# Solutions to RMM/Set-1

**1.** (*d*) $\cot^{-1}x$

**2.** (*d*) $3 \times 1$

**3.** (*b*) As, $(A + A')' = A' + (A')' = A' + A = A + A'$ [$\because$ Matrix addition is commutative]

$\therefore$ $A + A'$ is symmetric matrix.

**4.** (*c*)

$$|A| = 3(0 - 1) + 1(0 - 1) + 2(0 - 4)$$

$$= -3 - 1 - 8 = -12$$

Now, $$|A^{-1}| = \frac{1}{|A|} = \frac{-1}{12}$$

**5.** (*d*)

$$A + B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$$

Let $D = A + B$, then $|D| = \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} = -1 - 2 = -3 \neq 0$

So, $D^{-1}$ exists as $|D| \neq 0$.

Let $D_{ij}$ be the cofactors of each element in $|D|$.

Now, $$D_{11} = (-1)^2\,(1) = 1,\ D_{12} = (-1)^3\,(2) = -2$$

$$D_{21} = (-1)^3\,(1) = -1,\ D_{22} = (-1)^4\,(-1) = -1$$

$\therefore$ $$\text{Adj}(D) = \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$\therefore$ $$D^{-1} = \frac{\text{Adj}(D)}{|D|} = \frac{-1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$\therefore$ $$(A + B)^{-1} = \frac{-1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

**6.** (*d*) if matrix is singular, then

$$\begin{vmatrix} 4+3k & 3 \\ 1+2k & 2 \end{vmatrix} = 0$$

$\Rightarrow$ $$8 + 6k - 3 - 6k = 0$$

$\Rightarrow$ $$5 = 0, \text{ false.}$$

$\therefore$ matrix is not singular for any $k$.

**7.** (*a*) Since $f$ is continuous at $x = 1$, then $\lim\limits_{x \to 1} f(x) = f(1) \Rightarrow \lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1} = k$

$\Rightarrow \lim\limits_{x \to 1}(x + 1) = k \Rightarrow k = 2$

**8.** (*d*) as

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t}$$

$$= -\tan t$$

$\Rightarrow$ $$\left.\frac{dy}{dx}\right]_{t=\frac{3\pi}{4}} = 1$$

**9.** (*c*) As $$y = x^2 \quad ...(i)$$

$$\Rightarrow \quad \frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$\Rightarrow \quad x = \frac{1}{2} \qquad \left[\because \frac{dx}{dt} = \frac{dy}{dt}\right]$$

So, from (*i*), we get

$$y = \frac{1}{4}$$

$\therefore$ Point is $\left(\frac{1}{2}, \frac{1}{4}\right)$.

**10.** (*b*) $$(y - px)^2 = a^2p^2 + b^2$$

$\Rightarrow (x^2 - a^2)p^2 - 2xyp - b^2 + y^2 = 0$

$$\text{Degree} = 2, \text{ as } p^2 = \left(\frac{dy}{dx}\right)^2$$

**11.** (*a*) $$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2026}x}{\cos^{2026}x + \sin^{2026}x}dx \quad ...(i)$$

$$\therefore \quad I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2026}\left(\frac{\pi}{2} - x\right)}{\cos^{2026}\left(\frac{\pi}{2} - x\right) + \sin^{2026}\left(\frac{\pi}{2} - x\right)}dx$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos^{2026}x}{\sin^{2026}x + \cos^{2026}x}dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{2026}x + \cos^{2026}x}{\sin^{2026}x + \cos^{2026}x}dx$$

$$= \int_0^{\frac{\pi}{2}} dx = \Big[x\Big]_0^{\pi/2} = \frac{\pi}{2}$$

**12.** (*d*) Let $2^x = t \Rightarrow 2^x \cdot \log_e 2\, dx = dt$

$$\therefore \quad \int \frac{2^x}{\sqrt{1-4^x}}dx = \frac{1}{\log_e 2} \cdot \int \frac{1}{\sqrt{1-t^2}}dt = \frac{1}{\log_e 2} . \sin^{-1}t + C$$

$$= \frac{1}{\log_e 2} \cdot \sin^{-1}(2^x) + C \Rightarrow p = \frac{1}{\log_e 2}$$

**13.** (*d*) We know that, $$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \quad k^2 + k^2 + k^2 = 1$$

$$\Rightarrow \quad 3k^2 = 1$$

$$\Rightarrow \quad k = \pm \frac{1}{\sqrt{3}}$$

**14.** (*a*) If D is mid point of BC.

$$\text{Vector along } \overrightarrow{AD} = \frac{3\hat{i} + 0\hat{j} + 5\hat{k}}{2}$$

$$|\overrightarrow{AD}| = \sqrt{\frac{9}{4} + \frac{25}{4}}$$

$$= \sqrt{\frac{34}{4}}$$

$$= \frac{\sqrt{34}}{2} \text{ units}$$

**15.** (*a*) On evaluating the determinant,

$$\Delta = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$$

$$= |\vec{a}|^2|\vec{b}|^2 - \{|\vec{a}|\,|\vec{b}|\cos\theta\}^2$$

$$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$$

$$= |\vec{a}|^2\,|\vec{b}|^2\sin^2\theta$$

$$= \{|\vec{a}|\,|\vec{b}|\sin\theta\}^2$$

$$= |\vec{a}\times\vec{b}|^2$$

$$= (\vec{a}\times\vec{b})^2$$

**16.** (*a*) As for (0, 0), $0 + 0 - 2 \le 0$, true, but feasible region does not contain (0, 0).

**17.** (*b*) For (3, 5), $x - y \le 0$ is true

**18.** (*c*)

**19.** (*b*) For Assertion (A):

$\sin^{-1}(1) + \tan^{-1}(\sqrt{3}) - \sec^{-1}(2) = \frac{\pi}{2} + \frac{\pi}{3} - \frac{\pi}{3} = \frac{\pi}{2}$

So, A is true.

Also, R is true but R is not correct explanation of A.

**20.** (*c*) A is true, but R is false.

**21.** (*a*)

$$\tan^{-1}\left[2\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left[2\sin\left(\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

**OR**

(*b*) We have, $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = 3\pi$ ...(*i*)

Now, range of $\cos^{-1} x$ is $[0, \pi]$.

$\therefore$ Equation (*i*) is satisfied when $\cos^{-1} p = \pi, \quad \cos^{-1} q = \pi, \quad \cos^{-1} r = \pi$

$\Rightarrow \quad p = \cos\pi, \quad q = \cos\pi, \quad r = \cos\pi$

$\Rightarrow \quad p = -1, \quad q = -1 \quad r = -1$

$\therefore \quad pq + qr + rp = (-1)\times(-1) + (-1)\times(-1) + (-1)\times(-1)$

$= 1 + 1 + 1 = 3$

**22.**

$$y = \log_2(\sqrt{x-a}+\sqrt{x-b}) = \frac{\log(\sqrt{x-a}+\sqrt{x-b})}{\log 2}$$

$$\frac{dy}{dx} = \frac{1}{\log 2}\cdot\frac{1}{(\sqrt{x-a}+\sqrt{x-b})}\cdot\left\{\frac{1}{2\sqrt{x-a}}\cdot 1+\frac{1}{2\sqrt{x-b}}\cdot 1\right\}$$

$$= \frac{1}{\log 2}\cdot\frac{1}{(\sqrt{x-a}+\sqrt{x-b})}\cdot\frac{1}{2}\left\{\frac{\sqrt{x-b}+\sqrt{x-a}}{\sqrt{x-a}\,\sqrt{x-b}}\right\}$$

$$= \frac{1}{2\log 2\cdot\left(\sqrt{x-a}\cdot\sqrt{x-b}\right)}$$

**23.** (*a*)

$$\int_{-1}^{5} |x-3|\,dx$$

Let $f(x) = |x-3|$

Now, $f(x) = \begin{cases} -(x-3), \text{ when } x < 3 \\ (x-3), \text{ when } x \geq 3 \end{cases}$

So,

$$\int_{-1}^{5} |x-3|\,dx = -\int_{-1}^{3}(x-3)\,dx + \int_{3}^{5}(x-3)\,dx = -\left[\frac{x^2}{2}-3x\right]_{-1}^{3}+\left[\frac{x^2}{2}-3x\right]_{3}^{5}$$

$$= -\left[\left(\frac{9}{2}-9\right)-\left(\frac{1}{2}+3\right)\right]+\left[\left(\frac{25}{2}-15\right)-\left(\frac{9}{2}-9\right)\right]$$

$$= \left(\frac{9}{2}+\frac{7}{2}\right)+\left(-\frac{5}{2}+\frac{9}{2}\right) = 8+2 = 10$$

**OR**

(*b*)

$$\text{Area} = \int_0^4 \sqrt{x}\,dx = \int_0^4 x^{1/2}dx = \left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^4$$

$$= \frac{2}{3}\left[x^{3/2}\right]_0^4 = \frac{2}{3}[8-0] = \frac{16}{3} \text{ sq units}$$

**24.** $\because$ $f(x+y) = f(x)\cdot f(y)\ \forall\ x, y \in R$

$\therefore$ $f(1+0) = f(0)\,f(1) \Rightarrow f(1) = f(0)\,f(1)$

$\Rightarrow$ $f(0) = 1$

Now,

$$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{f(1)\,f(h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{2f(h)-2}{h} = 2\lim_{h\to 0}\frac{f(h)-1}{h}\quad[\because f(1) = 2 \text{ given}]$$

$$= 2\lim_{h\to 0}\frac{f(h)-f(0)}{h}\quad[\because f(0) = 1]$$

$$= 2f'(0) = 2\times 3\quad[\because f'(0) = 3 \text{ given}]$$

$$= 6$$

**25.**

$$\text{Area of parallelogram} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 4 & 3 & -5 \end{vmatrix}$$

$$= \left[\hat{i}(-5-6)-\hat{j}(-5-8)+\hat{k}(3-4)\right]$$

$$= \left|-11\hat{i}+13\hat{j}-\hat{k}\right| = \sqrt{(-11)^2+13^2+(-1)^2}$$

$$= \sqrt{121+169+1} = \sqrt{291} \text{ sq units}$$

**26.** (*a*) We have $y = (\sin^{-1} x)^2$

Differentiating both sides, w.r.t. $x$, we get

$$\frac{dy}{dx} = 2(\sin^{-1} x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \qquad \sqrt{1-x^2}\frac{dy}{dx} = 2(\sin^{-1} x)$$

Differentiating again w.r.t. $x$, on both sides,

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1}{2}\frac{(-2x)}{\sqrt{1-x^2}} \cdot \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow \qquad (1-x^2)\frac{d^2y}{dx^2} - x \cdot \frac{dy}{dx} = 2$$

$$\Rightarrow \qquad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

**OR**

(*b*) $$\sin(xy) + \frac{x}{y} = x^2 - y$$

Differentiating both sides w.r.t. $x$, we get

$$\cos(xy)\left\{x\frac{dy}{dx} + y\right\} + \frac{y - x\frac{dy}{dx}}{y^2} = 2x - \frac{dy}{dx}$$

$$\Rightarrow \qquad y^2\cos(xy)\left\{x\frac{dy}{dx} + y\right\} + y - x\frac{dy}{dx} = 2xy^2 - y^2\frac{dy}{dx}$$

$$\Rightarrow \qquad xy^2\cos(xy)\frac{dy}{dx} + y^3\cos(xy) + y - x\frac{dy}{dx} = 2xy^2 - y^2\frac{dy}{dx}$$

$$\Rightarrow \qquad [xy^2\cos(xy) - x + y^2]\frac{dy}{dx} = 2xy^2 - y^3\cos(xy) - y$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{2xy^2 - y^3\cos(xy) - y}{xy^2\cos(xy) - x + y^2}$$

**27.** Given $xy = c^2$ ...(*i*)

Let $Z = ax + by$

$\Rightarrow$ $Z = ax + \frac{bc^2}{x}$ ...(*ii*)

Differentiating both sides w.r.t. $x$, we get

$$\frac{dZ}{dx} = a - \frac{bc^2}{x^2}$$

For minimum $Z$, $\frac{dZ}{dx} = 0 \Rightarrow a - \frac{bc^2}{x^2} = 0$

$$\Rightarrow \qquad x^2 = \frac{bc^2}{a} \Rightarrow x = \pm\sqrt{\frac{bc^2}{a}}$$

Now, $$\frac{d^2Z}{dx^2} = 0 + \frac{2bc^2}{x^3} = \frac{2bc^2}{x^3}$$

$$\left.\frac{d^2Z}{dx^2}\right]_{x=\sqrt{\frac{bc^2}{a}}} = \frac{2bc^2}{\left(\frac{bc^2}{a}\right)^{3/2}} > 0 \text{ and } \left.\frac{d^2Z}{dx^2}\right]_{x=-\sqrt{\frac{bc^2}{a}}} = \frac{2bc^2}{-\left(\frac{bc^2}{a}\right)^{\frac{3}{2}}} < 0$$

Hence, for $x = \sqrt{\frac{bc^2}{a}}$, $Z$ is minimum.

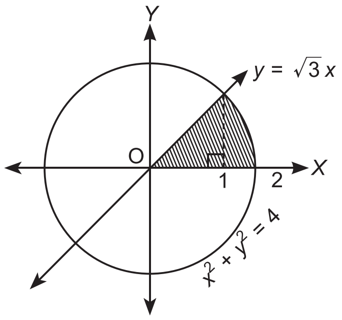
Substituting in (*ii*), we get

$$\text{Minimum } Z = a \cdot \sqrt{\frac{bc^2}{a}} + \frac{bc^2}{\sqrt{\frac{bc^2}{a}}} = 2 \times \sqrt{abc^2}$$

**28.** (*a*) Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get $x^2 + 3x^2 = 4$

$$\Rightarrow \qquad x^2 = 1$$

$$\Rightarrow \qquad x = \pm 1$$



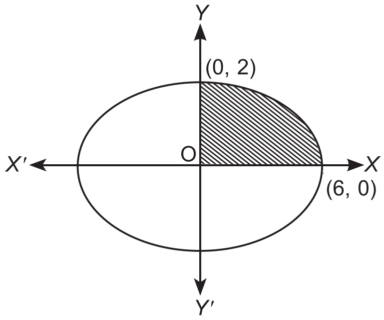
$$\text{Required area} = \sqrt{3}\int_0^1 x\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx$$

$$= \sqrt{3}\left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$$

$$= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6}\right] = \frac{2\pi}{3} \text{ sq units.}$$

**OR**

(*b*) Curve is symmetrical to both the axes.

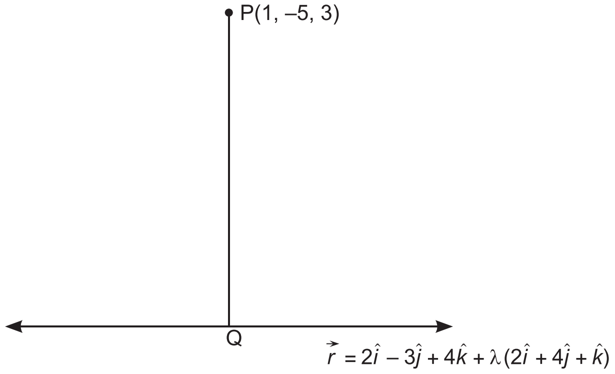


$$\text{Required area} = 4\int_0^6 y\,dx = 4 \times \frac{1}{3}\int_0^6 \sqrt{36 - x^2}\,dx$$

$$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

$$= \frac{4}{3}\left[18 \times \frac{\pi}{2} - 0\right] = 12\pi \text{ sq units}$$

**29.** (*a*)



The line passes through (1, –5, 3) and parallel to $z$-axis is

$$\vec{r} = \hat{i} - 5\hat{j} + 3\hat{k} + \mu\hat{k}, \ \mu \in R \qquad \ldots(i)$$

General point on the above line (*i*) is $P(1, -5, 3 + \mu)$

$$\text{Given line is } \vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k} + \lambda(2\hat{i} + 4\hat{j} + \hat{k}) \qquad \ldots(ii)$$

General point on the line is $(2 + 2\lambda, -3 + 4\lambda, 4 + \lambda)$

For intersection point of lines (*i*) and (*ii*)

$$1 = 2 + 2\lambda; \ -5 = -3 + 4\lambda; \ 3 + \mu = 4 + \lambda$$

$$1 = 2 + 2\lambda, \Rightarrow \lambda = -\frac{1}{2}$$

$\therefore$ Point of intersection is $\left(1, -5, \frac{7}{2}\right)$

$\because$ Distance between (1, –5, 3) and $\left(1, -5, \frac{7}{2}\right) = \frac{7}{2} - 3 = \frac{1}{2}$ unit

$\therefore$ Required distance $= \frac{1}{2}$ unit

**OR**

(*b*) We are given equation of line

$$\vec{r} = 2\hat{i} + \hat{k} + \mu(\hat{i} - \hat{j} + \hat{k}) \qquad ...(i)$$

General point on the line $P(2 + \mu, -\mu, 1 + \mu)$

Equation of line passes through (2, 1, –1) and (3, –1, 2) is

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k}) \qquad ...(ii)$$

General point on (*ii*) is $Q(2 + \lambda, 1 - 2\lambda, -1 + 3\lambda)$

For point of intersection of (*i*) and (*ii*) is

$$2 + \mu = 2 + \lambda; -\mu = 1 - 2\lambda; 1 + \mu = -1 + 3\lambda$$

$$\because \quad 2 + \mu = 2 + \lambda \quad \Rightarrow \quad \lambda = \mu$$

Using $\lambda = \mu$ in $-\mu = 1 - 2\lambda \Rightarrow -\mu = 1 - 2\mu$

$$\Rightarrow \quad \mu = 1 \quad \Rightarrow \quad \lambda = 1$$

Using $\lambda = 1, \mu = 1$ in $1 + \mu = -1 + 3\lambda$

$$\Rightarrow \quad 1 + 1 = -1 + 3 \Rightarrow 2 = 2 \text{ true}$$

$\therefore$ Point of intersection of lines (*i*) and (*ii*) is (3, –1, 2)

$\therefore$ Distance of point (3, –1, 2) from *z*-axis is

$$= \sqrt{3^2 + (-1)^2} = \sqrt{10} \text{ units}$$

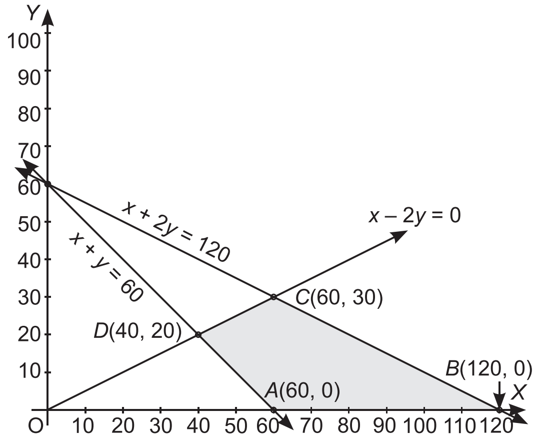
**30.** To minimise

$$Z = 5x + 10y$$

subject to the constraints

$$x \geq 0, y \geq 0, x - 2y \geq 0, x + y \geq 60, x + 2y \leq 120$$

Plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for minimum $Z$ are $A(60, 0)$, $B(120, 0)$, $C(60, 30)$ and $D(40, 20)$



| Corner Points | Z = 5x + 10y | Values | |
|---|---|---|---|
| $A(60, 0)$ | 300 + 0 | 300 | ← Minimum |
| $B(120, 0)$ | 600 + 0 | 600 | |
| $C(60, 30)$ | 300 + 300 | 600 | |
| $D(40, 20)$ | 200 + 200 | 400 | |

$\therefore$ $Z$ is minimum at $A(60, 0)$. Hence, for $x = 60$ and $y = 0$, $Z$ is minimum. Minimum value of $Z = 300$.

**31.** Let $E_1$ be the event that the die shows 1 or 3.

$E_2$ be the event that the die shows 2 or 6.

$E_3$ be the event that the die shows 4 or 5.

B be the event of choosing a black ball

$$\therefore \quad P(E_1) = \frac{2}{6} = \frac{1}{3} = P(E_2) = P(E_3)$$

$$P(B/E_1) = \frac{3}{5},\ P(B/E_2) = \frac{3}{6} = \frac{1}{2}$$

$$P(B/E_3) = \frac{4}{7}$$

$$\therefore \quad P(B) = P(E_1)\ P(B/E_1) + P(E_2)\ P(B/E_2) + P(E_3)\ P(B/E_3)$$

$$= \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{4}{7}$$

$$= \frac{1}{3}\left[\frac{3}{5} + \frac{1}{2} + \frac{4}{7}\right] = \frac{1}{3}\left[\frac{42+35+40}{70}\right] = \frac{117}{210}$$

$$= \frac{39}{70}$$

**32.** (*a*) Given
$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

and
$$B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Also, $$(AB)^{-1} = B^{-1} A^{-1} \qquad ...(i)$$

$A^{-1}$ is given, so we find $B^{-1}$.

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= 1(3) - 2\,(-1) - 2(2)$$

$$= 3 + 2 - 4 = 1 \neq 0.$$

Matrix formed by cofactors of each element in $|B|$. $B_{ij}$ is cofactor of element $b_{ij}$ in $|B|$

$$B_{11} = +(3-0) = 3\ ; \quad B_{12} = -(-1-0) = 1;$$

$$B_{13} = +(2-0) = 2$$

$$B_{21} = -(2-4) = 2\ ; \quad B_{22} = +(1-0) = 1;$$

$$B_{23} = -(-2-0) = 2$$

$$B_{31} = +(0+6) = 6\ ; \quad B_{32} = -(0-2) = 2;$$

$$B_{33} = +(3+2) = 5$$

$$\therefore \quad \text{adj } B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\Rightarrow \quad B^{-1} = \frac{\text{adj } B}{|B|}$$

$$= \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$\therefore$ From (*i*)

$$(AB)^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

**OR**

(*b*) Given $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= 1\,(1-4) - 2\,(2-4) + 2\,(4-2) = -3 + 4 + 4 = 5 \neq 0.$$

Let $A_{ij}$ be the cofactors of each element in $|A|$.

$$A_{11} = +(1-4) = -3\ ; \quad A_{12} = -(2-4) = 2\ ;$$
$$A_{13} = +(4-2) = 2$$
$$A_{21} = -(2-4) = 2\ ; \quad A_{22} = +(1-4) = -3\ ;$$
$$A_{23} = -(2-4) = 2$$
$$A_{31} = +(4-2) = 2\ ; \quad A_{32} = -(2-4) = 2;$$
$$A_{33} = +(1-4) = -3$$

$\therefore$ $$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

$\therefore$ $$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \qquad ...(i)$$

Consider $A^2 - 4A - 5I = O$

Multiplying both sides by $A^{-1}$,

$A^{-1}(AA) - 4A^{-1}A - 5A^{-1}I = A^{-1}O$

$\Rightarrow (A^{-1}A)A - 4I - 5A^{-1} = O \Rightarrow IA - 4I - 5A^{-1} = O$

$\Rightarrow IA - 4I = 5A^{-1} \Rightarrow A - 4I = 5A^{-1}$

$$\text{LHS} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1-4 & 2-0 & 2-0 \\ 2-0 & 1-4 & 2-0 \\ 2-0 & 2-0 & 1-4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = 5 \times \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = 5\,A^{-1} = \text{RHS}$$

Hence $A^2 - 4A - 5I = O$

**33.** $$I = \int \frac{x^3 + x}{x^4 - 9} = \int \frac{x^3}{x^4 - 9}dx + \int \frac{x}{x^4 - 9}dx$$

For first integral

Put $t = x^4 - 9 \Rightarrow dt = 4x^3dx \Rightarrow x^3dx = \dfrac{dt}{4}$

$$\therefore \qquad \int \frac{x^3}{x^4-9}dx = \frac{1}{4}\int \frac{dt}{t} = \frac{1}{4}ln|t| + C_1$$

$$= \frac{1}{4}ln|x^4-9| + C_1$$

For second integral

Put $x^2 = s \Rightarrow xdx = \frac{ds}{2}$

$$\therefore \qquad \int \frac{xdx}{x^4-9} = \frac{1}{2}\int \frac{ds}{s^2-3^2} = \frac{1}{12}ln\left|\frac{s-3}{s+3}\right| + C_2$$

$$= \frac{1}{12}ln\left|\frac{x^2-3}{x^2+3}\right| + C_2$$

$$\therefore \qquad I = \frac{1}{4}ln|x^4-9| + \frac{1}{12}ln\left|\frac{x^2-3}{x^2+3}\right| + C \text{ where } C = C_1 + C_2$$

**34.** Consider equation $2y\, e^{x/y}\, dx + (y - 2x\, e^{x/y})\, dy = 0$

$$\Rightarrow \qquad \frac{dx}{dy} = \frac{2x\, e^{x/y} - y}{2y\, e^{x/y}} \qquad \text{...}(i)$$

Let $F(x, y) = \dfrac{2x\, e^{x/y} - y}{2y\, e^{x/y}}$

$$F(\lambda x, \lambda y) = \frac{2\lambda x \,.\, e^{\frac{\lambda x}{\lambda y}} - \lambda y}{2\lambda y\, e^{\frac{\lambda x}{\lambda y}}} = \frac{2x \cdot e^{\frac{x}{y}} - y}{2y\, e^{\frac{x}{y}}} = F(x, y)$$

Hence, function is homogeneous, so corresponding differential equation is homogeneous.

Let $\dfrac{x}{y} = v \Rightarrow x = vy \Rightarrow \dfrac{dx}{dy} = v + y\dfrac{dv}{dy}$

From (*i*), we have

$$v + y\frac{dv}{dy} = \frac{2vy \cdot e^v - y}{2ye^v} = \frac{2ve^v - 1}{2e^v}$$

$$\Rightarrow \qquad y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v} = \frac{-1}{2e^v}$$

$$\Rightarrow \qquad e^v dv = -\frac{1}{2} \cdot \frac{dy}{y} \Rightarrow \int e^v dv = -\frac{1}{2}\int \frac{dy}{y}$$

$$\Rightarrow \qquad e^v = -\frac{1}{2}\log|y| + C$$

$$\Rightarrow \qquad e^{\frac{x}{y}} = -\frac{1}{2}\log|y| + C \qquad \text{...}(ii)$$

Given $x = 0$, when $y = 1$, then from (*ii*), we get

$$e^0 = -\frac{1}{2}\log|1| + C$$

$$\Rightarrow \qquad 1 = 0 + C \Rightarrow C = 1$$

Substituting $C = 1$ in (*ii*), we get

$$e^{\frac{x}{y}} = -\frac{1}{2}\log|y| + 1 \text{ is the particular solution.}$$

**35.** (*a*) $$\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}, \text{ where } \vec{\beta_1} \,||\, \vec{\alpha} \text{ and } \vec{\beta_2} \perp \vec{\alpha} \qquad \text{...}(i)$$

Let $$\vec{\beta_1} = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}) \qquad \text{...}(ii)$$

$$\therefore \qquad 2\hat{i} + \hat{j} - 4\hat{k} = \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{\beta_2} \qquad [\text{from } (i)]$$

$$\Rightarrow \qquad \vec{\beta_2} = (2-3\lambda)\hat{i} + (1-4\lambda)\hat{j} + (-4-5\lambda)\hat{k} \qquad \text{... }(iii)$$

Also $\vec{\beta_2} \perp \vec{\alpha} \Rightarrow \vec{\beta_2} \cdot \vec{\alpha} = 0$

$\Rightarrow \; 3(2-3\lambda) + 4(1-4\lambda) + 5(-4-5\lambda) = 0$

$$\Rightarrow \qquad 50\lambda = -10 \Rightarrow \lambda = -\frac{1}{5}$$

Substituting for $\lambda$ in (*ii*) and (*iii*), we get

$$\vec{\beta_1} = -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) = -\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j} - \hat{k}$$

and
$$\vec{\beta_2} = \left(2 + \frac{3}{5}\right)\hat{i} + \left(1 + \frac{4}{5}\right)\hat{j} + (-4+1)\hat{k}$$

$$\Rightarrow \qquad \vec{\beta_2} = \frac{13}{5}\hat{i} + \frac{9}{5}\hat{j} - 3\hat{k}$$

$$\therefore \qquad 2\hat{i} + \hat{j} - 4\hat{k} = \left(-\frac{3}{5}\hat{i} - \frac{4}{5}j - \hat{k}\right) + \left(\frac{13}{5}\hat{i} + \frac{9}{5}\hat{j} - 3\hat{k}\right)$$

**OR**

(*b*) Let line through the point $A(1, 2, -4)$ be $\dfrac{x-1}{a} = \dfrac{y-2}{b} = \dfrac{z+4}{c}$ ...(*i*)

If line (*i*) is perpendicular to the lines

$$\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and
$$\frac{x-1}{1} = \frac{y+2}{-3} = \frac{z-3}{5}$$

then $\quad 2a + 3b + 4c = 0$ ...(*ii*)

and $\quad a - 3b + 5c = 0$ ...(*iii*)

Solving (*ii*) and (*iii*), we get

$$\frac{a}{15+12} = \frac{-b}{10-4} = \frac{c}{-6-3} \quad \Rightarrow \quad \frac{a}{27} = \frac{b}{-6} = \frac{c}{-9}$$

DR's are 9, –2, –3

From (*i*), line in Cartesian form is $\dfrac{x-1}{9} = \dfrac{y-2}{-2} = \dfrac{z+4}{-3}$

Let $\dfrac{x-1}{9} = \dfrac{y-2}{-2} = \dfrac{z+4}{-3} = \lambda$ (say)

General point on the line is $(9\lambda + 1, -2\lambda + 2, -3\lambda - 4)$

Position vector of point on the line is

$$\vec{r} = (9\lambda + 1)\hat{i} + (-2\lambda + 2)\hat{j} + (-3\lambda - 4)\hat{k}$$

$\Rightarrow \qquad \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(9\hat{i} - 2\hat{j} - 3\hat{k})$ is equation of line in vector form.

**36.** (*i*) No, as $(x, x) \notin R$ for $x \in A$

(*ii*) No, as $(A, B) \in R$ but $(B, A) \notin R$

(*iii*) (*a*) $R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, B)\}$

$$\text{Domain} = \{A, B, C, D\}$$

$$\text{Range} = \{B, C, D, E\}$$

**OR**

(*iii*) (*b*) No as A has three images;

B and D has two images.

**37.** (*i*) Given $\pi r^2 h = 432\pi$

$\Rightarrow$ $r^2 h = 432$

(*ii*) The total surface area, $S = 2\pi rh + 2\pi r^2$

$$= 2\pi r \frac{432}{r^2} + 2\pi r^2 = \left(\frac{864\pi}{r} + 2\pi r^2\right) \text{ sq units}$$

(*iii*) (*a*) For minimum surface area, $\frac{dS}{dr} = 0$ and $\frac{d^2S}{dr^2} > 0$

$$\frac{dS}{dr} = \frac{-864\pi}{r^2} + 4\pi r$$

Now, $\frac{dS}{dr} = 0 \Rightarrow r^3 = 216 \Rightarrow r = 6$ units

Now, $\frac{d^2S}{dr^2} = \frac{1728\pi}{r^3} + 4\pi$

So, $\frac{d^2S}{dr^2} > 0$ for $r = 6$ units

**OR**

(*iii*) (*b*) $S_{\min} = \frac{864\pi}{6} + 2\pi(6)^2 = 144\pi + 72\pi = 216\pi$ sq units

**38.** Let $E_1$ : Event that Annu gets a prime number when a die is thrown.

$A$ : Event that she gets exact one head.

$E_2$ : Event that Annu gets non-prime number when a die is thrown.

$$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2}$$

$$P\left(\frac{A}{E_1}\right) = \frac{3}{8} \text{ and } P\left(\frac{A}{E_2}\right) = \frac{1}{2}$$

(*i*) $$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right) + P(E_2)\times P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{3}{8}}{\frac{1}{2}\times\frac{3}{8}+\frac{1}{2}\times\frac{1}{2}} = \frac{3}{7}$$

(*ii*) $$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\times P\left(\frac{A}{E_2}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right) + P(E_2)\times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{3}{8}+\frac{1}{2}\times\frac{1}{2}} = \frac{4}{7}$$