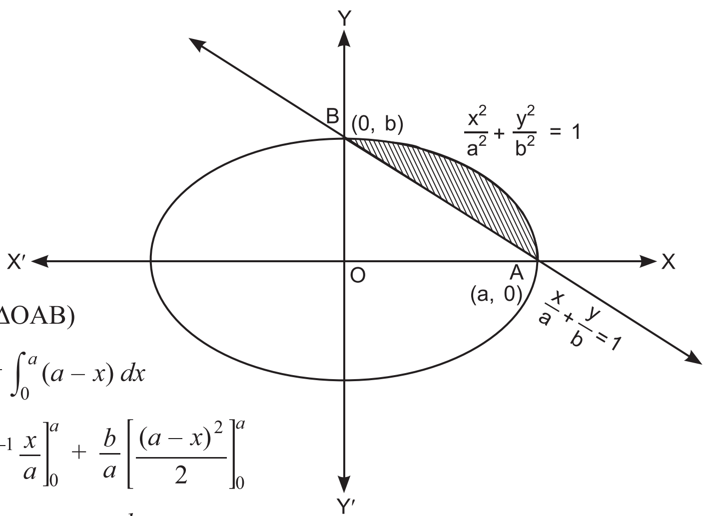
$$\Rightarrow \qquad (1 - x^2) \frac{d^2 y}{dx^2} = x \frac{dy}{dx} - p^2 y$$

32. Equation of AB is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \qquad y = b\left(1 - \frac{x}{a}\right)$$

$$\Rightarrow \qquad y = \frac{b}{a}(a - x)$$



Area of shaded region = ar(Quadrant OAB) – ar(ΔOAB)

$$= \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx - \frac{b}{a}\int_0^a (a - x)\, dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a + \frac{b}{a}\left[\frac{(a - x)^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1} 1\right] - \left(\frac{a^2}{2}\right)\frac{b}{a} = \frac{\pi}{4}ab - \frac{ab}{2}$$

$$= \left(\frac{\pi ab}{4} - \frac{ab}{2}\right) \text{ sq units}$$

33. $R = \{(a, b): |a - b| \text{ is divisible by } 5\}$

For Reflexive:

Let $(a, a) \in R$, then $(a, a) \in R$

$\Rightarrow \qquad a - a = 0$, which is divisible by 5.

∴ R is reflexive.

For Symmetric:

Let $(a, b) \in R$. Then $(a, b) \in R \Rightarrow$ 5 divides $(a - b) \Rightarrow a - b = 5\lambda$

$$\Rightarrow -(b - a) = 5\lambda$$

$\Rightarrow \quad (b - a) = -5\lambda$

$\therefore \quad (b, a) \in R$

$\therefore$ R is symmetric relation.

For Transitive: Let $(a, b) \in R$ and $(b, c) \in R$. Then,

$$(a, b) \in R \Rightarrow a - b = 5\lambda \qquad ...(i)$$

$$(b, c) \in R \Rightarrow b - c = 5k \qquad ...(ii)$$

By (*i*) and (*ii*), $\quad a - c = 5(\lambda + k)$

$\therefore \quad (a, c) \in R$

$\therefore$ R is a transitive relation.

Hence, R is an equivalence relation on Z.

OR

$$f(x) = x^2 + x + 1$$

For one-one function:

Let $\quad x_1, x_2 \in N$

Now, $\quad f(x_1) = f(x_2)$

$\Rightarrow \quad x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow \quad x_1^2 - x_2^2 + x_1 - x_2 = 0$

$\Rightarrow \quad (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0$

$\Rightarrow \quad (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\therefore \quad x_1 - x_2 = 0 \qquad [x_1 + x_2 + 1 \neq 0 \text{ as } x_1, x_2 \in N]$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one

For onto:

Let $y \in N$ (co-domain).

Then, $\quad f(x) = y$

$\Rightarrow \quad x^2 + x + 1 = y$

$\Rightarrow \quad x^2 + x + (1 - y) = 0$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{4y - 3}}{2}$$

$$\Rightarrow \quad x = \frac{-1 + \sqrt{4y - 3}}{2}$$

On taking $y = 16 \in N$(co-domain), the corresponding value of $x \notin N$(domain).

So, every element in the co-domain doesn't have its pre-image in domain. So, f is not onto.

34. $$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$\Rightarrow$ $$|A| = 1 \times 4 + 1\,(2 + 3) + 1 \times (2 - 1)$$

$$= 4 + 5 + 1 = 10 \neq 0, \;\therefore\; A^{-1} \text{ exits}$$

Let A_{ij} be the cofactors of a_{ij} in $|A|$.

$$A_{11} = 4, \quad A_{12} = -5, \quad A_{13} = 1$$

$$A_{21} = 2, \quad A_{22} = 0, \quad A_{23} = -2$$

$$A_{31} = 2, \quad A_{32} = 5, \quad A_{33} = 3$$

$$\text{adj. } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$\therefore$ $$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

The given system of equations is

$$x + 2y + z = 4$$

$$-x + y + z = 0$$

$$x - 3y + z = 2$$

Matrix form: $$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

i.e. $$BX = C$$

$\Rightarrow$ $$X = B^{-1}C$$

Now, $$B = A'$$

$\therefore$ $$B^{-1} = (A^{-1})'$$

$\therefore$ $$B^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

Now, $$X = B^{-1}C$$

$$= \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

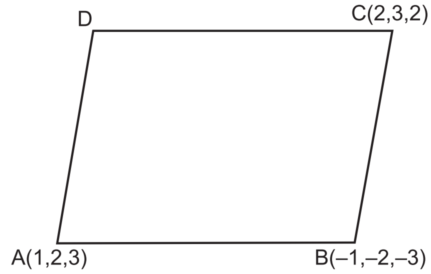
$\Rightarrow$ $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 - 0 + 2 \\ 8 + 0 - 4 \\ 8 + 0 + 6 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18/10 \\ 4/10 \\ 14/10 \end{bmatrix}$$

So, $x = \frac{18}{10}, \quad y = \frac{4}{10}, \quad z = \frac{14}{10}$

i.e. $x = 1.8, \quad y = 0.4, \quad z = 1.4$ is the solution.

35. Let coordinates of D be (α, β, γ).



In ||gm, diagonals bisect each other.

$\therefore$ Mid point of AC = Mid point of BD

$$\Rightarrow \left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = \left(\frac{\alpha-1}{2}, \frac{\beta-2}{2}, \frac{\gamma-3}{2}\right)$$

$\Rightarrow \alpha - 1 = 3, \quad \beta - 2 = 5, \quad \gamma - 3 = 5$

$\Rightarrow \alpha = 4, \quad \beta = 7, \quad \gamma = 8$

$\therefore$ Co-ordinates of D are (4, 7, 8).

Cartesian equation of a line passing through points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$\therefore$ Equation of CD is,

$$\frac{x-2}{4-2} = \frac{y-3}{7-3} = \frac{z-2}{8-2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-3}{4} = \frac{z-2}{6}$$

OR

We have, $\vec{r} = (8+3\lambda)\hat{i} - \hat{j}(9+16\lambda) + (10+7\lambda)\hat{k}$

$\Rightarrow \vec{r} = 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

Here, $\vec{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}$ and $\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$

Also, $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Here, $\vec{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$ and $\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$

Now, $\vec{a_2} - \vec{a_1} = 15\hat{i} + 29\hat{j} + 5\hat{k} - 8\hat{i} + 9\hat{j} - 10\hat{k} = 7\hat{i} + 38\hat{j} - 5\hat{k}$

Also, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$

$= \hat{i}(80-56) - \hat{j}(-15-21) + \hat{k}(24+48)$

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$

$|(\vec{b_1} \times \vec{b_2})| = \sqrt{576+1296+5184}$

$= \sqrt{7056} = 84$

Shortest distance between lines is given by,

$$\text{S.D.} = \left| \frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|} \right| = \left| \frac{(24\hat{i}+36\hat{j}+72\hat{k})\cdot(7\hat{i}+38\hat{j}-5\hat{k})}{84} \right|$$

$$= \left| \frac{168+1368-360}{84} \right| = \left| \frac{1176}{84} \right|$$

$= 14$ units

36. (*i*) Charge per student after increasing ₹ x per month = ₹ $(300 + x)$

Number of students becomes $(500 - x)$

Revenue function, $R(x) = (300 + x)(500 - x)$

$\Rightarrow R(x) = 150000 + 200x - x^2$

(*ii*) $$R(x) = 150000 + 200x - x^2$$

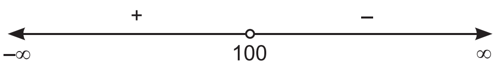
$$\Rightarrow \quad R'(x) = 200 - 2x$$

For critical point, $R'(x) = 0$

$$\Rightarrow \quad 200 - 2x = 0$$

$$\Rightarrow \quad x = 100$$

(*iii*) The changes in the sign of $R'(x)$ for different values of x are shown in the below figure



Clearly, $R'(x)$ changes sign for positive to negative as increases through 100.

So, $x = 100$ is a point of maxima.

So, maximum revenue = 150000 + 200 × 100 – 100 × 100 = ₹ 1,60,000

OR

(*iii*) $$R''(x) = -2 \qquad [\because R'(x) = 200 - 2x]$$

Now, $R''(x)]_{x = 100} = -2 < 0$

So, R is maximum at $x = 100$

Maximum revenue = 150000 + 200 × 100 – 100 × 100 = ₹ 1,60,000

37. (*i*) $$P(E/E_1) = 1$$

$$P(E/E_2) = \frac{1}{4}$$

(*ii*) $$P(E_1) = \frac{3}{4},\ P(E_2) = \frac{1}{4}$$

P(Student answers correctly)

$$= P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$$

$$= \frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{13}{16}$$

(*iii*) $$P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)}$$

$$= \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}}$$

$$= \frac{3}{3 + \frac{1}{4}} = \frac{12}{13}$$

OR

(*iii*) $$P(E_2/E) = \frac{P(E_2) \times P(E/E_2)}{P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)}$$

$$= \frac{\frac{1}{4} \times \frac{1}{4}}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}}$$

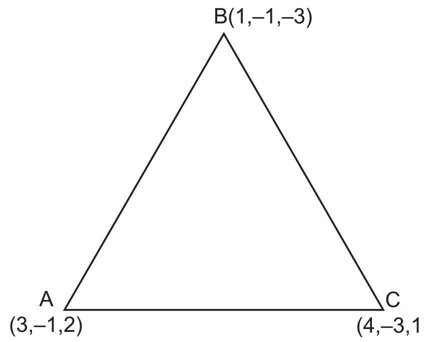
$$= \frac{\frac{1}{4}}{3 + \frac{1}{4}} = \frac{1}{13}$$

38. $\overrightarrow{AB}$ = P.V. of B – P.V. of A

$$= (\hat{i} - \hat{j} - 3\hat{k}) - (3i - \hat{j} + 2\hat{k}) = -2\hat{i} - 5\hat{k}$$

$\overrightarrow{AC}$ = P.V. of C – P.V. of A

$$= (4\hat{i} - 3\hat{j} + \hat{k}) - (3i - \hat{j} + 2\hat{k}) = \hat{i} - 2\hat{j} - \hat{k}$$



Vector $\perp$ to the plane of ΔABC, $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}(-10) - \hat{j}(7) + \hat{k}(4)$$

$$= -10\hat{i} - 7\hat{j} + 4\hat{k}$$

(*i*) $\therefore$ Required unit vector $= \frac{\vec{n}}{|\vec{n}|} = \left(\frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{100 + 49 + 16}}\right)$

$$= \frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}}$$

$$= -\frac{10}{\sqrt{165}}\hat{i} - \frac{7}{\sqrt{165}}\hat{j} + \frac{4}{\sqrt{165}}\hat{k}$$

(*ii*) Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$= \frac{1}{2}|\sqrt{165}|$$

$$= \frac{1}{2}\sqrt{165} \text{ sq units.}$$