Answers to RPH–DS2/Set-3

1. (*d*) Inside the hollow sphere, the electric field remains zero.

$$\therefore \quad E = -\frac{dV}{dx} = 0$$

$$\Rightarrow \quad V = \text{constant} \quad \therefore \; V = 500 \text{ V}$$

2. (*c*) We know that, drift velocity,

$$v_d = \frac{I}{neA}$$

$$\Rightarrow \quad v_d \propto \frac{1}{A}$$

∴ As area increases, drift velocity decreases.

3. (*a*)

4. (*c*)

5. (*a*)

6. (*c*)

7. (*b*)

8. (*b*) When a current carrying circular loop is placed in a magnetic field perpendicular to its plane then every portion of the loop will experience radially outward force. Hence, the loop expands.

9. (*a*) At resonance, series LCR circuit acts as purely resistive circuit and voltage across 'L' = Voltage across 'C'.

∴ Reading in V_2 = Reading in V_3.

10. (*c*)

11. (*c*)

12. (*d*)

13. (*d*) If both Assertion and Reason are false.

14. (*b*) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

15. (*b*) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

16. (*d*) If both Assertion and Reason are false.

17. (*a*) When $V_A > V_B$

In this situation both the diodes D_1 and D_2 are forward biased, hence the current will flow in the circuit.

As both the resistors are in parallel combination,

$$\therefore \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{100 \times 100}{100 + 100} = \frac{100 \times 100}{200} = 50 \; \Omega$$

(*b*) When $V_B > V_A$

In this situation both the diodes, will be in reverse bias and hence no current will flow in the circuit.

Therefore, equivalent resistance of the circuit will be infinite.

18. Using the equation of photoelectric effect,

For 1st situation, $$eV_0 = \frac{hc}{\lambda} - \phi_0$$

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \qquad \ldots(i) \left[\because \phi_0 = \frac{hc}{\lambda_0}\right]$$

For 2nd situation, $$\frac{eV_0}{4} = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$$

or $$eV_0 = 4\left[\frac{hc}{2\lambda} - \frac{hc}{\lambda_0}\right] \qquad \ldots(ii)$$

From equation (*i*) and (*ii*), we get

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = 4\left[\frac{hc}{2\lambda} - \frac{hc}{\lambda_0}\right]$$

or $$\frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{2}{\lambda} - \frac{4}{\lambda_0}$$

or $$-\frac{1}{\lambda} = -\frac{3}{\lambda_0}$$

$\therefore$ Threshold wavelength, $\lambda_0 = 3\lambda$

19. Following changes will be observed:

(*i*) In each diffraction order, the diffracted image of the slit gets dispersed into component colours of white light.

As fringe width $\propto \lambda$, so the red fringe with higher wavelength is wider than the violet fringe with smaller wavelength.

(*ii*) In higher order spectra the dispersion is more which causes overlapping of different colours.

20. Given: $N = 10^{21}$, $l = 50$ mm, $v_d = 0.25$ mm/s

$\because$ $$v_d = \frac{I}{neA}$$

Here, $$n = \frac{N}{Al} = \frac{10^{21}}{50A}$$

$\therefore$ $$I = neAv_d$$

$$I = \frac{10^{21}}{50A} \times 1.6 \times 10^{-19} \times A \times 0.25$$

$$I = \frac{16 \times 25}{50} \times 10^{-1} = 0.8 \text{ A}$$

21. We know that, $\sin i_C = \frac{1}{\mu}$

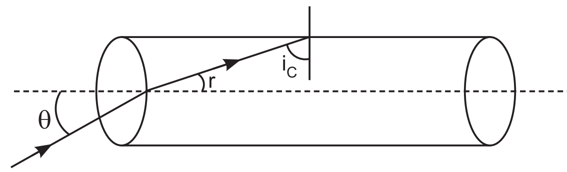
Given, $\mu = 2$

$\therefore \quad \sin i_C = \frac{1}{2}$

or $\quad i_C = 30°$

$\because \quad \sin r = \sin(90° - i_C) = \cos i_C = \cos 30° = \frac{\sqrt{3}}{2}$

Using Snell's law, $\frac{\sin\theta}{\sin r} = \frac{\mu_2}{\mu_1}$



or $\quad \frac{\sin\theta}{\sqrt{3}/2} = \frac{2}{1}$

or $\quad \sin\theta = 2 \times \frac{\sqrt{3}}{2}$

or $\quad \theta = \sin^{-1}(\sqrt{3})$

Or

Let I_0 be the intensity of light from each slit.

When slit is not painted,

$$I_{max} = (\sqrt{I_0} + \sqrt{I_0})^2 = 4I_0$$

and $$I_{min} = (\sqrt{I_0} - \sqrt{I_0})^2 = 0$$

When one of the slits is painted, it transmits one-fourth of the original intensity.

$$\therefore \quad I_{max} = \left[\sqrt{I_0} + \sqrt{\frac{I_0}{4}}\right]^2$$

$$= I_0\left[1 + \frac{1}{2}\right]^2 = \frac{9}{4}I_0 = 2.25I_0$$

and $$I_{min} = \left[\sqrt{I_0} - \sqrt{\frac{I_0}{4}}\right]^2$$

$$= I_0\left[1 - \frac{1}{2}\right]^2 = \frac{1}{4}I_0 = 0.25I_0$$

Hence, on painting one of the two slits, the intensity of maxima decreases from $4I_0$ to $2.25I_0$ and that of minima increases from 0 to $0.25I_0$.

The contrast between the bright and dark fringes decreases.

22. (*a*) Density of nuclear matter = 10^{17} kg m^{-3}

Density of ordinary matter = 10^3 kg m^{-3}

Ratio = $\frac{10^{17}}{10^3} = 10^{14}$

This shows that the density of nuclear matter is tremendously larger than that of ordinary matter because most of the space within an atom is empty and ordinary matter consists of such atoms having large amount of empty space.

(*b*) Let, A = mass number

R = Radius of a nucleus

m = Average mass of a nucleon

$\therefore$ Mass of nucleus = mA

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3$$

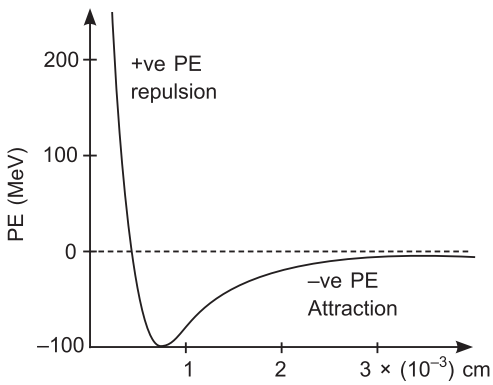
$$= \frac{4}{3}\pi(R_0 A^{1/3})^3 \qquad [\because R = R_0 A^{1/3}]$$

$$= \frac{4}{3}\pi R_0^3 A$$

$$\therefore \text{ Nuclear density,} \qquad \rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

This shows that the nuclear density is independent of mass number A.

(*c*)



23. (*a*) The plane passing through PQ, is perpendicular to the electric field and hence is an equipotential surface. Therefore, there is no potential difference between P and Q i.e., $\Delta V_{QP} = 0$

(*b*) $\because$ $$E = \frac{-\Delta V}{\Delta y}$$

or $$\Delta V_{RQ} = -E\Delta y = -500(4-1) = -1500 \text{ V}$$

(*c*) Since P and Q lie on same equipotential surface,

$\therefore$ $$V_P = V_Q$$

$\therefore$ $$\Delta V_{RP} = V_R - V_P$$

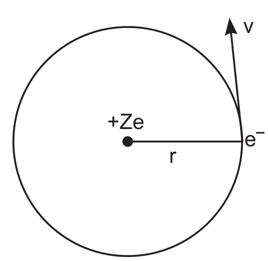
$$= V_R - V_Q = \Delta V_{RQ} = -1500 \text{ V}$$

24. Let r = radius of the orbit

m = mass of the electron

v = speed of the electron

As the e^- revolves around the nucleus, the centripetal force to the electron is provided by the electrostatic force between the electron and the nucleus.



$\therefore \quad F_{electric} = F_{centripetal}$

$$K\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

or $$mv^2 = \frac{KZe^2}{r}$$

or $$m^2v^2 = \frac{mKZe^2}{r} \quad \ldots(i)$$

By Bohr's quantisation condition,

$$mvr = \frac{nh}{2\pi}$$

or $$m^2v^2r^2 = \frac{n^2h^2}{4\pi^2} \quad \ldots(ii)$$

Now dividing equation (*ii*) by equation (*i*), we get

$$r^2 = \frac{n^2h^2r}{4\pi^2KZe^2m}$$

or $$r = \frac{n^2h^2}{4\pi^2KZe^2m}$$

Here, $$K = \frac{1}{4\pi\varepsilon_0}$$

$\therefore$ $$r = \frac{n^2h^2\varepsilon_0}{\pi mZe^2}$$

For hydrogen atom, $Z = 1$

$\therefore$ $$r = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

25. Using, $$R = R_0(1 + \alpha\Delta t)$$

At 100 °C, $$10 = R_0(1 + 100\ \alpha) \quad \ldots(i)$$

At 200 °C, $$12 = R_0(1 + 200\ \alpha) \quad \ldots(ii)$$

Dividing equation (*i*) by equation (*ii*), we get

$$\frac{10}{12} = \frac{R_0(1+100\ \alpha)}{R_0(1+200\ \alpha)}$$

or $$5(1 + 200\ \alpha) = 6(1 + 100\ \alpha)$$

$$5 + 1000\ \alpha = 6 + 600\ \alpha$$

or $$1000\ \alpha - 600\ \alpha = 6 - 5$$

or $$400\ \alpha = 1$$

or $$\alpha = \frac{1}{400}\ °C^{-1}$$

Using this in equation (*i*), we get

$$10 = R_0\left[1 + 100\left(\frac{1}{400}\right)\right]$$

$$10 = R_0\left[1 + \frac{1}{4}\right]$$

$$10 = \frac{5R_0}{4}$$

or $$R_0 = \frac{40}{5} = 8\ \Omega$$

∴ The temperature for 14 Ω,

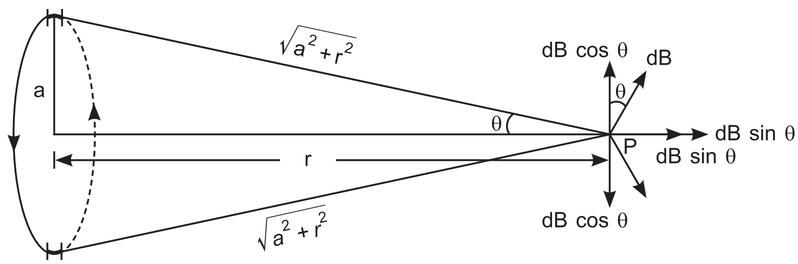
⇒ $$14 = 8\left(1 + \frac{1}{400}\Delta T\right)$$

or $$\left(\frac{14}{8} - 1\right) \times 400 = \Delta T$$

$$\frac{6}{8} \times 400 = \Delta T$$

$$\Delta T = 300\ °C$$

26. (*a*)



Consider a current carrying loop of radius '*a*'. Let at a distance '*r*' from the centre of the loop, there is a point '*P*' at which magnetic field is to be determined.

Let a small current element, then the magnetic field at '*P*' due to this element is (using Biot-Savart's law)

$$dB = \frac{\mu_0}{4\pi}\frac{Idl \sin 90°}{(\sqrt{a^2 + r^2})^2} = \frac{\mu_0}{4\pi}\frac{Idl}{a^2 + r^2}$$

On resolving *dB*, due to various current elements of the loop, we find that the vertical components being equal and opposite get canceled whereas the horizontal components are added.

∴ Net magnetic field at '*P*' due to the current carrying loop is given by

$$B = \int_0^{2\pi a} dB \sin\theta$$

$$= \int_0^{2\pi a} \frac{\mu_0}{4\pi}\frac{Idl}{a^2 + r^2} \cdot \frac{a}{\sqrt{(a^2 + r^2)}}$$

$$= \frac{\mu_0}{4\pi}\frac{Ia}{(a^2 + r^2)^{3/2}}\int_0^{2\pi a} dl$$

$$= \frac{\mu_0}{4\pi}\frac{Ia}{(a^2 + r^2)^{3/2}}(2\pi a)$$

$$B = \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{(a^2 + r^2)^{3/2}}$$

(*b*) Given: radius of loop = *a*

∴ Magnetic field at the centre of the loop is

$$B = \frac{\mu_0 I a^2}{2a^3}$$

$$= \frac{\mu_0 I}{2a} \quad \ldots(i)$$

But magnetic field on the axis at distance '*a*' from the centre is given by,

$$B' = \frac{\mu_0 I a^2}{2(a^2 + a^2)^{3/2}} = \frac{\mu_0 I a^2}{2 \times 2^{3/2} a^3}$$

$$= \frac{\mu_0 I}{4\sqrt{2}\,a} \quad \ldots(ii)$$

Taking ratio of B and B', we get

$$\frac{B}{B'} = \frac{\mu_0 I}{2a} \times \frac{4\sqrt{2}\,a}{\mu_0 I} = 2\sqrt{2} : 1$$

27. (*a*) Microwaves < X-rays < Gamma rays

(*b*) (*i*) Microwaves are produced by special vacuum tube like klystron and magnetron.

(*ii*) X-rays are produced by sudden deceleration of fast moving electrons by a metal target.

(*iii*) Gamma rays are produced by radioactive decay of the nucleus.

(*c*) Ultraviolet radiation

28. Given: $\varepsilon_0 = 8$ V, $\nu = \frac{30}{\pi}$ Hz, $R = 8\ \Omega$, $P = 0.4$ W

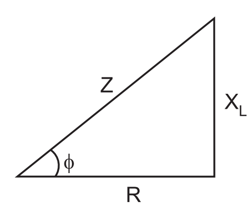
Here, $\omega = 2\pi\nu$

$$= 2 \times \pi \times \frac{30}{\pi} = 60 \text{ rad/s}$$

Also, $\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}} = \frac{8}{\sqrt{2}}$

Power factor, $\cos\phi = \frac{R}{Z}$

Here, $Z = \sqrt{X_L^2 + R^2}$



∴ Power, $P = \varepsilon_{rms} I_{rms} \cos\phi$

$$= \varepsilon_{rms}\left(\frac{\varepsilon_{rms}}{Z}\right)\left(\frac{R}{Z}\right)$$

$$= \frac{\varepsilon_{rms}^2 R}{Z^2}$$

$$= \frac{\varepsilon_{rms}^2 R}{X_L^2 + R^2}$$

or $$0.4 = \frac{\left(\frac{8}{\sqrt{2}}\right)^2 \times 8}{X_L^2 + 8^2}$$

$$0.4\, X_L^2 + 25.6 = 32 \times 8$$

$$0.4\, X_L^2 = 256 - 25.6$$

$$X_L^2 = \frac{230.4}{0.4}$$

$$X_L^2 = 576$$

or $$X_L = \sqrt{576} = 24$$

$\because$ $$X_L = \omega L$$

$\therefore$ $$L = \frac{X_L}{\omega} = \frac{24}{60} = 0.4 \text{ H}$$

Or

(*a*) SI unit of self inductance is Henry.

1 Henry: One Henry is defined as the amount of inductance that generates a change of one volt and when the current is varying at the rate of one ampere per second.

(*b*) $l = 1$ m, $n = 20$ turns/cm $= 2000$ turns/m

$A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, $dI = 2\text{A} - 1 \text{ A} = 1 \text{ A}$, $dt = 0.2$ s

Using, $$|\varepsilon| = L\frac{dI}{dt}$$

$$= \mu_0 n^2 A l \frac{dI}{dt}$$

$$= 4\pi \times 10^{-7}(2000)^2 \times 10^{-4} \times 1 \times \frac{1}{0.2}$$

$$= 4 \times 3.14 \times 10^{-7} \times 4 \times 10^6 \times 5 \times 10^{-4}$$

$$= 8 \times 3.14 \times 10^{-4}$$

$$= 25.12 \times 10^{-4} = 2.51 \text{ mV}$$

29. (*i*) (*c*) (*ii*) (*d*)

(*iii*) (*b*) Given, $a = \text{A}$, $\phi = \frac{\pi}{2}$

Using, $$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$A' = 2a \cos\left(\frac{\phi}{2}\right)$$

$$= 2\text{A} \cos\left(\frac{\pi}{4}\right)$$

$$= 2\text{A}\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}\,\text{A}$$

(*iv*) (*a*)

Or

(*iv*) (*d*) Using, $I' = 4I\cos^2\left(\frac{\phi}{2}\right)$

Given, $I' = 2I$

Then, $2I = 4I\cos^2\left(\frac{\phi}{2}\right)$

$$\frac{1}{2} = \cos^2\left(\frac{\phi}{2}\right)$$

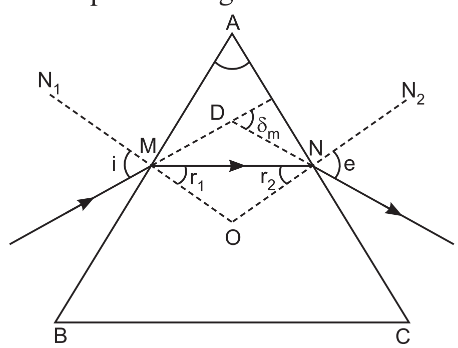
or $\cos^2\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$

or $\frac{\phi}{2} = \frac{\pi}{4}$

or $\phi = \frac{\pi}{2}$

30. (*i*) (*a*) (*ii*) (*c*) (*iii*) (*c*) (*iv*) (*c*) ***Or*** (*iv*) (*d*)

31. (*a*) **Prism:** A prism is a piece of transparent material, often glass or plastic, which consists of straight planes and is cut at precise angles.



In ΔDMN, $(i - r_1) + (e - r_2) = \delta$

or $(i + e)\,(r_1 - r_2) = \delta$ ...(*i*)

In quadrilateral AMON,

$$\angle A + \angle M + \angle N + \angle O = 360°$$

$$\angle A + 90° + 90° + \angle O = 360°$$

or $\angle A + \angle O = 180°$ ...(*ii*)

In ΔMNO, $\angle M + \angle N + \angle O = 180°$

or $r_1 + r_2 + \angle O = 180°$

From (*ii*) and (*iii*) equations, we get

or $r_1 + r_2 = \angle A$ ...(*iv*)

Using this in equation (*i*), we get

$(i + e) - \angle A = \delta$ ...(*v*)

For angle of minimum deviation,

$i = e$ and $r_1 = r_2 = r$ (say), $\delta = \delta_m$

$\therefore$ Equation (*iv*) and (*v*) can be written as,

$$r + r = \angle A$$

or $$r = \frac{A}{2} \qquad \ldots(vi)$$

And $$i + i = \delta_m + A$$

or $$i = \frac{\delta_m + A}{2} \qquad \ldots(vii)$$

By Snell's law, $$\frac{\sin i}{\sin r} = \mu$$

Using (*vi*) and (*vii*) equation, we get

$$\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

(*b*) Given: $\mu = \sqrt{3}$

To find: Angle of prism A

Formula used: $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}}$$

Putting values: $$\sqrt{3} = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin \frac{A}{2}}$$

Solving: $$\sqrt{3} = \frac{\sin A}{\sin \frac{A}{2}}$$

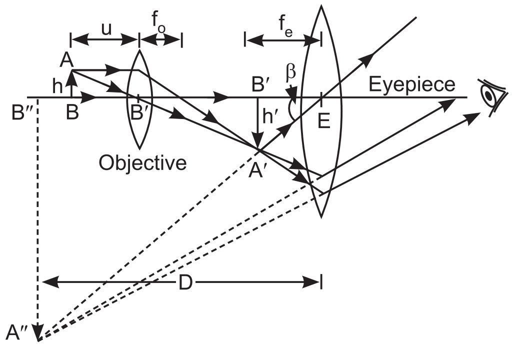
or $$\sqrt{3} = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin \frac{A}{2}}$$

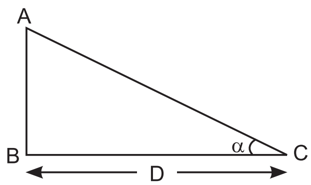
or $$\frac{\sqrt{3}}{2} = \cos \frac{A}{2}$$

or $$\frac{A}{2} = 30° \quad \text{or} \quad A = 60°$$

Or

(*a*)





Magnifying power: The magnifying power of a microscope is defined as the ratio of angle subtended (β) by the final image on the eye to the angle subtended (α) by the object on eye, when the object is placed at the least distance of distinct vision.

Magnifying power, $M = \dfrac{\beta}{\alpha}$

(*b*) Given: $f_o = 1.25$ cm, $f_e = 5$ cm, $M = 30$

To find: Length of microscope, L

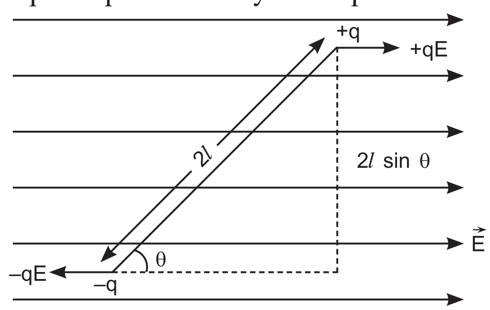
Formula used: $$M = \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$$

Putting values: $$30 = \frac{L}{1.25}\left(1 + \frac{25}{5}\right)$$

or $$30 = \frac{4}{5}L\,[1+5]$$

$$L = \frac{5 \times 30}{4 \times 6} = \frac{25}{4} = 6.25 \text{ cm}$$

32. (*a*) We know that the torque experienced by the dipole when placed in an electric field is



$$\tau = pE \sin\theta \qquad ...(i)$$

Where θ is the angle between dipole moment and electric field.

If the dipole is rotated by a small angle dθ then small amount of work done

$$dW = \tau d\theta$$

$$= pE \sin\theta \, d\theta \qquad \text{[Using equation } (i)\text{]}$$

∴ Total amount of work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is

$$W = \int_{\theta_1}^{\theta_2} pE \sin\theta d\theta$$

$$= pE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$W = -pE(\cos\theta_2 - \cos\theta_1)$$

This work done is stored in the form of potential energy,

$$U = -pE\ (\cos\theta_2 - \cos\theta_1)$$

If $\theta_1 = 90°$ and $\theta_2 = \theta$

Then $\qquad U = -pE\cos\theta$

or $\qquad U = -\vec{p}\cdot\vec{E}$

For stable equilibrium

$$\theta = 0°$$

∴ $\qquad U = -pE$

That is, potential energy is minimum.

For unstable equilibrium

$$\theta = 180°$$

∴ $\qquad U = -pE\cos 180°$

$$= +pE$$

That is, potential energy is maximum.

(*b*) Given: Side of the square = $\sqrt{2}$ m

Charges are 70 μC, – 30 μC, 10 μC and –40 μC

Diagonal of the square = $\sqrt{(\sqrt{2})^2+(\sqrt{2})^2}$ = 2 m

∴ Distance of each charge from the centre of the square

$$r = \frac{2}{2} = 1$$

∴ Potential at the centre of the square due to given four charges,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r}+\frac{q_2}{r}+\frac{q_3}{r}+\frac{q_4}{r}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 r}[q_1 + q_2 + q_3 + q_4]$$

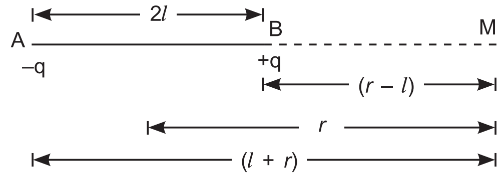
$$= \frac{9\times10^9}{1}[70 - 30 + 10 - 40] \times 10^{-6}$$

$$= 9 \times 10^9 \times 10 \times 10^{-6}$$

$$= 9 \times 10^4 \text{ V}$$

Or

(*a*) (*i*)



Let there is a dipole of dipole length 2*l*. At a distance '*r*' from the dipole three is a point 'M', at which net potential due to this dipole is to be determined.

Now, potential at 'M' due to –*q* charge is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(l+r)} \qquad ...(i)$$

and potential at 'M' due to +*q* charge is,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)} \qquad ...(ii)$$

∴ Net potential at 'M' due to the dipole AB is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{-q}{(l+r)} + \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{l+r}+\frac{1}{r-l}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r-l)+(l+r)}{(r^2-l^2)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{-r+l+l+r}{(r^2-l^2)}\right] = \frac{q(2l)}{4\pi\varepsilon_0(r^2-l^2)}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P}{(r^2-l^2)}$$

For short dipole,

$$r >> l$$

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

(*ii*) Here, $q = 3 \times 10^{-6}$ C, $2l = 2 \times 10^{-3}$ C, $r = 0.6$ m

$$\therefore \qquad p = q(2l)$$

$$= 3 \times 10^{-6} \times 2 \times 10^{-3} \text{ m}$$

$$= 6 \times 10^{-9} \text{ C-m}$$

Now, electric field, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{r^3}$

$$= \frac{9\times10^9\times2\times6\times10^{-9}}{(0.6)^3} = 500 \text{ NC}^{-1}$$

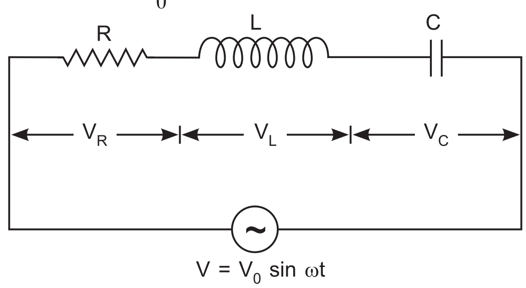
And electric potential,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{9\times10^9\times6\times10^{-9}}{(0.6)^2} = 150 \text{ V}$$

(*b*) Yes, there can be a potential difference. This is because potential of a conductor depends not only on the net charge carried by it, but also on its geometrical shape and size. So if the given conductors have different shape and size, they will have different potentials and hence potential difference.

33. (*a*) Let a resistance R, an inductance L and a capacitance C are connected in series to a source of alternating emf 'V' as shown below.

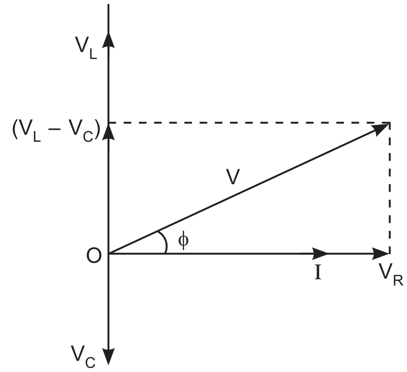
Where $V = V_0 \sin \omega t$



Let I be the current in the circuit at any instant then,

$$V_R = IR \quad ...(i)$$
$$V_L = IX_L \quad ...(ii)$$
$$V_C = IX_C \quad ...(iii)$$

The phasor diagram for series LCR circuit is shown below ($V_L > V_C$)



As V_L and V_C are in opposite directions, let $V_L > V_C$, therefore their resultant is $(V_L - V_C)$. By parallelogram law of vector addition, resultant of V_R and $(V_L - V_C)$ is equal to applied voltage 'V' and is given by diagonal of the parallelogram,

$$\therefore \quad V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

or
$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Clearly, $\sqrt{R^2+(X_L-X_C)^2}$ is the effective resistance of the series LCR circuit and is represent by 'Z' known as impedance of the circuit.

$$\therefore \qquad Z = \sqrt{R^2+(X_L-X_C)^2}$$

$$Z = \sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2} \qquad \left(\because\ X_L=\omega L \text{ and } X_C=\frac{1}{\omega C}\right)$$

The phase angle between voltage and current is given by

$$\tan\phi = \frac{V_L-V_C}{V_R} = \frac{IX_L-IX_C}{IR} = \frac{X_L-X_C}{R}$$

or

$$\phi = \tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

If the voltage and current are in same phase, then

$$\phi = 0$$

$$\Rightarrow \qquad X_L - X_C = 0$$

or

$$X_L = X_C$$

The circuit in this condition is called resonant circuit.

(*b*) The resonant frequency in a series LCR circuit is given by

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

or

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{L_2C_2}{L_1C_1}}$$

Here, $L_1 = L$, $C_1 = C$ and $L_2 = ?$, $C_2 = 4C$

Also, $\nu_1 = v_2 = \nu$ (Say)

$$\therefore \qquad \frac{\nu}{\nu} = \sqrt{\frac{L_2(4C)}{L(C)}}$$

or

$$1 = \sqrt{\frac{4L_2}{L}}$$

Squaring both sides, we get

$$1 = \frac{4L_2}{L}$$

or

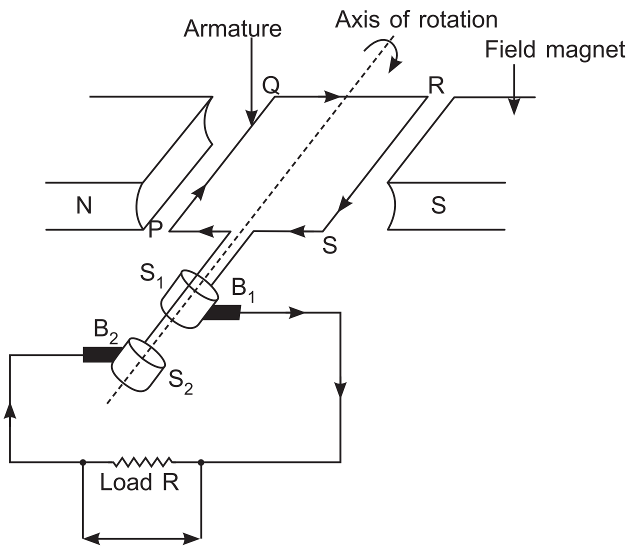
$$L_2 = \frac{L}{4}$$

Therefore, the inductance becomes one-fourth of its initial value.

Or

AC Generator: An a.c. generator is a device which converts mechanical energy into electrical energy.

Principle: Its working is based on the principal of electromagnetic induction. When a coil is rotated in a magnetic field then the flux linked with the coil charges and hence an induced emf is set up in coil which causes the flow of current in it.



Working: When the armature coil rotates, an induced current starts to flow through it.

Let initially coil *PQRS* be in the vertical position and is rotated in clockwise direction thus by Fleming's right hand rule, during first half rotation of the coil, the induced current flows in the direction of *SRQP* with brush B_1 acting as a positive and brush B_2 as negative terminal.

Whereas during second half rotation the direction of induced current is reversed and it flows along *PQRS*, so the brush B_1 functions as negative and brush B_2 as the positive terminal.

Thus the direction of current in the external circuit is reversed after every half cycle and hence, alternating current is produced by the generator.

Expression for induced emf:

Given, N = Number of turns in the coil

A = Cross-sectional area of each turn

B = Magnitude of magnetic field

ω = Angular velocity with which coil rotates

Let θ = Angle which normal to the coil makes with field $\vec{B}$ at any instant t

The magnetic flux linked with the coil at any instant 't' is

$$\phi = NBA\cos\theta = NBA\cos\omega t$$

As the coil is rotated in the magnetic field then by Faraday's law, the induced emf is given by,

$$\varepsilon = \frac{-d\phi_B}{dt} = -\frac{d}{dt}(NBA\cos\omega t)$$

$$= -NBA(-\omega\sin\omega t) = NBA\omega\sin\omega t$$

or $\varepsilon = \varepsilon_0 \sin\omega t$

Where $\varepsilon_0 = NBA\omega$, maximum induced emf.

$\therefore$ Induced emf is given by

$$\varepsilon = \varepsilon_0 \sin\omega t$$

which is the required expression.