Solutions to RMM/Set-3

1. (*d*) Let $y = \sin^{-1}\sqrt{x-1}$

$\because \quad -1 \le \sqrt{x-1} \le 1$

$\Rightarrow \quad 0 \le \sqrt{x-1} \le 1$

$\Rightarrow \quad 0 \le x - 1 \le 1$

$\Rightarrow \quad 1 \le x \le 2$

$x \in [1, 2]$

2. (*c*)

3. (*b*) Direction ratios of line through the points (1, – 1, 2) and (3, 4, – 2) are < 2, 5, –4 >.

Direction ratios of line through the points (0, 3, 2) and (3, 5, 6) are < 3, 2, 4 >.

Now $\quad a_1a_2 + b_1b_2 + c_1c_2 = 3 \times 2 + 2 \times 5 + 4 \times (-4) = 0$

$\therefore$ Lines are perpendicular.

4. (*c*) **5.** (*d*) **6.** (*a*)

7. (*b*) $x = \sin^2 t \Rightarrow \dfrac{dx}{dt} = 2\sin t\cos t = \sin(2t)$

$y = \cos^2 t \Rightarrow \dfrac{dy}{dt} = -2\cos t\sin t = -\sin(2t)$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sin 2(2t)}{\sin 2(2t)} = -1$$

8. (*c*) $[x]$ represents greatest integer function less than or equal to x

$\therefore x - [x]$ is defined at all integral points.

Now $f(x) = x - [x]$ is defined at all integral points

Consider 'n' be any integer.

$$\text{LHL} = \lim_{x\to n^-} f(x) = \lim_{x\to n^-} x - [x]$$

$$= \lim_{x\to n^-} x - \lim_{x\to n^-} [x] \qquad \left\{\begin{array}{l}\text{as } x \to n^- \\ \Rightarrow [x] \to (n-1)\end{array}\right\}$$

$$= n - (n-1) = 1$$

$$\text{RHL} = \lim_{x\to n^+} f(x) = \lim_{x\to n^+} x - [x] \qquad (\text{as } x \to n^+ \Rightarrow [x] \to n)$$

$$= \lim_{x\to n^+} x - \lim_{x\to n^+} [x]$$

$$= n - n = 0$$

$$\lim_{x\to n^-} f(x) \ne \lim_{x\to n^+} f(x)$$

$\Rightarrow \quad f(x)$ is not continuous at $x = n$

9. (*a*) Projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{(2\hat{i} - \hat{j} + \hat{k})\cdot(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}}$$

$$= \frac{2 - 2 + 2}{3} = \frac{2}{3}$$

10. (*c*) $$x\frac{dy}{dx} - y = x^4 - 3x$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$$

Comparing with standard form of linear differential equation, i.e. $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{-1}{x};\ Q = x^3 - 3$$

Now, $$IF = e^{\int P dx} = e^{\int -\frac{1}{x}dx}$$

$$= e^{-\log|x|} = \frac{1}{x}$$

11. (*c*)

12. (*a*) Required area $= \int_0^3 x\,dx = \left[\frac{x^2}{2}\right]_0^3 = \frac{9}{2}$ sq units

13. (*c*) as $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(*i*)

Using property: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad (ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_0^{\pi/2} 1\cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

14. (*d*) From graph area $= \int_0^4 \sqrt{y}\,dy$

15. (*a*) $\int e^{\sin x}\cos x\,dx = \int d(e^{\sin x}) = e^{\sin x} + C$

16. (*c*) as $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

17. (*c*)

$$P(A\cap B) = P(A) + P(B) - P(A\cup B)$$
$$= 0.4 + 0.3 - 0.5 = 0.2$$
$$P(\overline{A}\cap B) = P(B) - P(A\cap B)$$
$$= 0.3 - 0.2 = 0.1$$

18. (*d*) as, $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$ and $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$

19. (*d*) $$f(x) = \lambda x^2 - 5x + 5 \Rightarrow f'(x) = 2\lambda x - 5$$

Now, $$f'(1) = 5 \Rightarrow 2\lambda - 5 = 5 \Rightarrow \lambda = 5$$

So, *A* is false.

But, *R* is true.

20. (*c*) Assertion is true but the reason is false.

21. As, $(1 + \sin^2 x)dy + (1 + y^2) \cos x\, dx = 0$

$$\Rightarrow \quad \frac{1}{1+y^2}dy = -\left(\frac{\cos x}{1+\sin^2 x}\right)dx$$

$$\Rightarrow \quad \int \frac{dy}{1+y^2} = \int \frac{-\cos x}{1+\sin^2 x}dx$$

$$\Rightarrow \quad \tan^{-1} y = -\tan^{-1}(\sin x) + C \qquad ...(i)$$

When $x = \frac{\pi}{2}, y = 0$ then from (*i*), $0 = -\frac{\pi}{4} + C \Rightarrow C = \frac{\pi}{4}$

Substituting $C = \frac{\pi}{4}$ in (*i*), we get

$\tan^{-1} y + \tan^{-1}(\sin x) = \frac{\pi}{4}$ as the required solution.

22. Let A be the event, when sum of 9 appears on both dice.

$A = \{(3, 6), (4, 5), (5, 4) (6, 3)\}$

Let B be the event, such that second die exhibits prime number.

$B = \{(1, 2), (2, 2), (2, 3), (2, 5), (1, 3), (1, 5)$
$(3, 2), (3, 3), (3, 5), (4, 2), (4, 5), (5, 2)$
$(4, 3), (5, 3), (5, 5), (6, 2), (6, 3), (6, 5)\}$

Now, $A \cap B = \{(4, 5), (6, 3)\}$

$\therefore$ Probability of getting 9 as the sum when second die exhibits prime number is given by,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}.$$

23. Given $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ defined on $R : \{1, 2, 3\} \to \{1, 2, 3\}$

For reflexive: As $(1, 1), (2,2), (3, 3) \in R$.

Hence, reflexive

For symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$.

Hence, not symmetric.

For transitive: $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$. Hence, not transitive.

24. (*a*) Here,

$$\vec{b_1} = 3\hat{i} + 5\hat{j} + \hat{k} \text{ and } \vec{b_2} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b_1} \cdot \vec{b_2} = 3 - 10 + 3 = -4$$

$$|\vec{b_1}| = \sqrt{9+25+1} = \sqrt{35}$$

$$|\vec{b_2}| = \sqrt{1+4+9} = \sqrt{14}$$

Let θ be the angle between the lines

$$\therefore \quad \cos\theta = \left|\frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right| = \left|\frac{-4}{\sqrt{35}\sqrt{14}}\right| = \frac{4\sqrt{10}}{7\times 10} = \frac{2\sqrt{10}}{35}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{2\sqrt{10}}{35}\right)$$

OR

(*b*) We have equation of line

$$5x - 3 = 15y + 7 = 3 - 10z$$

$$\Rightarrow \quad \frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y + \frac{7}{15}}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}}$$

$$\Rightarrow \quad \frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y - \left(-\frac{7}{15}\right)}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}} = \lambda \text{ (say)}$$

$$\Rightarrow \quad x = \frac{3}{5} + \frac{1}{5}\lambda, y = -\frac{7}{15} + \frac{1}{15}\lambda,$$

$$z = \frac{3}{10} - \frac{1}{10}\lambda$$

Equation of line in vector form

$$\vec{r} = \left(\frac{3}{5} + \frac{1}{5}\lambda\right)\hat{i} + \left(-\frac{7}{15} + \frac{1}{15}\lambda\right)\hat{j} + \left(\frac{3}{10} - \frac{1}{10}\lambda\right)\hat{k}$$

$$\Rightarrow \quad \vec{r} = \frac{3}{5}\hat{i} - \frac{7}{15}\hat{j} + \frac{3}{10}\hat{k} + \lambda\left(\frac{1}{5}\hat{i} + \frac{1}{15}\hat{j} - \frac{1}{10}\hat{k}\right)$$

25. (*a*)

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 64$$

$$\because \quad |\text{adj } A| = |A|^2 = (64)^2 = 4096$$

OR

(*b*)

$$[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = [0 \quad 0]$$

$$\Rightarrow \quad [x - 2 \quad 0] = [0 \quad 0]$$

$$\Rightarrow \quad x - 2 = 0 \Rightarrow x = 2$$

26. (*a*) We have, $f(x) = x^4 - 32x^2 + ax + 10$

Differentiating w.r.t. '*x*', we get

$$f'(x) = 4x^3 - 64x + a$$

A.T.Q at $x = 1, f'(x) = 0$, so

$$0 = 4(1)^3 - 64(1) + a$$

$$\Rightarrow \quad 0 = 4 - 64 + a$$

$$\Rightarrow \quad a = 60$$

OR

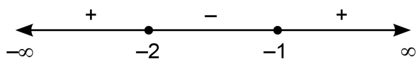
(*b*) $$y = 2x^3 + 9x^2 + 12x - 1$$

Differentiating w.r.t '*x*', we get

$$\frac{dy}{dx} = 6x^2 + 18x + 12 = 6(x^2 + 3x + 2)$$

$$= 6(x + 2)(x + 1)$$

Intervals	Sign of $f'(x)$	Nature of f
$(-\infty, -2)$	+ve	Strictly increasing
$(-2, -1)$	–ve	Strictly decreasing
$(-1, \infty)$	+ve	Strictly increasing



Sign of $f'(x)$ for different values of x

$\therefore$ In $(-2, -1)$, 'f' is strictly decreasing.

27. Put $x = \sin A$, $y = \sin B \Rightarrow A = \sin^{-1}x$, $B = \sin^{-1}y$

$\Rightarrow$ $$\sqrt{1-x^2} = \cos A, \sqrt{1-y^2} = \cos B$$

Now, $$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$$

$\Rightarrow$ $$\sin B \cos A + \sin A \cos B = 1$$

$\Rightarrow$ $$\sin (A + B) = 1 \Rightarrow A + B = \sin^{-1}(1) = \frac{\pi}{2}$$

$\Rightarrow$ $$\sin^{-1} x + \sin^{-1}y = \frac{\pi}{2}$$

Differentiating both the sides w.r.t. x, we get

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$\Rightarrow$ $$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

28. $f : R - \{3\} \to R - \{1\}$ such that

$$f(x) = \frac{x-2}{x-3}$$

One-one

Let $x, x_2 \in R - \{3\}$ such that

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$\Rightarrow$ $$x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$$

$\Rightarrow$ $$-x_1 = -x_2 \Rightarrow x_1 = x_2$$

$\Rightarrow$ f is one-one.

Onto

Let $$y = f(x) = \frac{x-2}{x-3}$$

$\Rightarrow$ $$xy - 3y = x - 2 \Rightarrow xy - x = 3y - 2$$

$\Rightarrow$ $$x(y - 1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}$$

Now, $$f(x) = f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3}$$

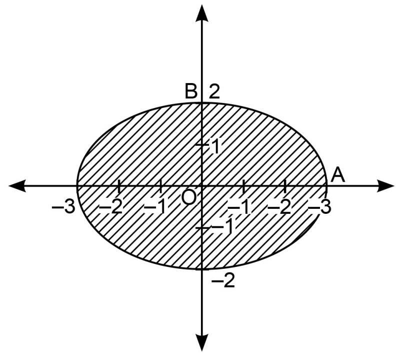
$$= \frac{3y-2-2y+2}{3y-2-3y+3} = \frac{y}{1} = y$$

So, for $y \in R - \{1\}$, $\exists x = \dfrac{3y-2}{y-1} \in R - \{3\}$ such that $f(x) = y$

$\Rightarrow f$ is onto.

Hence f is one-one and onto.

29. (*a*) We have $4x^2 + 9y^2 = 36 \Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$



$$\text{Required area A} = 4 \text{ area of OABO}$$

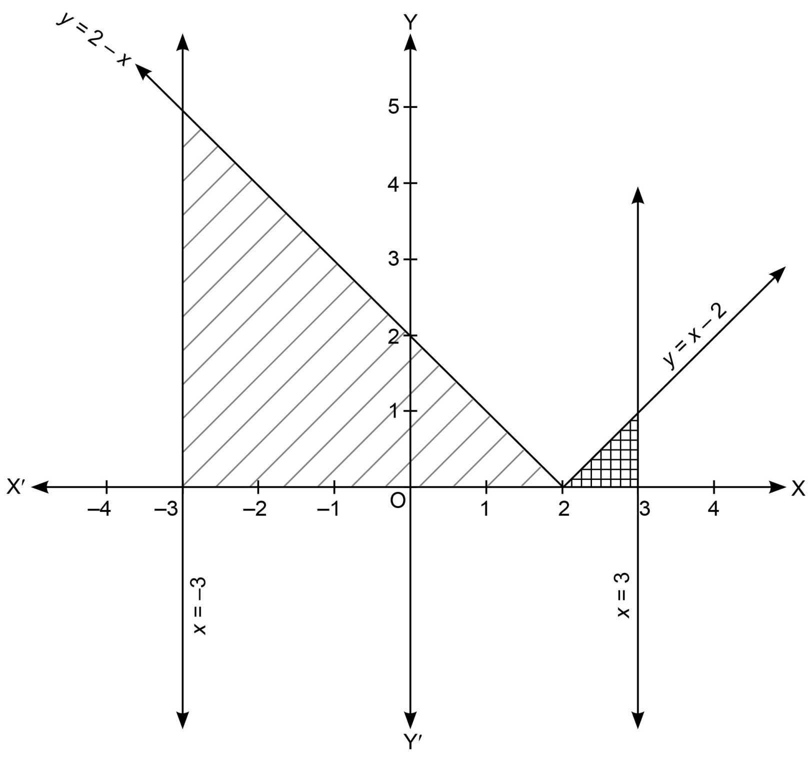
$$= 4 \times \frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx = \frac{8}{3}\int_3^8 \sqrt{9-x^2}\,dx$$

$$= \frac{8}{3}\left[\frac{x\sqrt{9-x^2}}{2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$$

$$= \frac{8}{3}\left[0 + \frac{9}{2} \times \sin^{-1}(1) - 0\right]$$

$$= \frac{8}{3} \times \frac{9}{2} \times \frac{\pi}{2} = 6\pi \text{ sq. units.}$$

(*b*)



$$\int_{-3}^{3} |x-2|\,dx = \int_{-3}^{2} -(x-2)\,dx + \int_2^3 (x-2)\,dx$$

$$= \int_{-3}^{2} (2-x)\,dx + \int_2^3 (x-2)\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_{-3}^{2} + \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= \left[(4-2) - \left(-6 - \frac{9}{2}\right)\right] + \left[\left(\frac{9}{2} - 6\right) - (2-4)\right]$$

$$= \left[2+\frac{21}{2}\right]+\left[\left(\frac{9-12}{2}\right)+2\right]$$

$$= 2+\frac{21}{2}-\frac{3}{2}+2 = 4+\frac{18}{2} = 4+9 = 13$$

30. (*a*) Let $y = x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{x\cos x}\right)+\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) \quad ...(i)$$

Consider $u = x^{x\cos x}$ $\Rightarrow \log u = x\cos x\cdot\log x$

$$\Rightarrow \quad \frac{1}{u}\cdot\frac{du}{dx} = x\cos x\cdot\frac{1}{x}+x\log x\,(-\sin x)+\log x\cdot 1\cdot\cos x$$

$$\Rightarrow \quad \frac{d}{dx}(x^{x\cos x}) = x^{x\cos x}\left[\cos x - x\log x\cdot\sin x+\log x\cos x\right] \quad ...(ii)$$

Consider, $$\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) = \frac{(x^2-1)\cdot\frac{d}{dx}(x^2+1)-(x^2+1)\cdot\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\,2x-(x^2+1)2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2} \quad ...(iii)$$

Substituting from (*ii*) and (*iii*) in (*i*), we get

$$\frac{dy}{dx} = x^{x\cos x}\left[\cos x - x\log x\cdot\sin x+\log x\cdot\cos x\right]-\frac{4x}{(x^2-1)^2}.$$

OR

(*b*) Given function $f(x) = |x-3| = \begin{cases} x-3, & x\geq 3 \\ -x+3, & x<3\end{cases}$

For continuity at $x = 3$:

$$\underset{x=3}{\text{LHL}} = \lim_{h\to 0} f(3-h) = \lim_{h\to 0}\{-(3-h)+3\} = \lim_{h\to 0} h = 0$$

$$\underset{x=3}{\text{RHL}} = \lim_{h\to 0} f(3+h) = \lim_{h\to 0}\{(3+h)-3\}$$

$$= \lim_{h\to 0} h = 0$$

$$f(3) = 3-3 = 0$$

As $\underset{x=3}{\text{LHL}} = \underset{x=3}{\text{RHL}} = f(3)$, hence, function is continuous at $x = 3$.

For differentiability at $x = 3$:

$$\underset{x=3}{\text{LHD}} = \lim_{h\to 0}\frac{f(3-h)-f(3)}{-h} = \lim_{h\to 0}\frac{(-3+h+3)-(0)}{-h}$$

$$= \lim_{h\to 0}\frac{h}{-h} = \lim_{h\to 0}(-1) = -1$$

$$\underset{x=3}{\text{RHD}} = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{(3+h-3)-(0)}{h}$$

$$= \lim_{h\to 0}\frac{h}{h} = \lim_{h\to 0}(1) = 1$$

As $\underset{x=3}{\text{LHD}} \neq \underset{x=3}{\text{RHD}}$. Hence, function is not derivable (differentiable) at $x = 3$.

31. Let
$$I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x)\,dx \qquad ...(i)$$

$\Rightarrow$
$$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)dx\right] \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$\Rightarrow$
$$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right]dx$$

$\Rightarrow$
$$I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1 + \tan x}\right)dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get
$$2I = \int_0^{\frac{\pi}{4}} \left[\log(1 + \tan x) + \log\left(\frac{2}{1 + \tan x}\right)\right]dx$$

$\Rightarrow$
$$2I = \int_0^{\frac{\pi}{4}} [\log(1 + \tan x) + \log 2 - \log(1 + \tan x)]\,dx$$

$\Rightarrow$
$$2I = \log 2 \int_0^{\frac{\pi}{4}} dx$$

$\Rightarrow$
$$2I = \log 2 \times \Big[x\Big]_0^{\frac{\pi}{4}}$$

$\Rightarrow$
$$2I = \frac{\pi}{4}\log 2$$

$\Rightarrow$
$$I = \frac{\pi}{8}\log 2$$

32. Given matrix is
$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj } A$$

$$|A| = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix} = 1(-3) - 2(-2) + 0 = 1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}' = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$\therefore$
$$A^{-1} = \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \qquad ...(i)$$

Given equations are
$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

Matrix equation is
$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$A'X = B$$

Solution is $X = (A')^{-1}B$

$$= (A^{-1})'B$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}' \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 0, y = -5, z = -3$ is the solution.

33. (*a*)
$$x\frac{dy}{dx} = y(\log y - \log x + 1)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right) \quad ...(i)$$

It is a homogeneous differential equation

Put $\frac{y}{x} = v \Rightarrow y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Using the value of $\frac{y}{x}$ and $\frac{dy}{dx}$ in (*i*), we get

$$v + x\frac{dv}{dx} = v(\log v + 1) = v\log v + v$$

$$\Rightarrow \quad x\frac{dv}{dx} = v\log v \Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int\frac{dv}{v\log v} = \int\frac{dx}{x}$$

$$\Rightarrow \quad \int d(\log(\log v)) = \int d(\log x)$$

$$\Rightarrow \quad \log(\log v) = \log x + \log C$$

$$\Rightarrow \quad \log(\log v) = \log Cx$$

$$\Rightarrow \quad \log v = Cx \Rightarrow \log\frac{y}{x} = Cx$$

$$\Rightarrow \quad y = xe^{Cx}$$

OR

(*b*)
$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$$

It is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$

where, $$P = \frac{2x}{1+x^2}, Q = \frac{4x^2}{1+x^2}$$

$\therefore$ $$\text{IF} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$$

$\therefore$ solution is given by

$$y(1+x^2) = \int (1+x^2) \times \frac{4x^2}{1+x^2} dx$$

$$= 4\int x^2 dx = 4 \times \frac{x^3}{3} + C$$

$\Rightarrow$ $$y(1+x^2) = \frac{4}{3}x^3 + C$$

Given $y(1) = 2$

$\Rightarrow$ $$4 = \frac{32}{3} + C \Rightarrow C = 4 - \frac{32}{3} = \frac{-20}{3}$$

$\therefore$ solution is

$$y = \frac{4}{3}\frac{x^3}{(1+x^2)} - \frac{20}{3(1+x^2)}$$

$$= \frac{4x^3 - 20}{3(1+x^2)} = \frac{4(x^3-5)}{3(1+x^2)}$$

34. (*a*) Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$

In vector form, these equations are written as

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

and $$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})$$

Here $$\vec{a_1} = \hat{i} - \hat{j} \text{ and } \vec{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{a_2} = -\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b_2} = 5\hat{i} + \hat{j}$$

$$\vec{a_2} - \vec{a_1} = -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j} = -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(0-5) + \hat{k}(2-15) = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$\therefore$ $$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (-2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 5\hat{j} - 13\hat{k})$$

$$= (2 + 15 - 26) = 17 - 26 = -9$$

Shortest distance $$= \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| = \left| \frac{-9}{\sqrt{1+25+169}} \right| = \left| \frac{-9}{\sqrt{195}} \right| = \frac{9}{\sqrt{195}} \text{ units}$$

Here, shortest distance is not zero, so lines are not intersecting.

OR

(*b*) Let line through the point $A(1, 2, -4)$ be $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ ...(*i*)

If line (*i*) is perpendicular to the lines

$$\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $\frac{x-1}{1} = \frac{y+2}{-3} = \frac{z-3}{5}$

then $2a + 3b + 4c = 0$ ...(*ii*)

and $a - 3b + 5c = 0$ ...(*iii*)

Solving (*ii*) and (*iii*), we get

$$\frac{a}{15+12} = \frac{-b}{10-4} = \frac{c}{-6-3} \Rightarrow \frac{a}{27} = \frac{b}{-6} = \frac{c}{-9}$$

DR's are 9, –2, –3

From (*i*), line in Cartesian form is $\frac{x-1}{9} = \frac{y-2}{-2} = \frac{z+4}{-3}$

Let $\frac{x-1}{9} = \frac{y-2}{-2} = \frac{z+4}{-3} = \lambda$ (say)

General point on the line is $(9\lambda + 1, -2\lambda + 2, -3\lambda - 4)$

Position vector of point on the line is

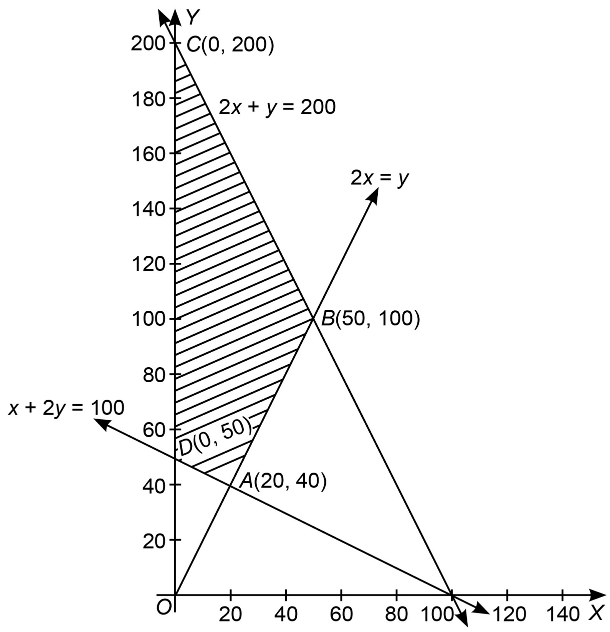
$$\vec{r} = (9\lambda+1)\hat{i} + (-2\lambda+2)\hat{j} + (-3\lambda-4)\hat{k}$$

$\Rightarrow$ $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(9\hat{i} - 2\hat{j} - 3\hat{k})$ is equation of line in vector form.

35. To maximise $Z = x + 2y$

subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$



On plotting the graph of inequations, we notice shaded portion is feasible solution. Possible points for maximum Z are $A(20, 40)$, $B(50, 100)$, $C(0, 200)$ and $D(0, 50)$.

Points	$Z = x + 2y$	Values
A(20, 40)	20 + 80	100
B(50, 100)	50 + 200	250
C(0, 200)	0 + 400	400 ← Maximum
D(0, 50)	0 + 100	100

$\therefore$ Z is maximum for $C(0, 200)$, i.e. $x = 0, y = 200$.

36. (*i*) Length = $(18 - 2x)$ cm, breadth = $(18 - 2x)$ cm, height = x cm

(*ii*) Volume V = length × breadth × height = $x(18 - 2x)^2$ cm^3

(*iii*) (*a*) As, $V = x(18 - 2x)^2$

$$\Rightarrow \frac{dV}{dx} = x \cdot 2(18-2x)(-2) + (18-2x)^2$$
$$= (18 - 2x)(18 - 6x)$$

For maximum or minimum volume, $\frac{dV}{dx} = 0$

$$\Rightarrow (18 - 6x)(18 - 2x) = 0$$
$$\Rightarrow x = 3 \text{ or } x = 9$$

Now, $x = 9$ is rejected as length and breadth becomes 0 for $x = 9$.

Now, $$\frac{d^2V}{dx^2} = (18 - 2x)(-6) + (18 - 6x)(-2)$$

$$\left.\frac{d^2V}{dx^2}\right]_{x=3} = (18 - 2 \times 3)(-6) + (-2)(18 - 6 \times 3) = -72 < 0$$

$\therefore$ Volume is maximum at $x = 3$.

OR

(*iii*) (*b*) Maximum Volume $= 3(18 - 6)^2 = 432 \text{ cm}^3$

37. (*i*) $$P(A\bar{B}\,\bar{C}) + P(\bar{A}\,B\,\bar{C}) + P(\bar{A}\,\bar{B}\,C) = \frac{1}{3}\times\frac{5}{7}\times\frac{5}{8}+\frac{2}{3}\times\frac{2}{7}\times\frac{5}{8}+\frac{2}{3}\times\frac{5}{7}\times\frac{3}{8}$$

$$= \frac{25+20+30}{168} = \frac{75}{168} = \frac{25}{56}$$

(*ii*) $P(\bar{A}\,\bar{B}\,\bar{C}) = \frac{2}{3}\times\frac{5}{7}\times\frac{5}{8} = \frac{25}{84}$

(*iii*) (*a*) $P(C) = \frac{3}{8}$.

OR

(*iii*) (*b*) $1 - P(\bar{A}\,\bar{B}\,\bar{C}) = 1 - \frac{25}{84} = \frac{59}{84}$.

38. $A(0, 0, 0)$, $B(4, 0, 0)$, $C(4, 4, 0)$, $D(0, 4, 0)$, $E(0, 0, 4)$, $F(4, 0, 4)$, $G(4, 4, 4)$ and $H(0, 4, 4)$.

(*i*) DR's of EC $= \langle 4, 4, -4\rangle$

DR's of AG $= \langle 4, 4, 4\rangle$

(*ii*) DR's of HB $= \langle 4, -4, -4\rangle$

DR's of DF $= \langle 4, -4, 4\rangle$

Let 'θ' be the angle between HB and DF

$$\cos\theta = \frac{4\times 4 + \{(-4)\times(-4)\} + 4\times\{-4\}}{\sqrt{4^2+(-4)^2+(-4)^2}\,\sqrt{4^2+(-4)^2+4^2}} = \frac{1}{3}$$

$$\therefore \qquad \theta = \cos^{-1}\left(\frac{1}{3}\right)$$