Solutions to RMT/Set-1

1. (*c*) $p(-1) = a(-1)^2 + b(-1) + c = a - b + c = 0$ (given),

$\therefore$ One zero $(\alpha) = -1$

$\alpha\beta$ = product of zeroes $= \frac{c}{a} \Rightarrow (-1)\cdot\beta = \frac{c}{a}$

$\Rightarrow \beta = \frac{-c}{a}$

2. (*d*) We know, the system of equations is inconsistent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ According to the question,

$$\frac{3}{(2k-1)} = \frac{1}{(k-1)} \neq \frac{1}{(2k+1)}$$

$$\therefore \quad \frac{3}{(2k-1)} = \frac{1}{(k-1)}$$

$$\Rightarrow \quad 3k - 3 = 2k - 1 \Rightarrow k = 2$$

3. (*d*) There are two tangents at a point on the circumference of the circle.

4. (*a*) The given sequence is an AP.

Here, $a = x - 7, d = (x - 2) - (x - 7) = -2 + 7 = 5.$

$$\therefore \quad a_{15} = a + 14d$$
$$= (x - 7) + 14(5)$$
$$= x - 7 + 70$$
$$= x + 63$$

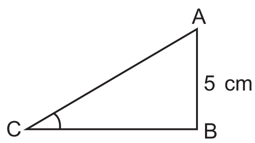
5. (*c*) Area of semicircle $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2}\pi\left(\frac{d}{2}\right)^2 = \frac{1}{8}\pi d^2$$

6. (*d*) $\sin C = \frac{AB}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{5}{AC}$

$\Rightarrow \quad AC = 10$ cm



7. (*d*) $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{AB}{DE} = \frac{\text{Perimeter}(\Delta ABC)}{\text{Perimeter}(\Delta DEF)}$$

$$\therefore \quad \frac{AB}{8} = \frac{60}{48} \Rightarrow AB = 10 \text{ cm}$$

8. (*d*) $x^2 + (a + 1)x + b$

$\because x = 2$ is a zero and $x = -3$ is another zero

$\therefore (2)^2 + (a + 1)\cdot 2 + b = 0$

and $(-3)^2 + (a + 1)(-3) + b = 0$

$\Rightarrow 4 + 2a + 2 + b = 0$ and $9 - 3a - 3 + b = 0$

$\Rightarrow 2a + b = -6$ ...(*i*) and $-3a + b = -6$ ...(*ii*)

Solving (*i*) and (*ii*), we get $5a = 0$

$\Rightarrow a = 0$ and $b = -6.$

9. (*c*) 2 + 4 + 5 + 71 = 82

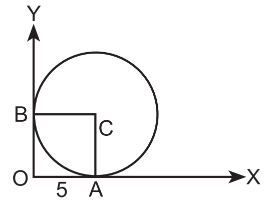
10. (*b*) OA = OB ⇒ OB = 5 units

AC = BC [Radii]

⇒ OACB is a square.

⇒ AC = OA = 5

⇒ Diameter = 10 units.



11. (*a*) Zeroes of quadratic polynomials are $\frac{3}{5}$ and $-\frac{1}{2}$.

∴ quadratic polynomial = $k\,[x^2$ – (sum of zeroes)x + product of zeroes]

$$= k\left[x^2 - \left\{\frac{3}{5} + \left(\frac{-1}{2}\right)\right\}x + \frac{3}{5} \times \left(-\frac{1}{2}\right)\right]$$

$$= k\left(x^2 - \frac{x}{10} - \frac{3}{10}\right) = \frac{k}{10}(10x^2 - x - 3)$$, where k is any constant.

12. (*c*) $(\tan A + \cot A)^2 = 4^2$

$\Rightarrow \tan^2 A + \cot^2 A + 2 = 16$

$\Rightarrow \tan^2 A + \cot^2 A = 14$

$\Rightarrow (\tan^2 A + \cot^2 A)^2 = (14)^2$

$\Rightarrow \tan^4 A + \cot^4 A + 2 = 196$

$\Rightarrow \tan^4 A + \cot^4 A = 194$

13. (*d*) Height of cylinder = 14 cm

Radius of cylinder = r

∴ Curved surface area = $2\pi rh$

$$\Rightarrow 88 = 2 \times \frac{22}{7} \times r \times 14$$

$$\text{Diameter} = 2r = \frac{88 \times 7}{22 \times 14} = 2 \text{ cm}$$

14. (*a*) $\frac{3}{26}$

15. (*c*) Let the ordinate of other end be y.

Now, $17 = \sqrt{(11-3)^2 + (y-4)^2}$

$\Rightarrow \pm 289 = 64 + (y-4)^2$

$\Rightarrow y - 4 = \pm 15 \Rightarrow y = -11$ or 19

16. (*b*) centred at the class marks of the classes.

17. (*c*) As, distance of (4, a) from the x-axis = $\frac{1}{2}$ × distance of (4, a) from the y-axis.

$\Rightarrow a = \frac{1}{2} \times 4 = 2$ units

18. (*c*) Probability of winning a game = 0.07

∴ Probability of losing the game = 1 – Probability of winning the game = 1 – 0.07 = 0.93

19. (*b*) Since, HCF × LCM = Product of two numbers

⇒ 17 × LCM = 5780

$$\Rightarrow \text{LCM} = \frac{5780}{17} = 340$$

∴ Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

20. (*c*) Here $l + b + h = 19$ cm and $\sqrt{l^2+b^2+h^2} = 5\sqrt{5}$ cm

$$\Rightarrow \quad l^2 + b^2 + h^2 = 125 \text{ cm}^2$$

Now, $$(l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + lh)$$

$$\Rightarrow \quad (19)^2 = 125 + 2(lb + bh + lh)$$

$$\Rightarrow \quad 361 - 125 = 2(lb + bh + lh)$$

$$\Rightarrow \quad \text{Surface area} = 236 \text{ cm}^2$$

$\Rightarrow$ Assertion (A) is true but reason (R) is false.

21. (*a*) Let numbers be $2x$ and $3x$.

LCM of two numbers $= 2 \times 3 \times x = 6x$

$$\Rightarrow \quad 6x = 180 \Rightarrow x = 30$$

$\therefore$ Numbers are $2 \times 30 = 60$ and $3 \times 30 = 90$

Now, $$\text{HCF} \times \text{LCM} = a \times b$$

$$\Rightarrow \quad \text{HCF} \times 180 = 60 \times 90$$

$$\Rightarrow \quad \text{HCF} = \frac{60 \times 90}{180} = 30$$

OR

(*b*) A.T.Q. greatest number will divide $445 - 4$, $572 - 5$ and $699 - 6$

$\Rightarrow$ greatest number will be HCF of 441, 567 and 693

Now $$441 = 3^2 \times 7^2$$

$$567 = 3^4 \times 7$$

$$693 = 3^2 \times 7 \times 11$$

$$\therefore \quad \text{HCF} = 3^2 \times 7 = 63$$

$\therefore$ 63 is the required number.

22. (*a*) Possible outcomes are

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total elementary events = 36

(*i*) Outcomes of doublet = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}, i.e. 6

$$\text{P(getting a doublet)} = \frac{6}{36} = \frac{1}{6}$$

(*ii*) Outcomes of getting a sum 10 = {(4, 6), (5, 5), (6, 4)}, i.e. 3

$$\text{P(getting a sum 10)} = \frac{3}{36} = \frac{1}{12}$$

OR

(*b*) Two different dice are tossed. Therefore, total outcomes are 36.

(*i*) Favourable outcomes for even number on both dice = 9, i.e. {(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}.

$\therefore$ Probability of getting even number on both dice $= \frac{9}{36} = \frac{1}{4}$

(*ii*) Favourable outcomes that the sum of the numbers appearing in two dice is 5 are {(1, 4), (2, 3), (3, 2), (4, 1)} i.e. 4.

$\therefore$ Probability of getting sum of numbers appearing on two dice is 5 $= \frac{4}{36} = \frac{1}{9}$.

23.

$$2 \sin 2\theta = \sqrt{3}$$

$$\Rightarrow \quad \sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60°$$

$$\therefore \quad \sin 2\theta = \sin 60°$$

$$\therefore \quad 2\theta = 60° \Rightarrow \theta = 30°$$

24. Let points be A (2, –5) and B (–2, 9)

Let P (x, 0) be the point on x-axis.

$$\therefore \quad PA = PB$$

$$\Rightarrow \quad \sqrt{(x-2)^2 + (0+5)^2} = \sqrt{(x+2)^2 + (0-9)^2}$$

$$\Rightarrow \quad (x-2)^2 + 25 = (x+2)^2 + 81$$

$$\Rightarrow \quad x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$\Rightarrow \quad -4x - 4x = 81 - 25$$

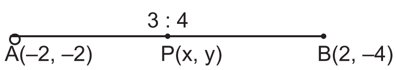
$$\Rightarrow \quad -8x = 56 \Rightarrow x = -7$$

$\therefore$ The required point is (– 7, 0).

25. $AP = \frac{3}{7}AB$ and $BP = AB - AP$

$$= \frac{AB}{1} - \frac{3}{7}AB = \frac{7AB - 3AB}{7} = \frac{4AB}{7}$$

$$\frac{AP}{BP} = \frac{\frac{3}{7}AB}{\frac{4}{7}AB} = 3 : 4$$

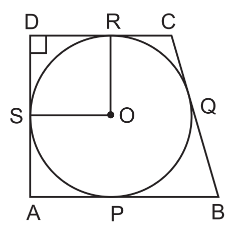


$$x = \frac{3(2) + 4(-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$$

$$y = \frac{3(-4) + 4(-2)}{3+4} = \frac{-12-8}{7} = -\frac{20}{7}$$

Hence, the coordinates of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

26. (*a*) $BP = BQ$ (Tangents from B)



$\because$ BP = 27 cm,

$\therefore$ BQ = 27 cm and BC = 38 cm

Now, CQ = 38 cm – 27 cm = 11 cm; CQ = CR $\Rightarrow$ CR = 11 cm

$\because$ CD = DR + CR $\Rightarrow$ 25 cm = DR + 11 cm $\Rightarrow$ DR = 14 cm ...(*i*)

Also, $\angle$D = 90°

$\because$ OR $\perp$ DC [Radius from point of contact of tangent]

$\therefore$ $\angle$ORD = 90°

Similarly $\angle$OSD = 90°

Also, RD = DS [Tangents from D]

$\Rightarrow$ OR = RD = 14 cm

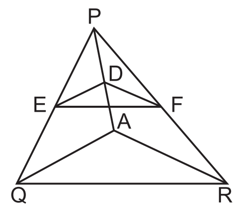
$\therefore$ ORDS is a square.

$\therefore$ Radius of circle = 14 cm

OR

(*b*) **Given:** In ΔPQR, DE || AQ and DF || AR.

To prove: EF || QR



Proof: In ΔPAQ, ED || QA (Given)

$\therefore$ $\frac{PE}{EQ} = \frac{PD}{DA}$ (BPT) ...(*i*)

In ΔPAR,

DF || AR (Given)

$\therefore$ $\frac{PD}{DA} = \frac{PF}{FR}$ (BPT) ...(*ii*)

From (*i*) and (*ii*), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore$ EF || QR.

27. Let α and β be the zeroes of the polynomial $2x^2 - 5x - 3$.

Then, sum of zeroes $(\alpha + \beta) = \frac{5}{2}$ ...(*i*)

And product of zeroes $(\alpha\beta) = \frac{-3}{2}$ ..(*ii*)

Let 2α and 2β be the zeroes of the polynomial $x^2 + px + q$.

Then $2\alpha + 2\beta = -p$

$\Rightarrow$ $2(\alpha + \beta) = -p$

$\Rightarrow$ $2 \times \frac{5}{2} = -p$ [From (*i*)]

So $p = -5$

And $2\alpha \times 2\beta = q$

$\Rightarrow$ $4\alpha\beta = q$

So $q = 4 \times \left(\frac{-3}{2}\right) = -6$ [From (*ii*)]

Hence, $p = -5$ and $q = -6$

28. Let the actual speed of the train be x km/h and let the actual time taken be y hours.

Distance covered is xy km. If the speed is increased by 6 km/h, then time of journey is reduced by 4 hours i.e. when speed is $(x + 6)$ km/h, time of journey is $(y - 4)$ hours.

$\therefore$ Distance covered $= (x + 6)(y - 4)$

$\Rightarrow$ $xy = (x + 6)(y - 4)$

$\Rightarrow$ $-4x + 6y - 24 = 0$

$\Rightarrow$ $-2x + 3y - 12 = 0$ ...(*i*)

Similarly $\quad xy = (x - 6)(y + 6)$

$\Rightarrow \quad 6x - 6y - 36 = 0$

$\Rightarrow \quad x - y - 6 = 0 \quad$ ...(*ii*)

Solving (*i*) and (*ii*), we get $x = 30$ and $y = 24$

$$\text{Distance} = xy = (30 \times 24)\text{km}$$
$$= 720 \text{ km.}$$

Hence, the length of the journey is 720 km.

29. $$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \quad ...(i)$$

Squaring both sides of (*i*), we get

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{xy}{ab}\sin\theta\cos\theta = 1 \quad ...(ii)$$

Now, $$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1 \quad ...(iii)$$

Squaring both sides of (*iii*), we get

$$\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - \frac{2xy}{ab}\sin\theta\cos\theta = 1 \quad ...(iv)$$

Adding (*ii*) and (*iv*), we get

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{x}{a}\frac{y}{b}\cos\theta.\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\frac{y}{b}\cos\theta\sin\theta = 2$$

$$\Rightarrow \quad \frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\Rightarrow \quad \frac{x^2}{a^2}\times 1 + \frac{y^2}{b^2}\times 1 = 2 \quad (\text{Using } \cos^2\theta + \sin^2\theta = 1)$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$ Hence proved.

30. (*a*) Volume of remaining solid = Volume of cube – volume of cone

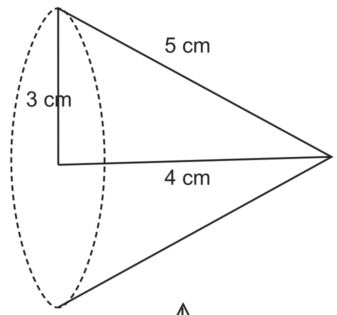
$$= 7^3 - \frac{1}{3}\pi r^2 h$$
$$= 7^3 - \frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 7$$
$$= 343 - 66$$
$$= 277 \text{ cm}^3$$

Surface area of remaining solid = area of 6 faces of cube + CSA of cone – area of base of cone

$$= 6\text{ side}^2 + \pi rl - \pi r^2$$
$$= \left(6\times 7^2 + \frac{22}{7}\times 3\times\sqrt{58} - \frac{22}{7}\times 9\right)\text{cm}^2 \quad \left[l = \sqrt{h^2 + r^2} = \sqrt{7^2 + 3^2} = \sqrt{58}\text{ cm}\right]$$
$$= \left(294 + \frac{66}{7}\sqrt{58} - \frac{198}{7}\right)\text{cm}^2$$
$$= \frac{1}{7}\left(1860 - 66\sqrt{58}\right)\text{cm}^2$$
$$= \frac{6}{7}\left(310 - 11\sqrt{58}\right)\text{cm}^2$$

OR

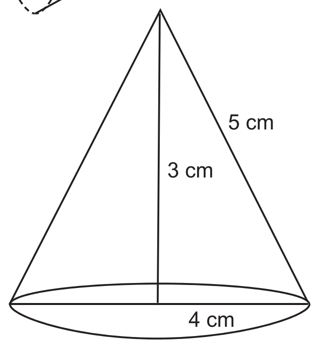
(*b*) When revolved about the side 4 cm, $r = 3$ cm, $h = 4$ cm



$$\text{Volume } V_1 = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 3^2 \times 4$$

When revolved about the side 3 cm, $r = 4$ cm, $h = 3$ cm



$$\text{Volume } V_2 = \frac{1}{3} \times \pi \times 4^2 \times 3$$

$$\frac{V_1}{V_2} = \frac{\frac{1}{3} \times \pi \times 3^2 \times 4}{\frac{1}{3} \times \pi \times 4^2 \times 3} = 3 : 4$$

CSA in first case

$$S_1 = \pi rl \qquad \left(l = \sqrt{3^2 + 4^2} = 5\right)$$

$$= \pi \times 3 \times 5 = 15\pi \text{ cm}^2$$

CSA in second case

$$S_2 = \pi \times 4 \times 5$$

$$= 20\pi \text{ cm}^2$$

$$\text{Difference} = 20\pi \text{ cm}^2 - 15\pi \text{ cm}^2$$

$$= 5\pi \text{ cm}^2$$

$$= 5 \times 3.14 \text{ cm}^2$$

$$= 15.70 \text{ cm}^2$$

31. Let us assume $3 + \sqrt{2}$ is a rational number.

$\therefore 3 + \sqrt{2} = \frac{p}{q}$, where p and q are integers and $q \neq 0$

$\Rightarrow \quad \sqrt{2} = \frac{p - 3q}{q}$

As, 3, p and q are integers.

$\Rightarrow \quad \frac{p - 3q}{q}$ is integer

$\Rightarrow \quad \sqrt{2}$ is rational, which is a contradiction.

So, our assumption that $3 + \sqrt{2}$ is rational, is wrong.

Hence, $3 + \sqrt{2}$ is an irrational number.

32. (*a*) Let number of students be x.

Number of apples = 300

$\Rightarrow$ Apples distributed per student $= \frac{300}{x}$

If 10 more students are added then number of students $= x + 10$

Then number of apples per student $= \frac{300}{x + 10}$

According to the question,

$$\frac{300}{x} - \frac{300}{x + 10} = 1$$

$$\Rightarrow \quad \frac{300(x + 10) - 300x}{x(x + 10)} = 1$$

$\Rightarrow \quad x^2 + 10x - 3000 = 0$

$\Rightarrow \quad (x + 60)(x - 50) = 0$

$\Rightarrow \quad x = -60$ (rejected) or $x = 50$

$\therefore$ Number of students = 50

OR

(*b*) Let C.P. of tea set be ₹ x.

$$\text{Loss} = ₹\frac{5}{100} \times x = ₹\ \frac{x}{20}$$

and C.P. of lemon set be ₹ y.

$$\text{Profit} = ₹\frac{15}{100} \times y = ₹\ \frac{3y}{20}$$

According to the question,

$$\frac{3y}{20} - \frac{x}{20} = 7 \Rightarrow 3y - x = 140 \quad ...(i)$$

Also, if gain on tea set = ₹ $\frac{5}{100} \times x$ = ₹ $\frac{x}{20}$

and gain on lemon set = ₹ $\frac{10}{100} \times y$ = ₹ $\frac{y}{10}$

According to the question,

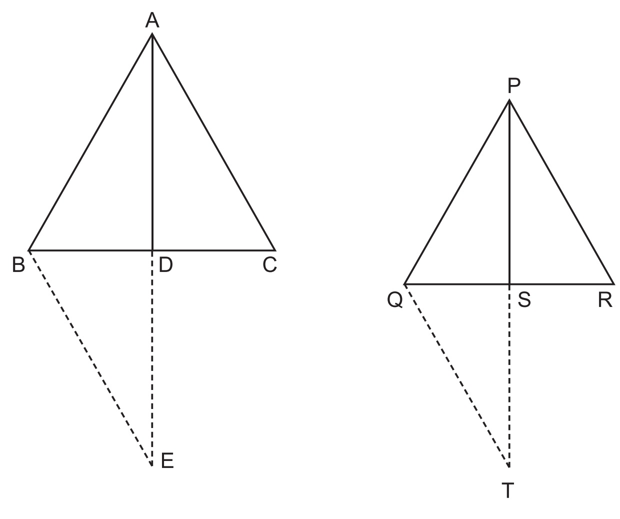
$$\frac{x}{20} + \frac{y}{10} = 13 \Rightarrow x + 2y = 260 \quad ...(ii)$$

On solving equations (*i*) and (*ii*), we get

$$x = 100 \text{ and } y = 80$$

$\therefore$ Cost price of tea set = ₹ 100

33. Given: In ΔABC and ΔPQR, AD and PS are median respectively and $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PS}$



To prove: $\Delta ABC \sim \Delta PQR$

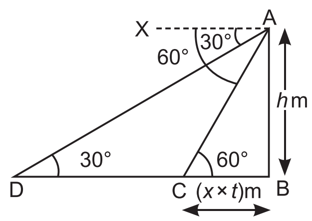
Construction: Extend AD to point E such that AD = DE and join BE.

In ΔPQS extend PS to point T such, that PS = ST and join QT.

Proof: In ΔBDE and ΔADC

$$BD = DC \quad \text{(D is the mid point)}$$
$$\angle ADC = \angle BDE \quad \text{(Vertically opposite angles)}$$
$$AD = DE \quad \text{(From construction)}$$
$\therefore$ $$\Delta BDE \cong \Delta CDA$$
$$BE = AC \quad \text{(By CPCT) ...}(i)$$
Similarly $$\Delta QST \cong \Delta RSP$$
So, $$QT = RP \quad \text{(By CPCT)...}(ii)$$
Now, $$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PS} \quad \text{(Given)}$$
$$\frac{AB}{PQ} = \frac{BE}{QT} = \frac{2AD}{2PS} \quad \text{[From } (i) \text{ and } (ii)\text{]}$$
$$\frac{AB}{PQ} = \frac{BE}{OT} = \frac{AE}{PT} \quad \text{(Since AD = DE, PS = ST)}$$
$$\Delta ABE \sim \Delta PQT \quad \text{...}(iii)$$
$$\angle BAD = \angle QPS \quad \text{...}(iv)$$
Similarly $$\angle DAC = \angle SPR$$
On adding (iii) and (iv), we get $\angle BAD + \angle DAC = \angle QPS + \angle SPR$
$$\Rightarrow \quad \angle BAC = \angle QPR$$
$$\Rightarrow \quad \frac{AB}{AC} = \frac{QP}{PR}$$
$$\Rightarrow \quad \Delta ABC \sim \Delta PQR$$

34. Let the height of the tower AB be h m.



Given: $$\angle XAD = \angle ADB = 30^\circ$$
and $$\angle XAC = \angle ACB = 60^\circ$$
Let the speed of the car be x m/sec

Distance $$CD = 6 \times x = 6x \text{ m}$$
Let the time taken from C to B = t sec.

Distance BC = $x \times t$ m [$\because$ Distance = Speed × Time]

In ΔABD, $$\frac{AB}{DB} = \tan 30^\circ$$
$$\Rightarrow \quad \frac{h}{DC + CB} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad \frac{h}{6x + xt} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad h = \frac{(6+t)x}{\sqrt{3}} \quad \text{... }(i)$$
In ΔABC, $$\frac{AB}{BC} = \tan 60^\circ \Rightarrow \frac{h}{tx} = \sqrt{3}$$
$$\Rightarrow \quad h = xt \times \sqrt{3} \quad \text{... }(ii)$$
From equations (i) and (ii), we get
$$\frac{(6+t)x}{\sqrt{3}} = tx \times \sqrt{3}$$
$$\Rightarrow \quad (6 + t)x = tx \times 3 \Rightarrow 6x + tx = tx \times 3$$
$$\Rightarrow \quad 6x = 3tx - tx \Rightarrow 6x = 2tx \Rightarrow 6 = 2t$$
$$\Rightarrow \quad \frac{6}{2} = t \Rightarrow t = 3$$
Hence, time taken to reach from C to B = 3 sec.

35. (*a*)

Classes	Frequency	Cumulative frequency	
0 – 20	6	6	
20 – 40	8	14	
40 – 60	10	24	
60 – 80	12	36	← Median class
80 – 100	6	42	
100 – 120	5	47	
120 – 140	3	50	
	$n = 50$		

$$\therefore \quad \frac{n}{2} = 25$$

Median class = (60 – 80)

Here $l = 60, f = 12, cf = 24, h = 20$.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 60 + \left(\frac{25 - 24}{12}\right) \times 20$$

$$= 60 + \frac{1 \times 5}{3}$$

$$= \frac{180 + 5}{3} = \frac{185}{3} = 61.6$$

Modal class = (60 – 80) as its frequency is maximum (12).

Here $h = 20, l = 60, f_1 = 12, f_0 = 10, f_2 = 6$.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{12 - 10}{2 \times 12 - 10 - 6}\right) \times 20$$

$$= 60 + \frac{2}{8} \times 20 = 65$$

Now, Mode = 3 Median – 2 Mean

$\Rightarrow$ 65 = 3 (61.6) – 2 Mean

$\Rightarrow$ 2 Mean = 184.8 – 65

2 Mean = 119.8

$$\Rightarrow \quad \text{Mean} = \frac{119.8}{2} = 59.9$$

$\therefore$ Mean = 59.9;

Median = 61.6;

Mode = 65

OR

(*b*)

Weight (in kg)	Number of students (f_i)	*cf*
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30
Total	30	

Here, $\frac{n}{2} = \frac{30}{2} = 15,$

$\therefore$ Median class = 55 – 60,

Here, $l = 55, f = 6, cf = 13, h = 5$

$$\text{Median weight} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 55 + \left(\frac{15-13}{6}\right) \times 5$$

$$= 55 + \frac{5}{3} = 55 + 1.67$$

$$= 56.67 \text{ kg}$$

36. (*i*) Since each row is increasing by 10 seats, so it is an AP with first term $a = 30$, and common difference $d = 10$.

So number of seats in 10th row

$$a_{10} = a + 9d = 30 + 9\times10 = 120$$

(*ii*) If number of rows = 17

then the middle row is the 9th row

$a_9 = a + 8d = 30 + 80 = 110$ seats

(*iii*) (*a*) $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 1500 = \frac{n}{2}[2 \times 30 + (n-1)10]$

$\Rightarrow \quad 3000 = 50n + 10n^2$

$\Rightarrow \quad n^2 + 5n - 300 = 0$

$\Rightarrow \quad n^2 + 20n - 15n - 300 = 0$

$\Rightarrow \quad (n + 20)(n - 15) = 0$

$\Rightarrow \quad n = -20$ (rejecting), $n = 15$

So number of rows = 15

OR

(*iii*) (*b*) Number of seats already put up to the 10th row = S_{10}

$$S_{10} = \frac{10}{2}\{2 \times 30 + (10-1)10\}$$

$$= 5(60 + 90) = 750$$

So, the number of seats still required to be put = 1500 –750 = 750

37. (*i*) Radius of sphere = 30 cm

$$\text{Volume of spherical part} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 30 \times 30 \times 30$$

$$= 113142.86 \text{ cm}^3$$

(ii) The height and the diameter of the conical section of the tent are equal.

$$\text{Therefore } r = \frac{h}{2}$$

$$\text{Volume of conical part} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times \frac{h}{2} \times \frac{h}{2} \times h$$

$$= \frac{\pi h^3}{12} \text{ cm}^3$$

(iii) (a) The height of cylindrical part $= \frac{3}{2}h$

$$\text{Volume of cylinder} = \pi \times \frac{h}{2} \times \frac{h}{2} \times \frac{3}{2}h$$

$$= 3\pi\frac{h^3}{8}$$

$$\therefore \quad \frac{\text{Volume of conical part}}{\text{Volume of cylindrical part}} = \frac{\pi\frac{h^3}{12}}{3\pi\frac{h^3}{8}} = \frac{2}{9}$$

Volume of conical part : Volume of cylindrical part = 2 : 9

OR

(iii) (b) There are two horizontal planks (top and bottom).

Length of horizontal plank = 105 cm

Width of horizontal plank = 45 cm

Thickness of horizontal plank = 2 cm

Volume of planks = 2 × length × width × height

Volume of planks = 2 × 105 × 45 × 2 = 18900 cm^3

38. (i) $$\frac{AO}{OC} = \frac{OB}{OD} \Rightarrow \frac{x+15}{x-12} = \frac{x-7}{x-2} \Rightarrow (x + 15)(x - 2) = (x - 12)(x - 7)$$

$$\Rightarrow \quad x^2 + 13x - 30 = x^2 - 19x + 84$$

$$\Rightarrow \quad 32x = 114 \Rightarrow x = \frac{57}{16}$$

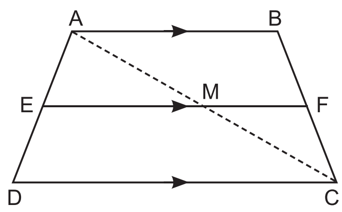
(ii) Since $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$

$$\therefore \quad \Delta CED \sim \Delta AEB$$

$$\therefore \quad \frac{CE}{AE} = \frac{ED}{EB} = \frac{DC}{AB} \text{ or } \frac{AE}{EC} = \frac{BE}{DE} = \frac{AB}{DC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB}{4} \quad \Rightarrow \quad AB = 2 \text{ cm.}$$

(*iii*) (*a*) Join AC, which cut EF at M.



In ΔADC, EM || DC

$$\frac{AE}{ED} = \frac{AM}{MC} \qquad ...(i)$$

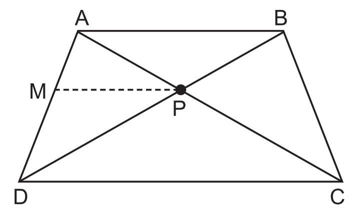
In ΔABC, MF || AB

$$\therefore \qquad \frac{AM}{MC} = \frac{BF}{FC} \qquad ...(ii)$$

From (*i*) and (*ii*), $\frac{AE}{ED} = \frac{BF}{FC} \Rightarrow AE \times FC = ED \times BF$

OR

(*iii*) (*b*) Join PM || AB || DC



In ΔADC,

$$\frac{AM}{MD} = \frac{AP}{PC} \qquad ...(i)$$

In ΔDAB,

$$\frac{AM}{MD} = \frac{PB}{DP} \qquad ...(ii)$$

From (*i*) and (*ii*), we get

$\frac{AP}{PC} = \frac{PB}{DP} \Rightarrow AP \times DP = PB \times PC$