Solutions to RMT/Set-2

1. (*c*) α and β are the zeroes of the polynomial $x^2 + x + 1$.

$\therefore \quad \alpha + \beta = -1$

and $\quad \alpha\beta = 1$

Now $\quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{1} = -1$

2. (*c*) $x - y = 2$ ...(*i*)

Adding $x + y = 4$ ...(*ii*)

$2x = 6 \Rightarrow x = 3$

$\therefore \quad y = 1$

$\Rightarrow \quad a = 3, b = 1$

3. (*b*)

4. (*c*) $x^2 - 5x + 1 = 0$

$\Rightarrow \quad x^2 + 1 = 5x$

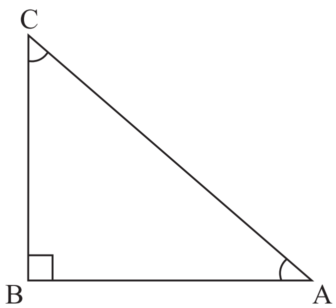
$\Rightarrow \quad \frac{x^2+1}{x} = 5$

$\Rightarrow \quad \frac{x^2}{x} + \frac{1}{x} = 5$

$\Rightarrow \quad x + \frac{1}{x} = 5$

5. (*a*) $4\pi r^2$

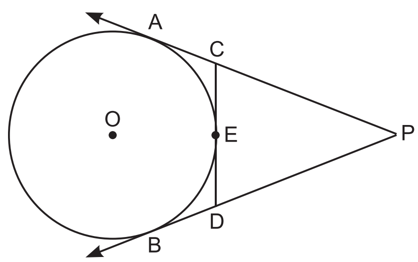
6. (*a*) We have,



$\sin A = \frac{7}{25} = \frac{BC}{AC}$

Now, $\quad \cos C = \frac{BC}{AC} = \frac{7}{25}$

7. (*a*) Perimeter of ΔPCD = PC + CE + ED + PD



= (PC + AC) + (DB + PD) [As CE = AC, ED = BD]

= PA + PB = 2PA

[Tangents drawn from an external point are equal PA = PB]

∴ Perimeter of ΔPCD = 2PA = 2 × 14 cm = 28 cm

8. (*c*) $(1 - p)$ is a root

$\therefore (1-p)^2 + p(1-p) + 1 - p = 0$

$\Rightarrow (1-p)[1 - p + p + 1] = 0$

$\Rightarrow \quad (1-p)(2) = 0 \Rightarrow p = 1$

Equation become $x^2 + x = 0$

One root = 0 and another root = – 1

∴ roots are 0 and –1.

9. (*b*) Class marks = 13.5

Class size = 3

Lower limit = $13.5 - \frac{3}{2} = 12$

10. (*d*) $\because$ DE $\parallel$ BC

$\therefore \angle ABC = 70^\circ$. (Corresponding angles)

Using angle sum property of triangle

$\angle ABC + \angle BCA + \angle BAC = 180^\circ$

$\Rightarrow \angle BCA = 180^\circ - 70^\circ - 50^\circ = 60^\circ$.

11. (*d*) Let $p(x) = x^2 - 1$

Here, $a = 1, b = 0, c = -1$

$$\alpha + \beta = \frac{-b}{a} = -\frac{0}{1} = 0$$

12. (*c*) $$\sec^2\theta = 1 + \tan^2\theta = \frac{\cot^2\theta + 1}{\cot^2\theta}$$

$$\Rightarrow \quad \sec\theta = \frac{\sqrt{\cot^2\theta + 1}}{\cot\theta}$$

13. (*d*) $$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

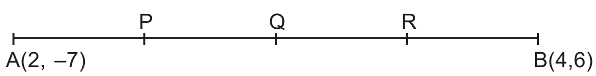
$$= \frac{1}{2}\left(\frac{\theta\pi r}{180^\circ}\right) r = \frac{lr}{2}$$

$$\Rightarrow \quad 20\pi = \frac{1}{2} \times 5\pi \times r \Rightarrow r = 8 \text{ cm}$$

14. (*b*) Favourable cases : {(5, 5), (5, 6), (6, 5), (6, 6)}

$$\text{Probability} = \frac{4}{36} = \frac{1}{9}$$

15. (*a*)

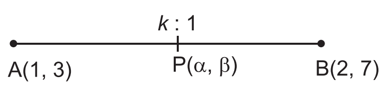


Point R divides AB in ratio 3 : 1.

$\therefore$ Coordinates of R $\left(\frac{12+2}{4}, \frac{18-7}{4}\right)$, i.e., R$\left(\frac{7}{2}, \frac{11}{4}\right)$

16. (*c*) 6

17. (*c*)



Let P(α, β) divides the line-segment AB in the ratio of k:1.

So, $\alpha = \frac{2k+1}{k+1}$; $\beta = \frac{7k+3}{k+1}$

Suppose P lies on line $3x + y - 9 = 0$.

Now, $3\alpha + \beta - 9 = 0$

$$\Rightarrow 3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} = 9$$

$$\Rightarrow \quad 13k + 6 = 9k + 9 \Rightarrow k = \frac{3}{4}$$

So, required ratio is 3:4.

18. (*b*) Total number of cards = 52

Card drawn is red and a king = 2 (as there are 2 red kings)

$\therefore$ probability of drawing a card which is red and a king = $\frac{2}{52} = \frac{1}{26}$

19. (*b*) Since, HCF × LCM = Product of two numbers.

$\Rightarrow$ $17 \times \text{LCM} = 5780$

$\Rightarrow$ $\text{LCM} = \frac{5780}{17} = 340$

$\therefore$ Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

20. (*d*) When cubes are joined end to end, it will form a cuboid.

$$l = 5 \text{ cm} + 5 \text{ cm} = 10 \text{ cm}, \; b = 5 \text{ cm} \quad \text{and} \quad h = 5 \text{ cm}$$

$$\text{Total surface area} = 2(lb + bh + lh)$$

$$= 2(10 \times 5 + 5 \times 5 + 10 \times 5) \text{ cm}^2$$

$$= 2 \times 125 \text{ cm}^2 = 250 \text{ cm}^2$$

Hence, assertion (A) is false but reason (R) is true.

21. (*a*) $\sin(A + B) = 1 = \sin 90°$

So $\quad A + B = 90° \quad$ ...(*i*)

$$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30°$$

So $\quad A - B = 30° \quad$ ...(*ii*)

On solving (*i*) and (*ii*), we get $\angle A = 60°$ and $\angle B = 30°$

OR

(*b*) $$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Dividing the numerator and denominator of LHS by cos θ, we get

$$\frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Which on simplification (or comparison) gives $\tan\theta = \sqrt{3}$

$\therefore \quad \theta = 60°$

22. (*a*) Possible outcomes are {HH, HT, TH, TT}.

Favourable outcomes of getting at most one head are {TT, HT, TH}.

$\therefore$ P(getting at most one head) = $\frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{3}{4}$

OR

(*b*) Let the total number of marbles in the jar be *x*.

Probability of selecting a blue marble + Probability of selecting a black marble + Probability of selecting a green marble = 1

$$\Rightarrow \quad \frac{1}{5} + \frac{1}{4} + \frac{11}{x} = 1$$

$$\Rightarrow \quad \frac{11}{x} = 1 - \frac{1}{5} - \frac{1}{4} \Rightarrow \frac{11}{x} = \frac{20 - 4 - 5}{20}$$

$$\Rightarrow \quad \frac{11}{x} = \frac{11}{20} \Rightarrow x = 20$$

Hence, total number of marbles in the jar = 20

23. For units digit to be 0, 7^n should have 2 and 5 as its prime factors, but 7^n does not contain 2 and 5 as its prime factors. Hence 7^n will not end with digit 0 for $n \in \mathbb{N}$.

24. Let points be A(5, –2), B(6, 4) and C(7, –2).

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$AC = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4+0} = 2 \text{ units}$$

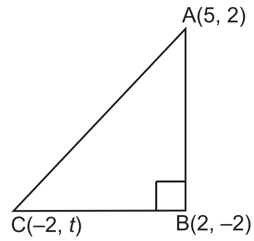
Here, $AB = BC$

$\therefore$ ΔABC is an isosceles triangle.

25.

$$AB = \sqrt{(5-2)^2 + [2-(-2)]^2}$$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25}$$



$$BC = \sqrt{[2-(-2)]^2 + (-2-t)^2}$$

$$= \sqrt{(2+2)^2 + (t+2)^2}$$

$$= \sqrt{(4)^2 + t^2 + 4 + 4t}$$

$$= \sqrt{16 + t^2 + 4 + 4t}$$

$$= \sqrt{t^2 + 4t + 20}$$

$$AC = \sqrt{(5+2)^2 + (2-t)^2}$$

$$= \sqrt{(7)^2 + 4 + t^2 - 4t}$$

$$= \sqrt{49 + 4 + t^2 - 4t} = \sqrt{t^2 - 4t + 53}$$

By Pythagorous Theorem,

$$(AC)^2 = (AB)^2 + (BC)^2$$

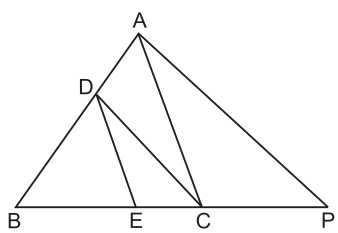
$\Rightarrow$ $t^2 - 4t + 53 = 25 + t^2 + 4t + 20$

$\Rightarrow$ $-4t - 4t = 45 - 53$

$\Rightarrow$ $-8t = -8$

$\Rightarrow$ $t = \dfrac{8}{8} = 1$

26. (*a*) In ΔABC, DE || AC



$\Rightarrow$ $\dfrac{BD}{AD} = \dfrac{BE}{EC}$ ...(*i*) [BPT Theorem]

In ΔABP, DC || AP

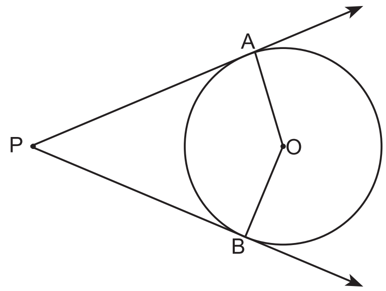
$\Rightarrow$ $\dfrac{BD}{AD} = \dfrac{BC}{CP}$ ...(*ii*) [BPT Theorem]

From (*i*) and (*ii*)

$$\frac{BE}{EC} = \frac{BC}{CP}$$

OR

(*b*) PA and PB are tangents to a circle with centre O from an external point P.



$\therefore$ OA $\perp$ PA and OB $\perp$ PB (Radius through point of contact of the tangents is perpendicular to the tangent)

$\Rightarrow \angle OAP = 90°$ and $\angle OBP = 90°$

In quadrilateral OAPB,

$$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$$

$$\Rightarrow \quad 90° + 90° + \angle APB + \angle AOB = 360°$$

$$\Rightarrow \quad \angle APB + \angle AOB = 180°$$

$\Rightarrow \angle APB$ and $\angle AOB$ are supplementary angles.

27. Let breadth of the rectangular mango grove be x m

then, the length of rectangular mango grove be $2x$ m

ATQ $\quad x \times 2x = 800$ or $2x^2 = 800$

$\Rightarrow \quad x^2 = 400$

$\Rightarrow \quad x = \pm 20$ [–20 is rejected]

Hence, breadth = 20 m

and length = 2 × 20 = 40 m

So, it is possible to design a rectangular mango grove whose length is twice its breadth.

28. α and β are zeroes of polynomial $3x^2 - 5x + 1$

$\therefore \quad \alpha + \beta = \dfrac{5}{3}$ and $\alpha\beta = \dfrac{1}{3}$

Zeroes of required polynomial are 3α and 3β.

$\therefore \quad$ Sum of zeroes $= 3\alpha + 3\beta = 3(\alpha + \beta)$

$$= 3 \times \frac{5}{3} = 5$$

Product of zeroes $= 3\alpha \times 3\beta = 9\alpha\beta$

$$= 9 \times \frac{1}{3} = 3$$

$\therefore \quad$ Polynomial is x^2 – (sum of zeroes)x + product of zeroes i.e. $x^2 - 5x + 3$.

29. $\tan\theta = \dfrac{p}{q}$

Now, $\quad$ LHS $= \dfrac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$

Dividing numerator and denominator by $\cos\theta$, we get

$$\frac{p\tan\theta - q}{p\tan\theta + q} = \frac{p \times \frac{p}{q} - q}{p \times \frac{p}{q} + q} = \frac{p^2 - q^2}{p^2 + q^2} = \text{RHS}$$

30. (*a*) Apparent capacity of glass $= \pi r^2 h$

$$= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$$

Actual capacity of glass = apparent capacity – volume of hemispherical part

$$= 196.25 \text{ cm}^3 - \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 \text{ cm}^3$$

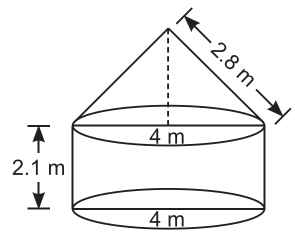
$$= 196.25 \text{ cm}^3 - 32.70 \text{ cm}^3 = 163.55 \text{ cm}^3$$

OR

(*b*) Radius of the cylindrical part $= \frac{4}{2} = 2\text{m}$

Height of the cylindrical part = 2.1 m

Curved surface area of the cylindrical part $= 2\pi rh$



$$= 2 \times \frac{22}{7} \times 2 \times 2.1 \text{ m}^2 = 26.4 \text{ m}^2$$

Curved surface area of the conical top $= \pi rl$

$$= \frac{22}{7} \times 2 \times 2.8 \text{ m}^2 = 17.6 \text{ m}^2$$

Total area of the canvas $= 26.4 \text{ m}^2 + 17.6 \text{ m}^2 = 44 \text{ m}^2$

Cost of canvas = ₹ 500 per m^2

Total cost = cost of canvas per metre square × total surface area of canvas

= ₹ 500 × 44 = ₹ 22,000

31. Let $\sqrt{3} = \frac{a}{b}$, where *a* and *b* are coprime integers and $b \neq 0$.

Squaring both sides, we get $3 = \frac{a^2}{b^2}$.

Multiplying with *b* on both sides, we get

$$3b = \frac{a^2}{b}$$

$$\text{LHS} = 3 \times b = \text{Integer}$$

$$\text{RHS} = \frac{a^2}{b} = \frac{\text{Integer}}{\text{Integer}} = \text{Rational number}$$

$\therefore$ LHS $\neq$ RHS

$\therefore$ Our supposition is wrong.

$\Rightarrow$ $\sqrt{3}$ is an irrational.

Let $15 + 17\sqrt{3}$ be a rational number.

$\therefore$ $15 + 17\sqrt{3} = \frac{a}{b}$, where *a* and *b* are coprime integers and $b \neq 0$

$\Rightarrow$ $17\sqrt{3} = \frac{a}{b} - 15$

$$\sqrt{3} = \frac{a - 15b}{17b}$$

$\frac{a-15b}{17b}$ is rational number but $\sqrt{3}$ is an irrational.

$\therefore$ $$\sqrt{3} \neq \frac{a-15b}{17b}$$

$\therefore$ Our supposition is wrong.

$\Rightarrow$ $15 + 17\sqrt{3}$ is an irrational number. Hence proved.

32. (*a*) Let the time taken by larger pipe alone to fill the tank be x hours.

Therefore, the time taken by the smaller pipe be $(x + 10)$ hours.

Water filled by larger pipe running for 4 hours = $\frac{4}{x}$ litres

Water filled by smaller pipe running for 9 hours = $\frac{9}{x+10}$ litres

According to the question, $\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$

$\Rightarrow$ $8x + 80 + 18x = x^2 + 10x$

$\Rightarrow$ $x^2 - 16x - 80 = 0$

$\Rightarrow$ $x^2 - 20x + 4x - 80 = 0$

$\Rightarrow$ $x(x - 20) + 4(x - 20) = 0$

$\Rightarrow$ $(x + 4)(x - 20) = 0$

$\Rightarrow$ $x = -4$ (x cannot be negative) or $x = 20$

Thus, $x = 20$ and $x + 10 = 30$

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours.

OR

(*b*) Let the usual speed of plane be x km/h

and the reduced speed of the plane be $(x - 200)$ km/h

Distance = 600 km [Given]

According to the question,

(Time taken at reduced speed) – (Schedule time) = 30 minutes = $\frac{1}{2}$ hours.

$\Rightarrow$ $$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$

$\Rightarrow$ $x^2 - 200x - 240000 = 0$

$\Rightarrow$ $x^2 - 600x + 400x - 240000 = 0$

$\Rightarrow$ $x(x - 600) + 400(x - 600) = 0$

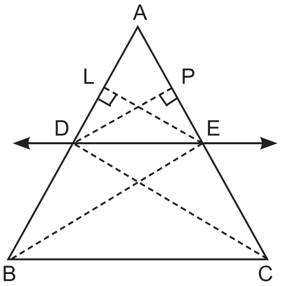
$\Rightarrow$ $(x - 600)(x + 400) = 0$

$\Rightarrow$ $x = 600$ or $x = -400$

But speed cannot be negative.

$\therefore$ The usual speed is 600 km/h and the scheduled duration of the flight = $\frac{600}{600}$ = 1 hour

33. BASIC PROPORTIONALITY THEOREM. In a triangle, a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, divides the other two sides in the same ratio.



Given: A triangle ABC, DE || BC, meeting AB at D and AC at E.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw EL ⊥ AD and DP ⊥ AE.

Proof : ΔBDE and ΔCDE are on the same base and between the same parallels BC and DE, hence equal in area,

i.e., $ar(\Delta BDE) = ar(\Delta CDE)$ ...(*i*)

Now, $$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}.AD.EL}{\frac{1}{2}.BD.EL} = \frac{AD}{BD}$$ ...(*ii*)

Similarly, $$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}AE \cdot DP}{\frac{1}{2}EC \cdot DP} = \frac{AE}{EC}$$ ...(*iii*)

Also, $$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$$ [Using (*i*)]

⇒ $$\frac{AD}{BD} = \frac{AE}{EC}$$ [From (*ii*) and (*iii*)]

We have

PQ || BC, using above theorem,

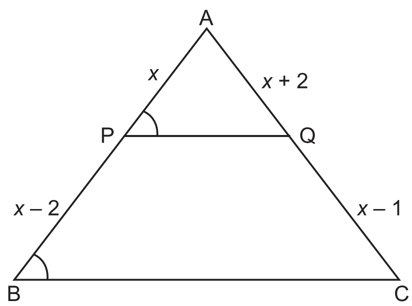
$$\frac{AP}{PB} = \frac{AQ}{QC}$$

⇒ $$\frac{x}{x-2} = \frac{x+2}{x-1}$$

⇒ $$x^2 - x = x^2 - 4$$

⇒ $$x = 4$$

∴ $$3x - 5 = 12 - 5 = 7$$

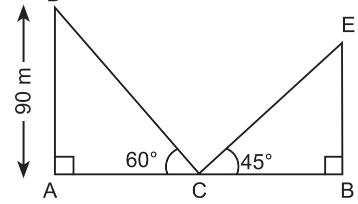


34. Let AB is path.

In right-angled ΔDAC, $\frac{DC}{AD} = \text{cosec } 60°$

⇒ $$\frac{DC}{90} = \frac{2}{\sqrt{3}}$$

⇒ $$DC = \frac{2}{\sqrt{3}} \times 90 \text{ m} = \frac{180}{\sqrt{3}} \text{m}$$



Now DC = CE [length of ladder]

$\therefore$ $CE = \frac{180}{\sqrt{3}}$ m

In right-angled ΔEBC, $\frac{BE}{CE} = \sin 45°$

$\Rightarrow$ $BE = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}}$ m $\Rightarrow$ BE = 73.47 m.

35. (*a*)

Income (in lakhs)	Number of persons (*f*)	Cumulative frequency
0 – 5	4	4
5 – 10	13	17
10 – 15	6	23
15 – 20	2	25
20 – 25	5	30

Here, $n = 30$. So, $\frac{n}{2} = \frac{30}{2} = 15$

$\therefore$ Median class is 5 – 10.

Then, $l = 5$, $cf = 4$, $f = 13$, $h = 5$

Using the formula, $$\text{median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 5 + \left(\frac{15-4}{13}\right) \times 5$$

$$= 5 + \frac{55}{13} = 5 + 4.23 = 9.23$$

OR

(*b*)

Length (in mm)	Number of leaves (f_i)	CI	x_i	d_i	$f_i d_i$
118 – 126	3	117.5 – 126.5	122	–27	–81
127 – 135	5	126.5 – 135.5	131	–18	–90
136 – 144	9	135.5 – 144.5	140	–9	–81
145 – 153	12	144.5 – 153.5	a = 149	0	0
154 – 162	5	153.5 – 162.5	158	9	45
163 – 171	4	162.5 – 171.5	167	18	72
172 – 180	2	171.5 – 180.5	176	27	54

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 149 + \frac{(-81)}{40} = 149 - 2.025 = 146.975$$

Average length of the leaves = 146.975 mm

36. (*i*) $50000x + 200y = 540000$, $900x + 10y = 11000$

(*ii*) Here $a_1 = 2, b_1 = 3, c_1 = -14$

and $a_2 = 5, b_2 = -p, c_2 = -14$

For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2}{5} \neq \frac{3}{-p} \Rightarrow p \neq -\frac{15}{2}.$

(*iii*) (*a*) $50000x + 200y = 540000$ ...(*i*)

$900x + 10y = 11000$ ...(*ii*)

On solving (*i*) and (*ii*), we get

$x = 10$ and $y = 200$

Total number of cows and hens = 10 + 200 = 210

OR

(*iii*) (*b*) Total number of legs = 10 × 4 + 200 × 2 = 440

37. (*i*) Volume of cylindrical cup $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10.5 \text{ cm}^3 = 404.25 \text{ cm}^3$$

(*ii*) Volume of hemispherical cup $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^3$$

$$= 89.83 \text{ cm}^3$$

(*iii*) (*a*) Volume of the cylindrical cup has more juice

More juice = $404.25 \text{ cm}^3 - 89.83 \text{ cm}^3 = 314.42 \text{ cm}^3$

OR

(*iii*) (*b*) Area of canvas = 551 m^2

Wasting incurred = 1 m^2

So used cloth area = $551 \text{ m}^2 - 1 \text{ m}^2 = 550 \text{ m}^2$

Then $\frac{22}{7} \times 7 \times l = 550 \Rightarrow l = 25 \text{ m}$

Hence slant height of the tent = 25 m

Now, $l^2 = r^2 + h^2$

$\Rightarrow (25)^2 = (7)^2 + h^2$

$\Rightarrow h^2 = 576$

$\Rightarrow h = 24 \text{ m}$

Hence, the height of the conical part $h = 24$ m

38. (*i*) The distance between two parallel tangents of a circle = diameter of the circle

= 2 × 6 cm

= 12 cm

(*ii*) Area of circle = πr^2

$\Rightarrow$ $25\pi = \pi r^2$

$\Rightarrow$ $r^2 = 25$

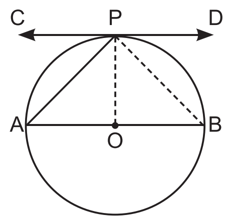
$\Rightarrow$ $r = 5$ cm

$\therefore$ distance between parallel tangents = $2r = 2 \times 5$ cm = 10 cm

(*iii*) (*a*) Tangents touch the circle but don't intersect the circle. Therefore maximum number of parallel tangents a circle can have is two.

OR

(*iii*) (*b*) Join PO and PB.



Now, $\angle CPO = 90°$ (Tangent makes right angle with radius)

In ΔAPO,

$\angle PAO = \angle PAB = 30°$

Also, OP = OA (Radii)

$\therefore$ $\angle PAO = \angle APO = 30°$ (Angles opposite to equal sides)

Now, $\angle CPA = \angle CPO - \angle APO$

$= 90° - 30°$

$= 60°$