Solutions to RMM/Set-2

1. (*d*) $\operatorname{cosec}^{-1}x$

2. (*a*) $A = [a_{ij}]_{m \times n}$

3. (*b*) by definition.

4. (*b*)
$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix} \Rightarrow |A| = \frac{a(1+bc)}{a} - bc$$
$$= 1 + bc - bc = 1$$

$\Rightarrow A^{-1}$ exists.

Now,
$$\text{adj}\,A = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

$\Rightarrow$
$$A^{-1} = \frac{1}{|A|}(\text{adj}\,A) = 1\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

5. (*a*) Points $P(3, -2)$, $Q(8, 8)$, $R(k, 2)$ are collinear.

$\therefore$
$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 3(8 - 2) + 2(8 - k) + 1(16 - 8k) = 0$

$\Rightarrow \quad 18 + 16 - 2k + 16 - 8k = 0$

$\Rightarrow \quad k = 5$

6. (*b*)
$$A\cdot\text{Adj}\,A = |A|\cdot I \text{ and } A^{-1} = \frac{1}{|A|}\text{Adj}\,A$$

$\Rightarrow \quad |A| = -4$

$\Rightarrow \quad K = -4$

$\Rightarrow \quad 16K = -64$

7. (*d*) as 'f' is not defined at $x = 0$. i.e. $f(0)$ does not exist.

8. (*a*)
$$y = e^{1 + \log x} = e^{\log e + \log x}$$

$\Rightarrow \quad y = e^{\log(ex)}$

$\Rightarrow \quad y = ex$

Differentiating both sides w.r.t x, we get
$$\frac{dy}{dx} = e$$

9. (*c*)
$$f'(x) = 1 + \sin x > 0 \text{ for } x \in R. \qquad \left\{1 + \sin x = \left(\cos\frac{x}{2} + \frac{\sin x}{2}\right)^2 \geq 0\right\}$$

As $0 \leq 1 + \sin x \leq 2$

$\therefore$ Always increasing.

10. (*a*) Degree = 2, order = 2

11. (*c*) $\int \frac{\tan x - 1}{\tan x + 1} dx = \int -\tan\left(\frac{\pi}{4} - x\right) dx$

$$= -\frac{\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|}{-1} + C$$

$$= \log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + C$$

12. (*b*) $\therefore \quad \vec{a} \cdot \vec{b} = 3 - 2 - 1 = 3 - 3 = 0$

$\Rightarrow \quad \vec{a} \perp \vec{b}$

13. (*d*) $3x + 1 = 6y - 2 = 1 - z$

$$3\left(x + \frac{1}{3}\right) = 6\left(y - \frac{1}{3}\right) = -(z - 1)$$

$$\frac{x + \frac{1}{3}}{\frac{1}{3}} = \frac{y - \frac{1}{3}}{\frac{1}{6}} = \frac{z - 1}{-1}$$

$$\frac{x + \frac{1}{3}}{2} = \frac{y - \frac{1}{3}}{1} = \frac{z - 1}{-6}.$$

14. (*a*) Given $|\vec{a}| = 1$, $|\vec{b}| = 1$ and $|\vec{a} + \vec{b}| = 1$

Since $|\vec{a} + \vec{b}| = 1 \Rightarrow |\vec{a} + \vec{b}|^2 = 1$

$\Rightarrow \quad (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1$

$\Rightarrow \quad \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} = 1$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$

$\Rightarrow \quad 1 + 1 + 2\vec{a} \cdot \vec{b} = 1$

$\Rightarrow \quad \vec{a} \cdot \vec{b} = -\frac{1}{2}$

$\therefore \quad \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{\frac{-1}{2}}{1 \times 1} = -\frac{1}{2}$

$\Rightarrow \quad \theta = 120°$ or $\frac{2\pi}{3}$

15. (*d*) as $|k\vec{a}| = |k||\vec{a}| = 2|k|$

Now, $-3 \le k \le 2$

$\Rightarrow \quad 0 \le |k| \le 3$

$\Rightarrow \quad 0 \le 2|k| \le 6$

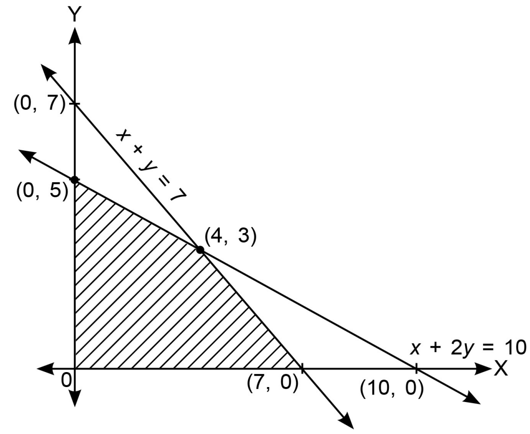
$\Rightarrow \quad 0 \le |k\vec{a}| \le 6$

So, $|k\vec{a}| \in [0, 6]$.

16. (*c*) Solving equations $x + y = 7$ and $x + 2y = 10$, we get

$y = 3, x = 4$

Plotting the graph of inequations, we see that shaded region is the feasible solution. The coordinates of corner points of shaded feasible region are (0, 0), (7, 0), (4, 3) and (0, 5).



Points	Values of $Z = 5x + 2y$
(0, 5)	10
(7, 0)	35 ← Maximum
(0, 0)	0
(4, 3)	26

17. (*b*) As (1, 2) does not satisfy inequation $3x \geq 5$.

18. (*a*)

19. (*b*) Assertion $R'(x) = 6x + 36$

$$R'(5) = 30 + 36 = 66, \text{ true}$$

Both A and R are true and R is not the correct explanation of A.

20. (*a*) Both A and R are true and R is the correct explanation of A.

21. (*a*)

$$\sin\left[\cos^{-1}\left(\cos\frac{7\pi}{4}\right)\right] = \sin\left[\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$= \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

OR

(*b*) For domain $-1 \leq 3x + 4 \leq 1$

$$\Rightarrow \quad -5 \leq 3x \leq -3$$

$$\Rightarrow \quad -\frac{5}{3} \leq x \leq -1$$

Domain $= \left[-\frac{5}{3}, -1\right]$

22.

$$y = \log(\log x^2)$$

Differentiating w.r.t. x both sides,

$$y_1 = \frac{1}{\log x^2} \cdot \frac{1}{x^2} \cdot 2x = \frac{2}{x \log x^2}$$

$$\Rightarrow \quad y_1 = \frac{2}{x \cdot 2 \log x} = \frac{1}{x \log x}$$

Differentiating again w.r.t. x both sides,

$$y_2 = \frac{x \log x \times 0 - 1\left\{x \times \frac{1}{x} + \log x\right\}}{(x \log x)^2} = \frac{-(1 + \log x)}{(x \log x)^2}$$

23. (*a*) Let $I = \int \frac{dx}{1+\tan x} = \int \frac{\cos x}{\sin x + \cos x}\,dx = \frac{1}{2}\int \frac{2\cos x}{\sin x + \cos x}\,dx = \frac{1}{2}\left[\int \frac{\cos x + \cos x + \sin x - \sin x}{\sin x + \cos x}dx\right]$

$$= \frac{1}{2}\left[\int \frac{\cos x + \sin x}{\sin x + \cos x}dx + \int \frac{\cos x - \sin x}{\sin x + \cos x}dx\right] = \frac{1}{2}\left[x + \int \frac{\cos x - \sin x}{\sin x + \cos x}dx\right]$$

$$= \frac{1}{2}\left[x + \int \frac{dt}{t}\right]$$

$$= \frac{1}{2}[x + \log|\cos x + \sin x|] + C$$

OR

(*b*) $$\text{Area} = \left|\int_{-3}^{0} -x^2\,dx\right| = \left|-\left[\frac{x^3}{3}\right]_{-3}^{0}\right|$$

$$= |-[0-(-9)]| = |-9| = 9 \text{ sq. units}$$

24. Let r be the base radius and h be the height of the cylinder at a particular instant of time 't'. Let V be its volume at that instant.

Given: $\frac{dr}{dt} = 2$ cm/s; $\frac{dh}{dt} = -3$ cm/s

We know that:

Volume of cylinder, $V = \pi r^2 h$

On differentiating both sides w.r.t 't', we get

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + h \times 2r\frac{dr}{dt}\right)$$

Now, $$\left[\frac{dV}{dt}\right]_{r=3,\,h=4} = \pi[9 \times (-3) + 4 \times 2 \times 3 \times 2] = \pi[-27 + 48]$$

$$= 21\pi \text{ cm}^3/\text{s}$$

So, volume is increasing at the rate of 21π cm^3/s.

25. From question

$$\vec{a} = \left(l\hat{i} + m\hat{j} + n\hat{k}\right) \qquad [\because \vec{a} \text{ is a unit vector}]$$

$$= \left(\cos\frac{\pi}{3}\hat{i} + \cos\frac{\pi}{4}\hat{j} + \cos\theta\,\hat{k}\right)$$

$\Rightarrow$ $$\vec{a} = \left(\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \cos\theta\,\hat{k}\right)$$

$\therefore$ $$|\vec{a}| = \left|\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \cos\theta\,\hat{k}\right|$$

$\Rightarrow$ $$1 = \sqrt{\frac{1}{4} + \frac{1}{2} + \cos^2\theta}$$

$\Rightarrow$ $$1 = \frac{3}{4} + \cos^2\theta$$

$\Rightarrow$ $$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$\Rightarrow \quad \cos\theta = \pm\frac{1}{2}$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \quad [\because \theta \text{ is an acute angle}]$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

26. (*a*) $$x = \frac{1}{y} \Rightarrow \frac{dx}{dy} = -\frac{1}{y^2} = -\frac{1}{y^2}\sqrt{\frac{1+y^4}{1+y^4}}$$

$$= -\sqrt{\frac{1}{y^4}\cdot\left(\frac{1+y^4}{1+y^4}\right)} = -\sqrt{\frac{\frac{1}{y^4}+1}{1+y^4}}$$

$$= -\sqrt{\frac{x^4+1}{1+y^4}} = -\sqrt{\frac{1+x^4}{1+y^4}}$$

OR

(*b*) Given $x^p y^q = (x+y)^{p+q}$

Taking logarithm both sides, we get

$p \log x + q \log y = (p+q)\log(x+y)$

Differentiating both sides w.r.t. x, we get $\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = (p+q)\left[\frac{1}{x+y}\left\{1+\frac{dy}{dx}\right\}\right]$

$\Rightarrow \quad \frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y} + \frac{p+q}{x+y}\frac{dy}{dx}$

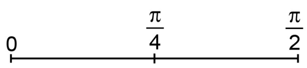
$\Rightarrow \quad \frac{p}{x} - \frac{p+q}{x+y} = \left(\frac{p+q}{x+y} - \frac{q}{y}\right)\frac{dy}{dx}$

$\Rightarrow \quad \frac{px+py-px-qx}{x(x+y)} = \frac{py+qy-qx-qy}{y(x+y)}\frac{dy}{dx}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{y}\frac{dy}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$ proved.

27. $f'(x) = 4\sin^3 x\cos x - 4\cos^3 x \sin x = -\sin 4x$ ...(*i*)

$f'(x) = 0 \Rightarrow \sin 4x = 0 \Rightarrow 4x = 0, \pi, 2\pi$



$\Rightarrow x = 0, \frac{\pi}{4}, \frac{\pi}{2}$

CASE I: $0 < x < \frac{\pi}{4} \Rightarrow 0 < 4x < \pi$

$\Rightarrow 4x \in$ I, IInd quadrant

From (*i*), $f'(x) < 0 \Rightarrow f$ is decreasing

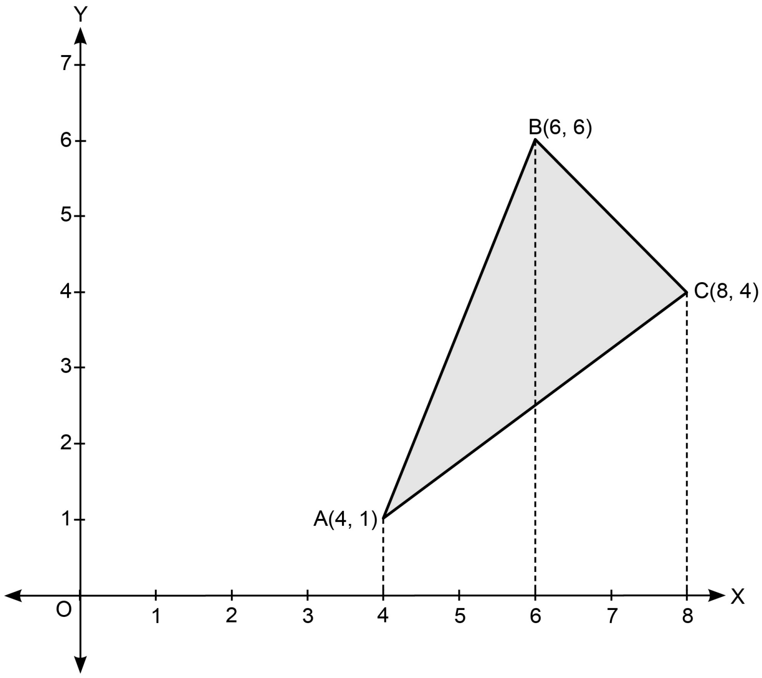
CASE II: $\frac{\pi}{4} < x < \frac{\pi}{2} \Rightarrow \pi < 4x < 2\pi$

$\Rightarrow 4x \in$ III, IVth quadrant.

From (*i*), $f'(x) > 0 \Rightarrow f$ is increasing.

$\therefore f$ is decreasing for $\left(0, \frac{\pi}{4}\right)$, increasing for $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

28. Plotting the points $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$ on graph, we notice, we have to find shaded area.



$$\text{Area } (\Delta ABC) = \int_4^6 y_{AB}\,dx + \int_6^8 y_{BC}\,dx - \int_4^8 y_{AC}\,dx$$

$$\text{Equation of } AB : y - 1 = \frac{6-1}{6-4}(x-4) \Rightarrow y - 1 = \frac{5}{2}(x-4)$$

$$\Rightarrow \quad y = \frac{5}{2}x - 10 + 1 \Rightarrow y = \frac{5}{2}x - 9$$

$$\text{Equation of } BC : y - 6 = \frac{4-6}{8-6}(x-6) \Rightarrow y - 6 = -1(x-6)$$

$$\Rightarrow \quad y - 6 = -x + 6 \Rightarrow y = -x + 12$$

$$\text{Equation of } AC : y - 1 = \frac{4-1}{8-4}(x-4) \Rightarrow y - 1 = \frac{3}{4}(x-4)$$

$$\Rightarrow \quad y = \frac{3}{4}x - 3 + 1 \Rightarrow y = \frac{3}{4}x - 2$$

$$\therefore \quad \text{Area} = \int_4^6 \left(\frac{5}{2}x - 9\right)dx + \int_6^8 (-x+12)\,dx - \int_4^8 \left(\frac{3}{4}x - 2\right)dx$$

$$= \left[\frac{5x^2}{4} - 9x\right]_4^6 + \left[-\frac{x^2}{2} + 12x\right]_6^8 - \left[\frac{3x^2}{8} - 2x\right]_4^8$$

$$= [(45 - 54) - (20 - 36)] + [(-32 + 96) - (-18 + 72)] - [(24 - 16) - (6 - 8)]$$

$$= [-9 + 16] + [64 - 54] - [8 + 2]$$

$$= 7 + 10 - 10$$

$$= 7 \text{ sq units}$$

29. (*a*) Here $\vec{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b_2} = 3\hat{i} + 2\hat{j} - \hat{k}$

$$\therefore \quad \vec{b_1} \cdot \vec{b_2} = 6 + 6 - 1 = 11$$

$$|\vec{b_1}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$

$$|\vec{b_2}| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$$

Let θ be the angle between the two lines.

$$\therefore \qquad \cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \left|\frac{11}{\sqrt{14}\,\sqrt{14}}\right| = \frac{11}{14}$$

$$\therefore \qquad \theta = \cos^{-1}\left(\frac{11}{14}\right)$$

OR

(*b*) Equation of line passes through A(0, –1, –1) and B(4, 5, 1) is

$$\vec{r} = -\hat{j} - \hat{k} + \lambda(4\hat{i} + 6\hat{j} + 2\hat{k}) \qquad ...(i)$$

and equation of line passes through C(3, 9, 4) and D(–4, 4, 4) is

$$\vec{r} = 3\hat{i} + 9\hat{j} + 4\hat{k} + \mu(-7\hat{i} - 5\hat{j}) \qquad ...(ii)$$

General point on the line (*i*) be $(4\lambda, -1 + 6\lambda, -1 + 2\lambda)$

General point on line (*ii*) be $(3 - 7\mu, 9 - 5\mu, 4)$

For point of intersection of (*i*) and (*ii*), we have

$$4\lambda = 3 - 7\mu \qquad ...(iii)$$

$$-1 + 6\lambda = 9 - 5\mu \qquad ...(iv)$$

$$-1 + 2\lambda = 4 \qquad ...(v)$$

From equation (*v*) $\qquad \lambda = \dfrac{5}{2},$

Using $\lambda = \dfrac{5}{2}$ in (*iv*), we get

$$\Rightarrow \qquad -1 + 15 = 9 - 5\mu$$

$$\Rightarrow \qquad 5 = -5\mu$$

$$\Rightarrow \qquad \mu = -1$$

Using $\lambda = \dfrac{5}{2}$ and $\mu = -1$ in equation (*iii*), we get

$$4 \times \frac{5}{2} = 3 - 7 \times -1$$

$$\Rightarrow \qquad 10 = 3 + 7$$

$$\Rightarrow \qquad 10 = 10 \text{ (true)}$$

$\therefore$ Lines (*i*) and (*ii*) intersect. **Proved**

30. (*a*) We have to maximise, $Z = 3x + 5y$

The given constraints: $\qquad x + 4y \le 24 \qquad ...(i)$

$$3x + y \le 21 \qquad ...(ii)$$

$$x + y \le 9 \qquad ...(iii)$$

$$x \ge 0, \quad y \ge 0 \qquad ...(iv)$$

Converting (*i*) and (*iii*) inequations to equations and solving, we get

$$\begin{array}{r} x + 4y = 24 \\ x + y = 9 \\ - \quad - \qquad - \\ \hline 3y = 15 \\ \hline \end{array}$$

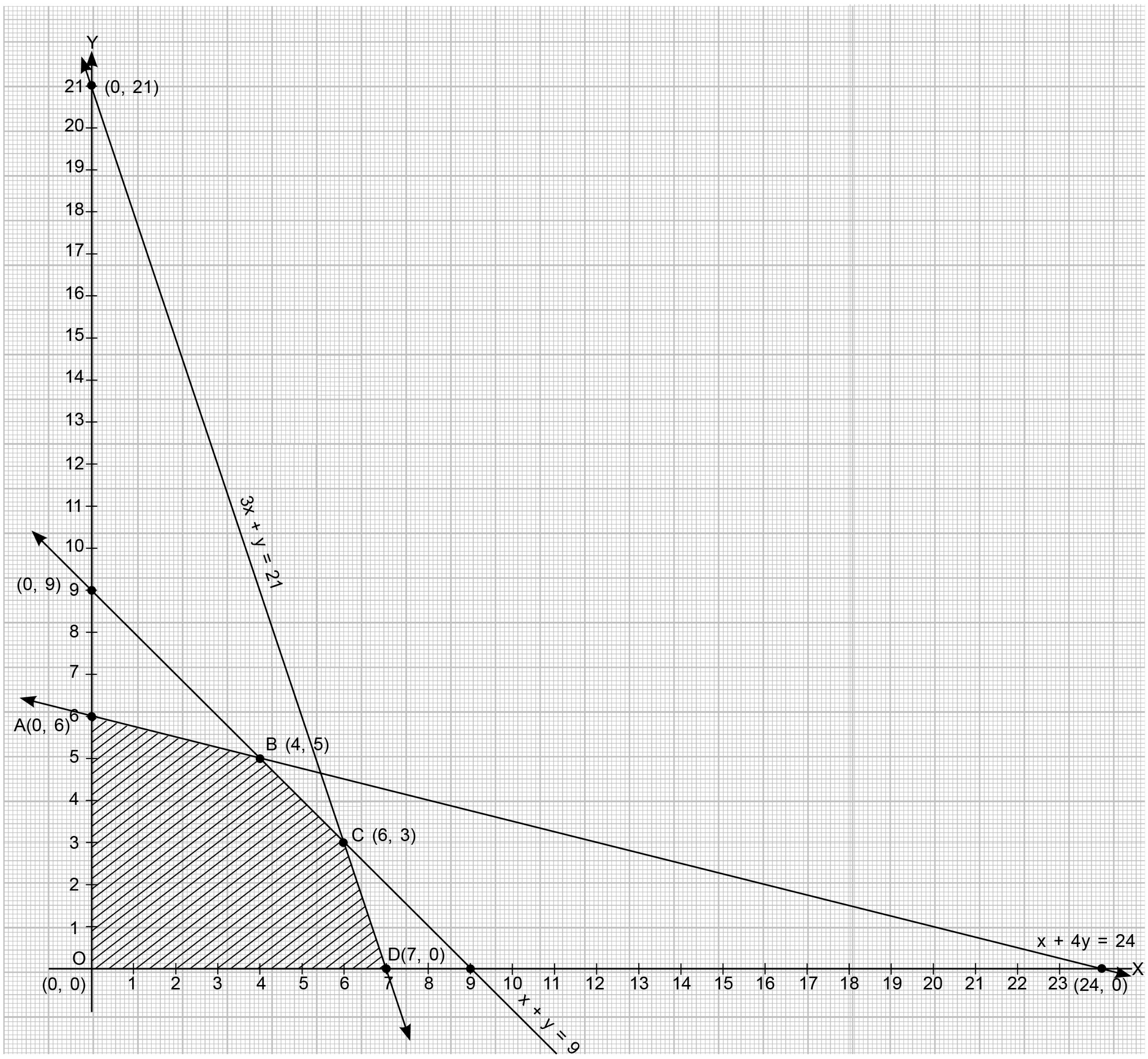
$$\Rightarrow \qquad y = 5, \text{ and } x = 4$$

Converting (*ii*) and (*iii*) inequations to equations and solving, we get

$$\begin{array}{r} x + y = 9 \\ 3x + y = 21 \\ - \quad - \quad - \\ \hline -2x = -12 \\ \hline \end{array}$$

$\Rightarrow$ $x = 6$, and $y = 3$

Converting (*i*) and (*ii*) inequations to equations and solving we get $x = \frac{60}{11}$ and $y = \frac{51}{11}$.



Corner Points	Values of $Z = 3x + 5y$
$O(0, 0)$	0
$A(0, 6)$	30
$B(4, 5)$	37 ← Maximum
$C(6, 3)$	33
$D(7, 0)$	21

$\therefore$ Maximum value $= 37$ at $x = 4$, $y = 5$

OR

(*b*) We have to minimise, $Z = 9x - 11y$

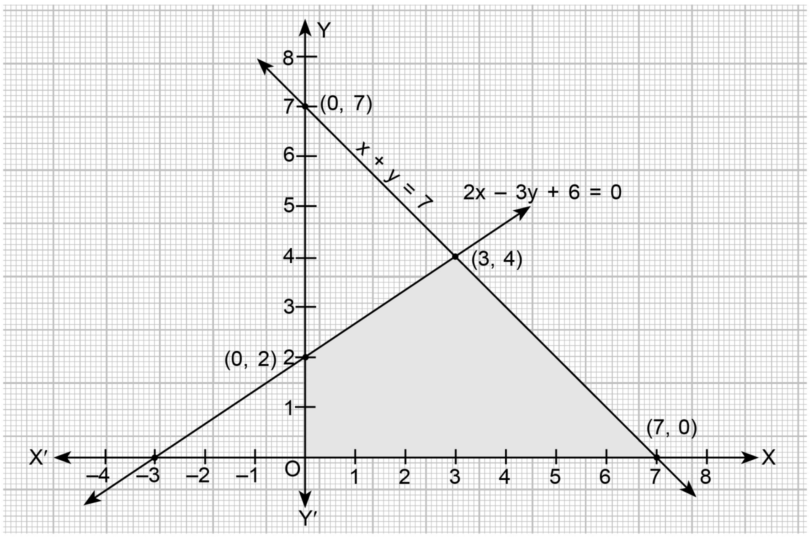
The given constraints are:

$$x + y \leq 7 \quad \text{...}(i)$$

$$2x - 3y + 6 \geq 0 \quad \text{...}(ii)$$

$$x, y \geq 0 \quad \text{...}(iii)$$

Let us graph the feasible region of the system of linear inequalities (*i*) to (*iii*). The shaded region is the feasible region.



Corner Points	Values of $Z = 9x - 11y$
(0, 0)	0
(7, 0)	63
(3, 4)	–17
(0, 2)	–22 ← Minimum

∴ Minimum value = –22 at $x = 0$, $y = 2$.

31. $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}, P(C) = \frac{1}{4}$

Problem is solved by exactly two students.

∴ $P(\text{Problem solved by exactly two students}) = P(AB\overline{C}) \text{ or } P(A\overline{B}C) \text{ or } P(\overline{A}BC)$

$$= \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{24}(3 + 1 + 2) = \frac{1}{4}$$

32. (*a*)

$$|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1 \text{ (expanding along } C_3)$$

Let A_{ij} be the cofactors of elements in $|A|$. Then,

$$A_{11} = \cos\alpha, \quad A_{12} = -\sin\alpha, \quad A_{13} = 0$$
$$A_{21} = \sin\alpha, \quad A_{22} = \cos\alpha, \quad A_{23} = 0$$
$$A_{31} = 0, \quad A_{32} = 0, \quad A_{33} = 1$$

$\therefore$ $$\text{adj } A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}' = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider $$A(\text{adj } A) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha + 0 & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha + 0 & 0+0+0 \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha + 0 & \sin^2\alpha + \cos^2\alpha + 0 & 0+0+0 \\ 0-0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 = |A|\, I_3$$

$$(\text{adj } A)\, A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha + 0 & -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha + 0 & 0+0+0 \\ -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha + 0 & \sin^2\alpha + \cos^2\alpha + 0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 = |A|\, I_3$$

$\therefore$ $A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_3$.

OR

(*b*) Consider $$A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

and $$B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I \qquad ...(i)$$

Consider equations

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

Corresponding matrix equation is

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$\Rightarrow$ $BX = C$ is matrix equation.

Its solution is $X = B^{-1}C$ ...(*ii*)

Now we can use result (*i*) as

$$AB = 8I \quad \Rightarrow \quad \left(\frac{1}{8}A\right)B = I$$

$$\Rightarrow \quad B^{-1} = \frac{1}{8}A$$

Now we can substitute B^{-1} in (*ii*) and proceed further by substituting for A and finding X and then x, y, z.

$$X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow \quad x = 3; y = -2; z = -1$$

33. (*a*) Let $$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$\Rightarrow$ $2x - 1 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$ ...(*i*)

Put $x = 1$ in (*i*), we get

$$2 - 1 = A \times 3 \times -2$$

$$\Rightarrow \quad 1 = -6A \Rightarrow A = \frac{-1}{6}$$

Put $x = -2$ in (*i*), we get

$$-5 = B \times -3 \times -5$$

$$\Rightarrow \quad B = \frac{-1}{3}$$

Put $x = 3$ in (*i*), we get

$$5 = C \times 2 \times 5$$

$$\Rightarrow \quad C = \frac{1}{2}$$

$$\therefore \quad \frac{2x-1}{(x-1)(x+2)(x-3)} = -\frac{1}{6}\times\frac{1}{x-1} - \frac{1}{3}\times\frac{1}{x+2} + \frac{1}{2}\times\frac{1}{x-3}$$

$$\therefore \quad I = \int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$$

$$= \frac{-1}{6}\int\frac{1}{x-1}dx - \frac{1}{3}\int\frac{dx}{x+2} + \frac{1}{2}\int\frac{dx}{x-3}$$

$$= \frac{-1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + C$$

OR

(*b*) $$I = \int_0^{\pi} x \log \sin x \, dx$$ ...(*i*)

Also $$I = \int_0^{\pi} (\pi - x)\log\sin(\pi - x)dx$$ [By P_4]

$\Rightarrow$ $$I = \pi\int_0^{\pi}\log\sin x\, dx - \int_0^{\pi} x \sin x \, dx$$ ...(*ii*)

Adding (i) and (ii), we get

$$2I = \pi\int_0^{\pi} \log \sin x \, dx$$

$$\Rightarrow \quad 2I = \pi \times 2\int_0^{\pi/2} \log \sin x \, dx = 2\pi\int_0^{\pi/2} \log \sin x \, dx \quad [\text{using } P_6]$$

$$\Rightarrow \quad I = \pi\int_0^{\pi/2} \log \sin x \, dx \quad ...(iii)$$

Let $$I_1 = \int_0^{\pi/2} \log \sin x \, dx \quad ...(iv)$$

Also $$I_1 = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right)dx = \int_0^{\pi/2} \log \cos x \, dx \quad ...(v) \; [\text{using } P_4]$$

$\therefore$ Adding (iv) and (v), we get

$$2I_1 = \int_0^{\pi/2} \log(\sin x \cos x) dx$$

$$\Rightarrow \quad 2I_1 = \int_0^{\pi/2} \log(\sin 2x) dx - \log 2\int_0^{\pi/2} dx$$

$$= \int_0^{\pi/2} \log(\sin 2x) dx - \log 2 \times \frac{\pi}{2}$$

Put $2x = t \Rightarrow dx = \frac{dt}{2}$

If $x = 0 \Rightarrow t = 0$, if $x = \frac{\pi}{2}$, $t = \pi$

$$\therefore \quad 2I_1 = \frac{1}{2}\int_0^{\pi} \log \sin t \, dt - \frac{\pi}{2}\log 2$$

$$\Rightarrow \quad 2I_1 = \frac{2}{2}\int_0^{\pi/2} \log \sin t \, dt - \frac{\pi}{2}\log 2 \quad [\text{using } P_6]$$

$$= \int_0^{\pi/2} \log \sin x \, dx - \frac{\pi}{2}\log 2 \quad [\text{using } P_0]$$

$$= I_1 - \frac{\pi}{2}\log 2$$

$$\Rightarrow \quad I_1 = \frac{-\pi}{2}\log 2$$

Using the value of I_1 in (iii), we get

$$I = \pi \times -\frac{\pi}{2}\log 2 = -\frac{\pi^2}{2}\log 2$$

34. $$\frac{dy}{dx} = \cos(x + y) + \sin(x + y) \quad ...(i)$$

Put $$x + y = t$$

$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dt}{dx} - 1$$

Using the values of $x + y$ and $\frac{dy}{dx}$ in (i), we get

$$\frac{dt}{dx} - 1 = \cos t + \sin t$$

$$\Rightarrow \quad \frac{dt}{dx} = 1 + \cos t + \sin t$$

$$\Rightarrow \quad \frac{dt}{1 + \cos t + \sin t} = dx$$

Integrating both sides, we get

$$\int \frac{dt}{1+\cos t+\sin t} = \int dx = x + C \qquad ...(ii)$$

For $$\int \frac{dt}{1+\cos t+\sin t} = \int \frac{dt}{1+\dfrac{1-\tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}}+\dfrac{2\tan\left(\frac{t}{2}\right)}{1+\tan^2\frac{t}{2}}}$$

$$= \int \frac{dt}{\dfrac{1+\tan^2\frac{t}{2}+1-\tan^2\left(\frac{t}{2}\right)+2\tan\left(\frac{t}{2}\right)}{1+\tan^2\frac{t}{2}}}$$

$$= \int \frac{\left(1+\tan^2\frac{t}{2}\right)dt}{2+2\tan\left(\frac{t}{2}\right)} = \int \frac{\sec^2\frac{t}{2}dt}{2\left(1+\tan\left(\frac{t}{2}\right)\right)}$$

Put $$1 + \tan\left(\frac{t}{2}\right) = z$$

$\Rightarrow$ $$\sec^2\left(\frac{t}{2}\right) \times \frac{dt}{2} = dz$$

$\Rightarrow$ $$\frac{1}{2}\sec^2\left(\frac{t}{2}\right)dt = dz$$

$\therefore$ $$\int \frac{1}{1+\cos t+\sin t}dt = \int \frac{dz}{z} = \log|z|$$

$$= \log\left|1+\tan\frac{t}{2}\right| = \log\left|1+\tan\left(\frac{x+y}{2}\right)\right|$$

Using in (*ii*), we get

$$\log\left|1+\tan\left(\frac{x+y}{2}\right)\right| = x + \text{C}$$

35. Let $\vec{d}$ be $x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\vec{d} \perp \vec{c}$, then

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + \hat{j} - \hat{k}) = 0$$

$\Rightarrow$ $$3x + y - z = 0 \qquad ...(i)$$

Also, $\vec{d} \perp \vec{b}$, then

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} - 4\hat{j} + 5\hat{k}) = 0$$

$\Rightarrow$ $$x - 4y + 5z = 0 \qquad ...(ii)$$

Also, $$\vec{d} \cdot \vec{a} = 21$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (4\hat{i} + 5\hat{j} - \hat{k}) = 21$$

$\Rightarrow$ $$4x + 5y - z = 21 \qquad ...(iii)$$

Subtracting (*i*) from (*iii*), we get

$$x + 4y = 21 \qquad ...(iv)$$

Multiply (*i*) by '5' and then adding (*ii*) to it, we get $15x + 5y - 5z + x - 4y + 5z = 0$

$\Rightarrow$ $$16x + y = 0 \qquad ...(v)$$

Solving (*iv*) and (*v*), we get

$x = \frac{-1}{3}$ and $y = \frac{16}{3}$

Put the values of 'x' and 'y' in (*i*),

we get $z = \frac{13}{3}$

$$\therefore \quad \vec{d} = \frac{1}{3}(-\hat{i} + 16\hat{j} + 13\hat{k})$$

36. (*i*) R = {(S_1, A), (S_2, B), (S_3, C), (S_4, D)}.

(*ii*) Yes, for each $x \in$ X, $\exists f(x) \in$ Y unique.

(*iii*) (*a*) For $(x, y) \in R$, $\nexists$ $(y, z) \in R$ such that $(x, z) \notin R$.

Hence R is a transitive relation.

OR

(*iii*) (*b*) If S_5 with scores 92 added to R, then we get a new relation

R_1 = {(S_1, A), (S_2, B), (S_3, C), (S_4, D), (S_5, A)}

is a relation from set

Z = {S_1, S_2, S_3, S_4, S_5} to Y = {A, B, C, D}.

Which is also a function. It is a many one function.

37. Given $$x = \frac{600 - p}{8}$$

$$\Rightarrow \quad 8x = 600 - p$$

$$\Rightarrow \quad p = 600 - 8x$$

$$R(x) = p \cdot x$$

$$= 600x - 8x^2$$

Cost function, $$C(x) = x^2 + 78x + 2500$$

(*i*) $$p = 600 - 8x$$

(*ii*) $$P(x) = R(x) - C(x)$$

$$= 600x - 8x^2 - x^2 - 78x - 2500$$

$$= -9x^2 + 522x - 2500$$

(*iii*) (*a*) $$P(x) = -9x^2 + 522x - 2500$$

Differentiating both sides w.r.t. x, we get

$$P'(x) = -18x + 522$$

For maximum or minimum profit,

put $$P'(x) = 0$$

$$\Rightarrow \quad 18x = 522$$

$$\Rightarrow \quad x = \frac{522}{18} = 29$$

$$P''(x) = -18 < 0$$

$\therefore$ $P(x)$ is maximum when $x = 29$.

OR

(*iii*) (*b*) $P(x) = -9x^2 + 522x - 2500$

Differentiating both sides w.r.t. x, we get

$$P'(x) = -18x + 522$$

Put $P'(x) = 0$, for critical point

$\therefore$ $-18x = -522 \Rightarrow x = 29$

Interval	sign of $P'(x)$
(0, 29)	+ve
(29, ∞)	–ve

$\therefore$ $P(x)$ is increasing in the interval (0, 29)

38. Let E_1: scooter driver is selected

E_2: truck driver is selected

E_3: car driver is selected

E: Ensured person meet with an accident

$$P(E_1) = \frac{2}{9}, \quad P(E_2) = \frac{4}{9}, \quad P(E_3) = \frac{3}{9}$$

$$P\left(\frac{E}{E_1}\right) = 0.01, \quad P\left(\frac{E}{E_2}\right) = 0.04; \quad P\left(\frac{E}{E_3}\right) = 0.02$$

(*i*) P(truck driver meeting with an accident) $= P(E_2)\, P(E/E_2)$

$$= \frac{4}{9} \times 0.04 = \frac{4}{225}$$

(*ii*)

$$P(E) = P(E_1)\, P(E/E_1) + P(E_2)\, \mathrm{P}(E/E_2) + P(E_3)\, P(E/E_3)$$

$$= \frac{2}{9} \times 0.01 + \frac{4}{9} \times 0.04 + \frac{3}{9} \times 0.02$$

$$= \frac{2 + 16 + 6}{900} = \frac{24}{900}$$

$$= \frac{2}{75}$$